Outage probability of macrodiversity system in Nakagami-m fading channels with correlated gamma shadowing

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***Abstract*—**This paper studies wireless communication system following microdiversity to mitigate the effects of short-term fading and macrodiversity processing to reduce shadowing effects. *N*-branch maximal-ratio combining (MRC) is implemented at the micro level (single base station) and selection combining (SC) with two base stations (dual diversity) is implemented at the macro level. Model in the paper assumes a Nakagami-m density function for the envelope of the received signal and a gamma distribution to model the average power to account for shadowing. Analytical expression for the outage probability, as an important performance measure, is derived. Various numerical results are graphically presented to illustrate the proposed mathematical analysis and to show the effects of various system parameters to the system performance, as well as enhancement due to use of the combination of micro- and macrodiversity.

***Keywords*—**Gamma shadowing, macrodiversity, microdiversity, Nakagami-*m* fading.

I. Introduction

In wireless communication systems, the received signal can be exposed to both short-term fading, which is the result of multipath propagation, and long-term fading (shadowing), which is the result of large obstacles and large deviations in terrain profile between transmitter and receiver [1]. The reliability of communication over the wireless channels can be improved using diversity techniques, such as space diversity techniques [2], [3]. The most popular space diversity techniques are selection combining (SC), equal-gain combining (EGC) and maximal-ratio combining (MRC). MRC is optimal combining technique in the sense that it achieves the highest output signal-to-noise ratio (SNR) regardless of the fading statistics on the diversity branches. However, MRC requires the knowledge of the channel fading amplitudes and phases of each diversity branch which must be continuously estimated by the receiver. These estimations require separate receiver chain for each branch of the diversity system increasing its complexity. EGC provides performance comparable to MRC, but with simpler implementation complexity. EGC does not require the estimation of the channel fading amplitudes since it combines signals from all branches with the same weighting factor. SC is the least complicated technique. It is reposed on processing only one of the diversity branches. SC combiner chooses the branch with the highest SNR, or equivalently, with the strongest signal assuming equal noise power among the branches. Diversity techniques at single base station (microdiversity) reduce the effects of short-term fading. Impairments due to shadowing can be mitigated using macrodiversity techniques which employ the processing of signals from multiple base stations. The use of composite micro- and macrodiversity has received considerable interest due to the fact that it simultaneously combats both short-term fading and shadowing.

Rayleigh, Rician and Nakagami-*m* statistical models are the most frequently used to describe fading envelope of the received signal. The Rayleigh distribution is frequently used to model multipath fading with no direct line-of-sight (LOS) path. The Rician distribution is often used to model propagation paths consisting of one strong direct LOS component and many random weaker components. The Nakagami-*m* distribution has gained widespread application in the modeling of physical fading radio channels. The primary justification of the use of Nakagami-*m* fading model is its good fit to empirical fading data. It is versatile and through its parameter *m*, we can model signal fading conditions that range from severe to moderate, to light fading or no fading. It includes the one-sided Gaussian distribution (m=0.5) and the Rayleigh distribution (m=1) as special cases. The average power, which is random variable due to shadowing, is usually modeled with lognormal distribution. A composite multipath/shadowed fading environment modeled either as Rayleigh-lognormal, Rician-lognormal or Nakagami-lognormal is considered in [4]-[7]. Unfortunately, the use of lognormal distribution to model the average power doesn't lead to a closed form solution for the probability density function (PDF) of the signal envelope at the receiver. This makes the analysis of system in shadowed fading environment very ponderous. Based on theoretical results and measured data, it was shown that gamma distribution does the job as well

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as lognormal [8], [9]. A compound fading model incorporates both short-term fading and shadowing which is modeled using gamma distribution instead of lognormal distribution [10]-[12]. Such an approach provides analytical solution for the PDF of the SNR facilitating the analysis of wireless systems.

In this paper, the system following micro- and macrodiversity reception in correlated gamma shadowed Nakagami-m fading channels is analyzed. Analytical expression for the outage probability, commonly used performance measure of wireless communication systems, is derived. Numerical results are graphically presented to show the effects of fading severity, shadowing severity, number of diversity branches at the micro level and corelation coefficient on outage probability, as well as enhancement due to use of the combination of micro- and macrodiversity.

II. System Model

System model considered in the paper is shown in Fig. 1. N-branch MRC receiver is implemented at the micro level (single base station) and SC receiver which involves the use of two geographically distributed base stations (dual diversity) is implemented at the macro level. The improvement in performance gained through the use of multiple antennas at the single base station is larger when the signals corresponding to each antenna are approximately independent, i.e. when the separation between antennas is on the order of one half of a wavelength [3]. Two base stations are treated to have nonzero correlation. It is realistic scenario because shadowing has a larger correlation distance and it is difficult to ensure that base stations operate independently, especially in microcellular systems [5].

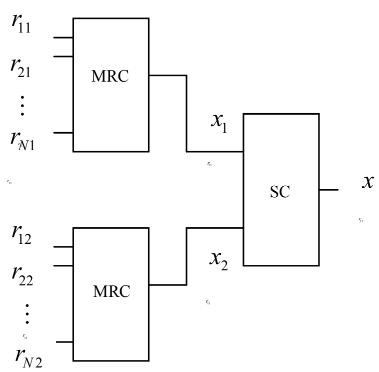

Fig. 1 System model

The PDF of the signal received by the ith antenna at the jth base station in the presence of Nakagami-m fading is

$$p_{r_{ij}}(r_{ij}) = \frac{2m^m r_{ij}^{2m-1}}{\Gamma(m)\Omega_j^{\ m}} \exp\left(-\frac{m}{\Omega_j} r_{ij}^2\right) \quad , \quad i=\overline{1,N}\ , \ j=1,2 \qquad (1)$$

where Ω_j is the average power of the signal at the jth base station and m is Nakagami fading parameter which describes fading severity ($m \geq 0.5$). As parameter m increases, the fading severity decreases. After transformation $x_{ij} = r_{ij}^2$, (1) becomes

$$p_{x_{ij}}(x_{ij}) = \frac{m^m x_{ij}^{m-1}}{\Gamma(m)\Omega_j^{\ m}} \exp\left(-\frac{m}{\Omega_j} x_{ij}\right) \quad , \quad i=\overline{1,N}\ , \ j=1,2. \qquad (2)$$

The result signal at the MRC output of the jth base station is the sum of squared envelopes of Nakagami-m faded signals, $x_j = \sum_{i=1}^{N} r_{ij}^{\ 2}$, or equivalently, $x_j = \sum_{i=1}^{N} x_{ij}$ with PDF given by [13, (71)]

$$p_{x_j}(x_j \mid y_j) = \frac{x_j^{\ M-1} M^M}{\Gamma(M) y_j^{\ M}} \exp\left(-\frac{M}{y_j} x_j\right), \quad j=1,2 \qquad (3)$$

where y_j is the total input power ($y_j = N\Omega_j$) and $M = Nm$.

The conditional nature of the PDF in (3) reflects the existence of shadowing with y_j being random variable. The joint PDF of y_1 and y_2 follows the correlated gamma distribution [14], [15]

$$p_{y_1 y_2}(y_1, y_2) = \frac{\rho^{-\frac{c-1}{2}} (y_1 y_2)^{\frac{c-1}{2}}}{\Gamma(c)(1-\rho) y_0^{c+1}} \exp\left(-\frac{y_1+y_2}{(1-\rho) y_0}\right) \times I_{c-1}\left(\frac{2\sqrt{\rho y_1 y_2}}{(1-\rho) y_0}\right) \qquad (4)$$

where ρ is the correlation between y_1 and y_2, c is the order of gamma distribution, y_0 is related to the average power of y_1 and y_2, $I_n(\cdot)$ is the first kind and nth order modified Bessel function and $\Gamma(\cdot)$ is gamma function. The severity of gamma shadowing is measured in terms of c. The lower value of c means the higher shadowing while the value of $c = \infty$ corresponds to a pure short-term fading channel. The relationship between the parameter c and standard deviation σ of shadowing in dB in the lognormal shadowing exists through [9,(6)], $\sigma(dB) = 4.3429\sqrt{\psi'(c)}$, where $\psi'(\cdot)$ is the trigamma function. The typical values of σ are between 2-12 dB.

Selection diversity is applied at the macrolevel. Namely, the base station with the larger average power is selected to provide service to the user. Using the concepts of probability, the PDF of the SNR after diversity combining at the micro- and macrolevel can be derived as

$$p_x(x)=\int_0^\infty dy_1\int_0^{y_1} p_{x_1}(x|y_1)\,p_{y_1y_2}(y_1,y_2)\ dy_2 +\int_0^\infty dy_2\int_0^{y_2} p_{x_2}(x|y_2)\,p_{y_1y_2}(y_1,y_2)\ dy_1 = 2\int_0^\infty p_{x_1}(x|y_1)dy_1\int_0^{y_1} p_{y_1y_2}(y_1,y_2)\ dy_2. \quad (5)$$

which, by substituting (3) and (4) and using [16, (8.445), (3.381/2) and (3.471/9)], yields

$$p_x(x)=\frac{1}{\Gamma(M)\Gamma(c)}\sum_{n,k=0}^{\infty}\frac{\rho^n M^{n+c+\frac{k+M}{2}}}{2^{n+c+\frac{k-M}{2}-2}(1-\rho)^{n+\frac{k+M}{2}}} \times\frac{x^{n+c+\frac{k+M}{2}-1}}{n!\Gamma(n+c)\,y_0^{n+c+\frac{k+M}{2}}\prod_{l=0}^{k}(n+c+l)} \times K_{2(n+c)+k-M}\left(2\sqrt{\frac{2Mx}{y_0(1-\rho)}}\right). \quad (6)$$

The CDF of the SNR at the jth base station output is

$$F_{x_j}(x_j|y_j)=\int_0^{x_j} p_{x_j}(t|y_j)dt,\quad j=1,2. \quad (7)$$

Substituting (3) in (7) and using [16, (3.381/2)], the CDF of x_j is

$$F_{x_j}(x_j|y_j)=\frac{1}{\Gamma(M)}\exp\left(-\frac{Mx_j}{y_j}\right)\sum_{k=0}^{\infty}\frac{1}{\prod_{l=0}^{k}(M+l)}\left(\frac{Mx_j}{y_j}\right)^{M+k}. \quad (8)$$

The CDF of the SNR at the output of a dual-port selection based macrodiversity can be obtained as

$$F_x(x)=\int_0^\infty dy_1\int_0^{y_1} F_{x_1}(x|y_1)\,p_{y_1y_2}(y_1,y_2)\ dy_2 +\int_0^\infty dy_2\int_0^{y_2} F_{x_2}(x|y_2)\,p_{y_1y_2}(y_1,y_2)\ dy_1 = 2\int_0^\infty F_{x_1}(x|y_1)dy_1\int_0^{y_1} p_{y_1y_2}(y_1,y_2)\ dy_2 \quad (9)$$

which, by substituting (4) and (8) and using [16, (3.381/2) and (3.471/9)] to solve integrals, becomes

$$F_x(x)=\frac{1}{\Gamma(M)\Gamma(c)}\sum_{n,i,k=0}^{\infty}\frac{\rho^n}{2^{n+c+\frac{i-k-M}{2}-2}(1-\rho)^{n+\frac{i+k+M}{2}}} \times\frac{(Mx)^{n+c+\frac{i+k+M}{2}}}{n!\Gamma(n+c)\,y_0^{n+c+\frac{i+k+M}{2}}\prod_{j=0}^{i}(n+c+j)\prod_{l=0}^{k}(M+l)} \times K_{2(n+c)+i-k-M}\left(2\sqrt{\frac{2Mx}{y_0(1-\rho)}}\right). \quad (10)$$

III. Outage Probability

The outage probability, P_{out}, is one of the widely accepted performance measure for diversity systems operating in fading environment. It is defined as the probability that the output SNR falls bellow a given outage threshold λ_{th}. The outage threshold is a protection value of the SNR above which the quality of service (QoS) is satisfactory. The outage probability can be obtained by replacing x with λ_{th} in (10) , i.e.

$$P_{out}=F_x(\lambda_{th}). \quad (11)$$

IV. Performance Evaluation Results and Discussion

In this section, performance evaluation results for the proposed communication system are presented using the previous mathematical analysis. Analytical expressions are in the form of the nested infinite multiple sums. But, these expressions converge rapidly, and therefore, they can be efficiently used. As an indicative example, the convergence rate of expression for the outage probability is examined in Table I. Table I summarizes the number of terms needed to be summed to achieve accuracy at the 4th significant digit. The number of the required terms depends strongly on ρ and λ_{th}. It is obvious that the number of terms increases as ρ and/or λ_{th} increases.

TABLE I
THE NUMBER OF TERMS NEEDED TO BE SUMMED IN (11) TO ACHIEVE ACCURACY AT THE 4TH SIGNIFICANT DIGIT
($m=1.8, \sigma=3$dB, $y_0=5$ dB and N=2)

λ_{th} (dB)	ρ = 0.2	ρ = 0.6
-5	7	12
5	11	15

In Fig. 2, the outage probability is plotted versus the normalized outage threshold, λ_{th}/y_0, for different fading and shadowing severity. As expected, system performance improves as severity of both fading and shadowing decreases (i.e. m increases and σ decreases). In addition, fading severity has a larger influence on the outage probability of system under a lighter shadowing.

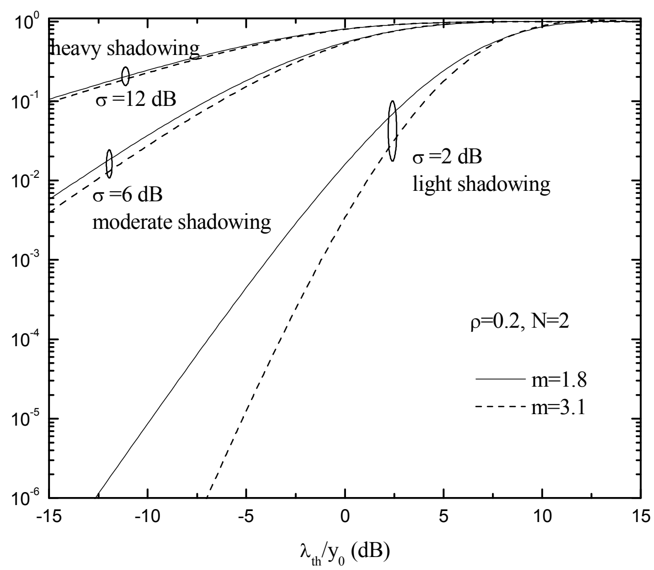

Fig. 2 Outage probability versus normalized outage threshold for different fading and shadowing severity

The outage probability versus the normalized outage threshold for different number of branches and correlation is plotted in Fig. 3. The system performance is better in the case of larger number of diversity branches at the base stations but influence of diversity branches number on system performance enhancement reduces with increment of that number. The system shows better performance for lower values of the correlation coefficient, i.e. for larger spatial separation between base stations. If the correlation is too high, then deep fades in the macrodiversity branches will occur simultaneously resulting in low improvement degree of considered space diversity system.

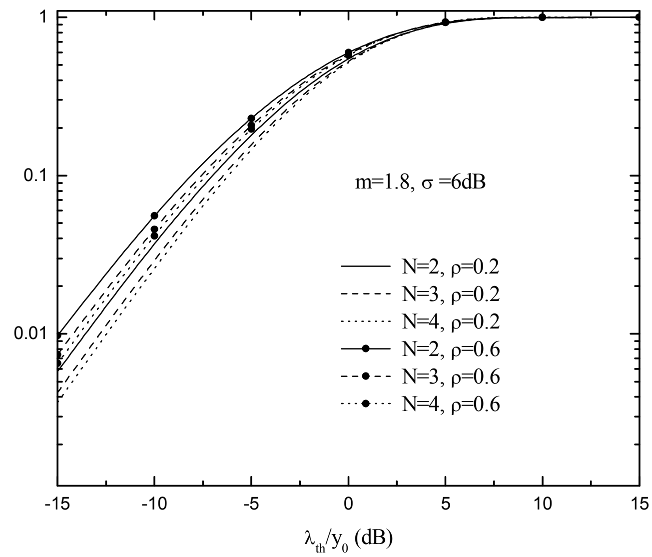

Fig. 3 Outage probability versus normalized outage threshold for different number of branches and correlation

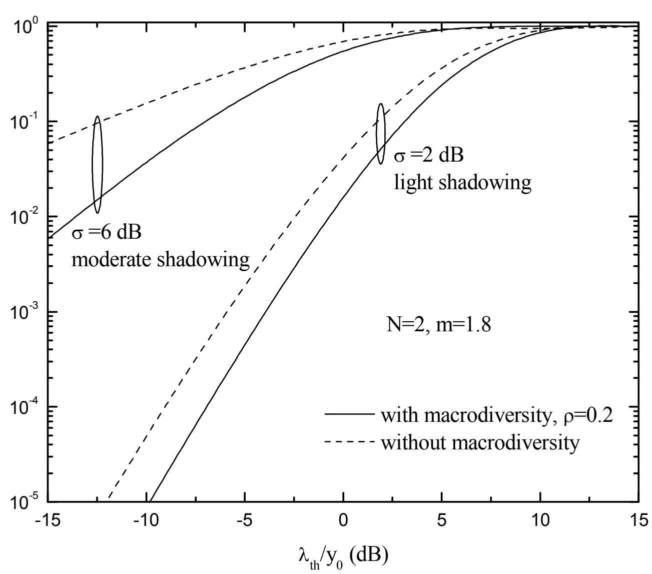

Fig. 4 Outage probability versus normalized outage threshold for systems with and without macrodiversity for different shadowing severity

In Fig. 4, we wanted to see and establish improvement obtained through macrodiversity. For example, for m=1.8 dB, N=2 and ρ=0.2, at an ABEP of 0.1, the macrodiversity gain for σ=2 dB is about 1.3 dB and for σ=6 dB, 5.4 dB. The macrodiversity gain here is defined as the reduction in the normalized outage threshold compared with the case having no macrodiversity. Therefore, the simultaneous use of both micro- and macrodiversity is more than reasonable, especially in environment under higher shadowing severity.

V. Conclusion

In this paper, system with micro- and macrodiversity reception was considered. The received signal envelope has a Nakagami-*m* distribution and it also suffers gamma shadowing. Microdiversity scheme is based on MRC and macrodiversity scheme is based on SC. Expression for the outage probability was analytically derived. This expression requires the summation of an infinite number of terms. However, the presented infinite-series representation converges for any value of the parameters and accordingly, it enables great accuracy of the evaluated and graphically presented results. They show that the system performance improves with an increase of the Nakagami-*m* factor, number of diversity branches at the micro level and order of gamma distribution while an increase of the correlation coefficient leads to deterioration of the system performance. Improvement achieved through macrodiversity is also established.

Appendix

The CDF of the instantaneous SNR at the output of single base station (the case of no macrodiversity) in shadowed Nakagami-m fading channels is

$$F_x(x)=\int_0^\infty F_x(x|y)\,p_y(y)\,dy \tag{12}$$

where $p_y(y)$ is the gamma PDF of the average power given by [15]

$$p_y(y)=\frac{y^{c-1}\exp(-y/y_0)}{\Gamma(c)\,y_0{}^{c}}. \tag{13}$$

Substituting (8) and (13) in (12) and after some straightforward manipulations, integral can be solved with the use of [16, (3.471/9)], resulting in analytical expression for the CDF of x

$$F_x(x)=\frac{2}{\Gamma(M)\Gamma(c)}\sum_{k=0}^{\infty}\frac{(Mx)^{\frac{c+M+k}{2}}}{y_0^{\frac{c+M+k}{2}}\prod_{l=0}^{k}(M+l)} \times K_{c-M-k}\left(2\sqrt{\frac{Mx}{y_0}}\right). \tag{14}$$

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