**SSC diversity receiver over correlated Nakagami-\(m\) fading channels in the presence of co-channel interference**

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**Abstract**—Performance analysis of switched-and-stay combining (SSC) diversity receiver operating over correlated Nakagami-\(m\) fading channels in the presence correlated Nakagami-\(m\) distributed co-channel interference (CCI) is presented. Novel infinite series expressions are derived for the output signal to interference ratio (SIR)’s probability density function (PDF) and cumulative distribution function (CDF). Capitalizing on them standard performance measures criterion like outage probability (OP) and average bit error probability (ABEP) for modulation schemes such as noncoherent frequency-shift keying (NCFSK) and binary differentially phase-shift keying (BDPSK) are efficiently evaluated. In order to point out the effects of fading severity and the level of correlation on the system performances, numerically obtained results, are graphically presented and analyzed.

**Keywords**— switched-and-stay combining; Nakagami-\(m\) fading channels; co-channel interference.

I. INTRODUCTION

In wireless communication systems various techniques for reducing fading effects and influence of the cochannel interference are used [1]. Space diversity reception, based on using multiple antennas is widely considered as a very efficient technique for mitigating fading and cochannel interference (CCI) effects. Increasing channel capacity and upgrading transmission reliability without increasing transmission power and bandwidth is the main goal of these techniques. Depending on complexity restriction put on the communication system and amount of channel state information available at the receiver, there are several principal types of combining techniques and division, that can be generally performed. Switch and stay combining (SSC) is the least complex diversity technique, and can be used in conjunction with various modulation schemes. In general case, with SSC diversity applied, the receiver selects a particular branch until its signal-to-noise ratio (SNR) drops below a predetermined threshold. When threshold is achieved, the combiner switches to another branch and stays there regardless of whether the SNR of that branch is above or below the predetermined threshold [1-3].

In cellular systems, where the level of the co-channel interference is sufficiently high as compared to the thermal noise, SSC selects a particular branch until its signal-to-interference ratio (SIR) drops below a predetermined SIR ratio (SIR-based switched diversity). When defined ratio is achieved, the combiner switches to another branch and stays there regardless of SIR of that branch.

The fading among the channels is correlated due to insufficient antenna spacing, which is a real scenario in practical diversity systems, resulting in a degradation of the diversity gain [4]. Therefore, it is important to understand how the correlation between received signals affects the system performance.

There are many distributions that well describe statistics of a mobile radio signal in the mobile radio environments. It has been found experimentally, that while the Rayleigh and Rice distributions can be indeed used to model the envelope of fading channels in many cases of interest, the Nakagami-\(m\) distribution offers a better fit for a wider range of fading conditions in wireless communications.

An approach to the performance analysis of SSC diversity receiver operating over correlated Ricean fading satellite channels can be found in [5]. Analysis of the SSC diversity receiver operating over correlated Weibull fading channels in terms of outage probability (OP), average bit error probability (ABER), and moment generating function (MGF) can be found in [6]. Dual-branch SSC diversity receiver with switching decision based on SIR, operating over correlated Ricean fading channels in the presence of correlated Nakagami-\(m\) distributed CCI, is presented in [7]. Moreover to the best author's knowledge, no specific analytical study of SSC involving assumed correlated model of Nakagami-\(m\)
fading for both desired signal and co-channel interference, has been reported in the literature.

In this paper, an approach to the performance analysis of proposed SSC diversity receiver over correlated Nakagami-\(m\) fading channels, in the presence of correlated CCI, will be presented. Novel infinite series expressions for probability density function (PDF) and cumulative distribution function (CDF) of the output SIR for SSC diversity will be derived. Numerical results for important performance measures, such as OP and ABER for modulation schemes such as noncoherent frequency-shift keying (NCFSK) and binary differentially phase-shift keying (BDPSK) will be shown graphically for different system parameters in order to point out the effects of fading severity and the level of correlation on the system performances.

II. SYSTEM MODEL

Nakagami fading (\(m\)-distribution) describes multipath scaterring with relatively large delay-time spreads, with different clusters of reflected waves [2]. It provides good fits to collected data in indoor and outdoor mobile-radio environments and is used in many wireless communications applications. In this paper, wireless communication system with triple SIR-based SC diversity is considered. The desired signal received by the \(i\)-th antenna can be written as [8]:

\[
D_i(t) = R_i e^{j\omega_i t} e^{j[2\pi f_c t + \Psi(t)]},
\]

where \(f_c\) is carrier frequency, \(\Phi(t)\) desired information signal, \(\Phi_i(t)\) the random phase uniformly distributed in [0,2\(\pi\)], and \(R_i(t)\), a Nakagami-\(m\) distributed random amplitude process given by [2]:

\[
f_{R_i}(t) = \frac{2t^{\alpha-1}}{\Gamma(m) \Omega^\alpha} \exp\left(-\frac{t^2}{\Omega}\right); \quad t \geq 0
\]

where \(\Gamma(*)\) is the Gamma function, \(\Omega = E(t^2)\), with \(E\) being the mathematical expectation operator, and \(m\) is the inverse normalized variance of \(t^2\), which must satisfy \(m \geq 1/2\), describing the fading severity. The resultant interfering signal received by the \(i\)-th antenna is:

\[
C_i(t) = r_i(t) e^{j\theta_i(t)} e^{j[2\pi f_c t + \Psi(t)]},
\]

where \(r_i(t)\) is also Nakagami-\(m\) distributed random amplitude process, \(\theta_i(t)\) is the random phase, and \(\Psi(t)\) is the information signal. This model refers to the case of a single co-channel interferer.

Now joint distributions of pdf for both desired and interfering signal correlated envelopes for dual-branch signal combiner could be expressed by [9]:

\[
\begin{align*}
    p_{R_i, R_j}(R_i, R_j) &= \frac{1}{\Gamma(m_i) \Omega_i^{\alpha_i}} \sum_{k_i=0}^{\infty} \frac{\Gamma(m_i + k_i + 1)}{\Gamma(m_i) \Gamma(k_i + 1)} R_i^{k_i} \frac{1}{1 + \rho_i} \exp\left(-\frac{R_i^2}{1 + \rho_i}\right) \times \frac{1}{\Gamma(m_j) \Omega_j^{\alpha_j}} \sum_{k_j=0}^{\infty} \frac{\Gamma(m_j + k_j + 1)}{\Gamma(m_j) \Gamma(k_j + 1)} R_j^{k_j} \frac{1}{1 + \rho_j} \exp\left(-\frac{R_j^2}{1 + \rho_j}\right) \\
    &\times \frac{1}{\Gamma(m_1 + m_2 + k_1 + k_2) \Gamma(m_1 + 1) \Gamma(m_2 + 1)} R_1^{k_1} R_2^{k_2} \Omega_1^{\alpha_1} \Omega_2^{\alpha_2} \exp\left(-\frac{R_1^2}{\Omega_1} - \frac{R_2^2}{\Omega_2}\right) \\
    &\times \frac{1}{\Gamma(m_1 + m_2 + k_1 + k_2 + 1) \Gamma(m_1 + m_2 + 1)} R_1^{k_1} R_2^{k_2} \Omega_1^{\alpha_1} \Omega_2^{\alpha_2} \exp\left(-\frac{R_1^2}{\Omega_1} - \frac{R_2^2}{\Omega_2}\right)
\end{align*}
\]

The power correlation coefficient \(\rho_d\) of desired signal is defined as \(\rho_d = C(R_1^2, R_2^2)/(V(R_1^2) V(R_2^2))^{1/2}\) and the power correlation coefficient \(\rho_i\) of interference is defined as \(\rho_i = C(r_1^2, r_2^2)/(V(r_1^2) V(r_2^2))^{1/2}\), where \(C(\cdot,\cdot)\) denotes the covariance operator.

Let \(z_1 = R_1^2/r_i^2\) and \(z_2 = R_2^2/r_i^2\) represent the instantaneous SIR on the diversity branches, respectively. The joint PDF of \(z_1\) and \(z_2\) can be expressed by [10]:

\[
f_{z_1, z_2}(z_1, z_2) = \frac{1}{4 \sqrt{z_1 z_2}} \int_0^\infty \int_0^\infty p_{R_1, R_2}(r_1, r_2) r_1 r_2 dr_1 dr_2
\]

Substituting (4) and (5) in (6), we obtain:

\[
f_{z_1, z_2}(z_1, z_2) = \sum_{k_1=0}^{\infty} \sum_{k_2=0}^{\infty} \sum_{m_1=0}^{\infty} \sum_{m_2=0}^{\infty} G_{m_1, m_2} \frac{m_1^{m_1 + k_1 + 1} m_2^{m_2 + k_2 + 1}}{1 - \rho_i} \frac{1}{\Gamma(m_1 + 1) \Gamma(m_2 + 1)} R_1^{k_1} R_2^{k_2} \Omega_1^{\alpha_1} \Omega_2^{\alpha_2} \exp\left(-\frac{R_1^2}{\Omega_1} - \frac{R_2^2}{\Omega_2}\right) \\
\times \frac{1}{\Gamma(m_1 + m_2 + k_1 + k_2 + 1) \Gamma(m_1 + m_2 + 1)} R_1^{k_1} R_2^{k_2} \Omega_1^{\alpha_1} \Omega_2^{\alpha_2} \exp\left(-\frac{R_1^2}{\Omega_1} - \frac{R_2^2}{\Omega_2}\right)
\]

where \(S_k = \Omega_{ik} / \Omega_{ik}\) being the average SIR’s at the \(k\)-th input branch of the dual-branch combiner and:

\[
G_{m_1, m_2} = \frac{\Gamma(m_1 + 1) \Gamma(m_2 + 1)}{\Gamma(m_1 + m_2 + k_1 + k_2 + 1) \Gamma(m_1 + m_2 + 1)} \frac{m_1^{m_1 + k_1 + 1} m_2^{m_2 + k_2 + 1}}{1 - \rho_i} \\
\times \frac{1}{\Gamma(m_1 + 1) \Gamma(m_2 + 1)} \frac{1}{\Gamma(m_1 + m_2 + k_1 + k_2 + 1) \Gamma(m_1 + m_2 + 1)}
\]
Let \( z_{\text{ssc}} \) represent the instantaneous SIR at the SSC output, and \( z \), the predetermined switching threshold for the both input branches. As it was previously explained SSC selects a particular branch \( z_i \) until its SIR drops below a \( z_{\tau} \). When this happens, the combiner switches to another branch. Following [11], the PDF of \( z_{\text{ssc}} \) is given by

\[
f_{\text{ssc}}(z) = \begin{cases} 
v_{\text{ssc}}(z) & z \leq z_{\tau}, \\ v_{\text{ssc}}(z) + f_{\tau}(z) & z > z_{\tau}, \end{cases}
\]

where \( v_{\text{ssc}}(z) \), according to [11], can be expressed as:

\[
v_{\text{ssc}}(z) = \int_{0}^{z_{\tau}} \int_{z_{2}}^{z_{\tau}} f_{\text{ssc}}(z, z_{2}) \, dz_{2} \, dz_{2}.
\]

Moreover, \( v_{\text{ssc}}(z) \) can be expressed as an infinite series:

\[
v_{\text{ssc}}(z) = \sum_{m_{k},m_{l}=0}^{\infty} G_{m_{k}} m_{k} m_{l} (1-\sqrt{\rho_{c}})^{m_{k}+l} (1-\sqrt{\rho_{d}})^{m_{k}} \times \frac{m_{k}+m_{l}+1}{(1-\sqrt{\rho_{c}})z_{2} + m_{k} (1-\sqrt{\rho_{d}})S_{1}} \times \frac{m_{k}+m_{l}+1}{(1-\sqrt{\rho_{c}})z_{2} + m_{k} (1-\sqrt{\rho_{d}})S_{1}}
\]

with \( B(z,a,b) \) denoting the incomplete Beta function [12]. In the same manner, the \( f_{\tau}(z) \) can be expressed as:

\[
f_{\tau}(z) = \frac{m_{m} m_{c} m_{z} S_{1}}{(m_{c} S_{1} + m_{z} z)^{m_{m} + m_{c}}} \frac{\Gamma(m_{m} + m_{c})}{\Gamma(m_{c}) \Gamma(m_{m})}
\]

(11)

Similar to (9), the CDF of the SSC output SIR, i.e. the \( F_{\text{ssc}}(z) \) is given by [11]:

\[
F_{\text{ssc}}(z) = \begin{cases} 
F_{z_{1},z_{2}}(z, z_{2}) - F_{z_{1},z_{2}}(z_{1}, z_{2}) & z \leq z_{\tau}, \\
F_{z_{1},z_{2}}(z_{1}, z_{2}) + F_{z_{1},z_{2}}(z_{1}, z_{2}) & z > z_{\tau}.
\end{cases}
\]

(12)

By substituting (7) in (12) and (11) in (13):

\[
F_{z_{1},z_{2}}(z, z_{2}) = \int_{0}^{z_{\tau}} \int_{0}^{z_{\tau}} f_{z_{1},z_{2}}(z, z_{2}) \, dz_{2} \, dz_{2}
\]

and (11) in

\[
F_{\tau}(z) = \int_{0}^{z_{\tau}} f_{\tau}(z) \, dz
\]

\[
F_{z_{1},z_{2}}(z, z_{2}) \text{ and } F_{\tau}(z)
\]

can be expressed as the following infinite series, respectively:

\[
F_{z_{1},z_{2}}(z, z_{2}) = \sum_{k_{i},l_{i}=0}^{\infty} G_{i} \times B \left( m_{z_{1}} \frac{m_{z_{1}}}{m_{z_{1}} + m_{z_{2}}}, \frac{m_{z_{2}} + m_{z_{1}} + 1}{(1-\sqrt{\rho_{c}})S_{1}} \right)
\]

\[
\times B \left( m_{z_{1}} \frac{m_{z_{1}}}{m_{z_{1}} + m_{z_{2}}}, \frac{m_{z_{2}} + m_{z_{1}} + 1}{(1-\sqrt{\rho_{c}})S_{1}} \right)
\]

\[
F_{\tau}(z) = \frac{m_{z_{1}}}{m_{z_{1}} + m_{z_{2}}}, \frac{m_{z_{2}} + m_{z_{1}} + 1}{(1-\sqrt{\rho_{c}})S_{1}}
\]

\[
F_{\tau}(z) = \frac{m_{z_{1}}}{m_{z_{1}} + m_{z_{2}}}, \frac{m_{z_{2}} + m_{z_{1}} + 1}{(1-\sqrt{\rho_{c}})S_{1}}
\]

Further simplification of the previous expression could be performed by bounding and approximating the above series with the generalized hypergeometric functions by using approach presented in [14] at the expense of more mathematical rigor. In Table 1, the number of terms to be summed in order to achieve accuracy at the desired significant digit is depicted. The terms need to be summed to achieve a desired accuracy depend strongly on the correlation coefficients, \( \rho_{c} \) and \( \rho_{d} \). It is obvious that number of the terms increases as correlation coefficients increase. Also, when \( \rho_{c} > \rho_{d} \), we need more terms for correct computation.
Table 1. Terms need to be summed in (15) to achieve accuracy at the 6th significant digit. We consider dual and triple branch selection combining diversity system.

<table>
<thead>
<tr>
<th>$S_i/z=10$dB</th>
<th>$S_t/z_t$</th>
<th>$m_{d}=1$</th>
<th>$m_{c}=1$</th>
<th>$m_{d}=1.2$</th>
<th>$m_{c}=1.5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho_{d}=0.3$</td>
<td>$\rho_{c}=0.2$</td>
<td>24</td>
<td>21</td>
<td>28</td>
<td>25</td>
</tr>
</tbody>
</table>

III. PERFORMANCE ANALYSIS

A. Outage probability (OP)

Since the outage probability (OP) is defined as probability that the instantaneous SIR of the system falls below a specified threshold value, it can be expressed in terms of the CDF of $z_{ssc}$, i.e., as:

$$P_{out} = P_{\bar{\xi}}(\xi < \gamma) = \int_{0}^{\gamma} p_{\bar{\xi}}(\gamma) d\gamma.$$  \hspace{1cm} (19)

where $\gamma$ is the specified threshold value. Using (10) and (11) the $P_{out}$ performances results have been obtained. These results are presented in the Fig. 1, as the function of the normalized outage threshold (dB) for several values of $\rho_{d}$, $\rho_{c}$, $m_{d}$, $m_{c}$. Normalized outage threshold (dB) is defined as being the average SIR’s at the input branch of the balanced dual-branch switched-and-stay combiner, normalized by specified threshold value $\gamma$. Results show that as the signal and interference correlation coefficient $\rho_{d}$ and $\rho_{c}$, increase and normalized outage threshold decreases, OP increases.

B. Average bit error probability

The ABER ($\overline{P}_{e}$) can be evaluated by averaging the conditional symbol error probability for a given SIR, i.e. $P_{e}(z)$, over the PDF of $z_{ssc}$, i.e. $f_{ssc}(z)$ [15]:

$$\overline{P}_{e} = \int_{0}^{\gamma} P_{e}(z)f_{ssc}(z)dz$$  \hspace{1cm} (20)

where $P_{e}(z)$ depends on applied modulation scheme. For a binary differentially coherent phase-shift keying (BDPSK) and non-coherent frequency shift keying (NCFSK) modulation schemes the conditional symbol error probability for a given SIR threshold can be expressed by $P_{e}(z) = 1/2 \exp[-\lambda z]$, where $\lambda=1$ for binary DPSK and $\lambda=1/2$ for NCFSK [16]. Hence, substituting (8) into (20) gives the following ABER expression for the considered dual-branch SSC receiver

$$\overline{P}_{e} = \int_{0}^{\gamma} P_{e}(z)\nu_{ssc}(z)dz + \int_{\gamma}^{\infty} P_{e}(z)f_{1}(z)dz$$  \hspace{1cm} (21)

Using the previously derived infinite series expressions, we present representative numerical performance evaluation results of the studied dual-branch SSC diversity receiver, such as ABER in case of two modulation schemes, DPSK and NCFSK.

Applying (21) on BDPSK and NCFSK modulation schemes, the ABER performance results have been obtained as a function of the average SIR’s at the input branches of the balanced dual-branch switched-and-stay combiner, i.e. $S_i=S_t/z_t$, for several values of $\rho_{d}$, $\rho_{c}$, $m_{d}$, $m_{c}$. These results are plotted in Fig. 2. It’s shown that while as the signal and interference correlation coefficient, $\rho_{d}$ and $\rho_{c}$, increase and the average SIR’s at the at the input branches increases, the ABER increases at the same time. It is very interesting to observe that for lower values of $S_i$, due to the fact that the interference is comparable to desired signal ABER deteriorates more severe when the fading severity of the signal and interferers changes. Finaly considering values from Fig. 2 better performance of BDPSK modulation scheme versus NCFSK modulation scheme are shown.
In this paper, the performance analysis of system with switch-and-stay combining, based on SIR over correlated Nakagami-\(m\) fading channels in the presence of co-channel interference, was obtained. Correlation model was observed for evaluating performances of proposed diversity system. The complete statistics for the SSC output SIR is given in the infinite series expressions form, i.e., PDF, CDF, OP. Using these new formulae, ABER was efficiently evaluated for some modulation schemes BDPSK and NCFSK. Numerical results are graphically presented.

REFERENCES