Takagi-Sugeno Fuzzy Control of a Synchronous Machine

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Abstract: This paper describes the results of an application of a PDC type T-S fuzzy control to synchronous machine. The non linear mathematical model for the proposed synchronous machine adopted in this work is described by T-S continuous fuzzy models. Next, the control law for T-S fuzzy control based on PDC design is studied. The stability and the stabilisation of the whole closed-loop model are investigated using the quadratic lyapunov function. These conditions were reformulated into terms of Linear Matrix Inequalities (LMI) problem by using a Schur complement. Simulation results for synchronous machine demonstrate the PDC controller’s effectiveness.

Key-Words: Takagi-Sugeno fuzzy model, continuous system, synchronous machine, Control PDC, quadratic stability, LMI

1 INTRODUCTION

Modern power systems are highly complex and non-linear and their operating conditions can vary over a wide range. The overall accuracy of the system is primarily decided by how correctly the synchronous machines within the system are modelled. Also, the control and the stabilisation of a synchronous machine are considered an interesting application area for control theory and engineering [1], [2], [3]. Steady state stability is defined as the capability of the power system to maintain synchronism after a gradual change in power caused by small disturbances. The great interest to the synchronous machine is linked to its large industrial application field as well as its properties of reliability, robustness and low cost. In the last two decades, many studies have been developed around fuzzy law control [4], [5], [6], [7], [8]. Control of synchronous machine is well known to be difficult owing to the fact that the dynamic model is nonlinear. We try in this paper to design a fuzzy feedback control that guarantee stability of the closed-loop system.

Recently, among nonlinear control theory, the Takagi–Sugeno (T–S) fuzzy model-based approach has nowadays become popular since it showed its efficiency to control complex nonlinear systems and has been used for many applications; see e.g. [9] and [10]. Indeed, Takagi and Sugeno have proposed a fuzzy model to describe nonlinear models [11] as a collection of linear time invariant models blended together with nonlinear functions. A control law, called “parallel distributed compensation” (PDC), can be synthesized as a collection of feedback gains that are connected using the same nonlinear functions [12]. The global linear fuzzy model is composed of set linear models witch are interpolated by membership functions.

Stability and stabilization analysis, for several kind of T–S fuzzy model, have been strongly investigated through Lyapunov direct method; see [12], [13], [14] and [15] and references therein. The problem can be presented in order to solve the feedback stabilization problem.

In this paper, based on the Takagi-Sugeno (TS) fuzzy model, we design a PDC fuzzy controller for the mathematical model of the Synchronous Machine. This approach guarantee a rigorous stability and minimum disturbance attenuation for the closed loop system. Stability conditions are formulated as an LMI problem using Quadratic Lyapunov functions. The proposed TS fuzzy modeling and stability analysis approach can also be applied to other types of power electronic converters. The rest of this paper is organized as follows. In section 2, we present an overview of dynamic Takagi Sugeno systems and PDC formulation in terms of LMI [16]. Section 3 deals with the description of the mathematical model of synchronous machine, which is transformed to a Takagi-Sugeno fuzzy model, and the fuzzy controller design via PDC with sufficient stability conditions reformulated into solving an LMI problem. Simulation results are shown. Finally, a conclusion is given.
2 TAKAGI-SUGENO FUZZY MODEL

2.1 Model Representation

The fuzzy model proposed by Takagi and Sugeno [11] is described by fuzzy IF-THEN rules, which represent local linear input–output relations of a nonlinear system. The i-th rules of the T-S fuzzy models for a continuous fuzzy system is written as follows:

Model Rule i:

\[ \text{IF } z_i(t) \text{ is } M^i_1 \text{ and} \ldots \text{ and } z_r(t) \text{ is } M^i_r \]

\[ \text{THEN } \begin{cases} \dot{x}(t) = A_i x(t) + B_i u(t) \\ y(t) = C_i x(t) \end{cases} \]

Where \( i = 1, 2, \ldots, r \). \( r \) is the number of IF-THEN rules.

\( x(t) \in \mathbb{R}^n \) is the state vector and \( u(t) \in \mathbb{R}^m \) is the input vector and \( y(t) \in \mathbb{R}^q \) is the output vector. \( A_i \in \mathbb{R}^{n \times n} \) and \( B_i \in \mathbb{R}^{n \times m} \) are system matrix and input matrix respectively. \( M^i_1, \ldots, M^i_r \) is the j-th fuzzy set of the i-th rule and \( z_i(t), \ldots, z_r(t) \) are known premises variables that may be functions of state variables, external disturbances, and/or time.

Let \( w_i^j(z_j) \) be the membership function of the j-th fuzzy set \( M^j_i \) in the i-th rule and \( w_i(z(t)) = \prod_{j=1}^{r} M^j_i(z_j(t)), i = 1, \ldots, r \).

Given a pair of \((x(t), u(t))\), the final output of the fuzzy systems is inferred as follows [15]:

\[ x(t) = \frac{\sum_{i=1}^{r} w_i(z(t))(A_i x(t) + B_i u(t))}{\sum_{i=1}^{r} w_i(z(t))} \quad (1) \]

\( z(t) \): the premise vector \( z(t) = (z_1(t), z_2(t), \ldots, z_r(t)) \).

Here the premise vector is independent of the input and often considered as a part of the state vector or as a linear combination of this one.

The weighting functions:

\[ h_i(z(t)) = \frac{w_i(z(t))}{\sum_{i=1}^{r} w_i(z(t))}, \quad i = 1, \ldots, r \quad (2) \]

For all \( i \). \( w_i^j(z_j(t)) \) is the grade of membership of \( z_j(t) \) in \( M^j_i \), From (2), note that

\[ \sum_{i=1}^{r} h_i(z(t)) = 1 \]

\[ h_i(z(t)) \geq 0 \quad \text{for } i = 1, \ldots, r \]

The global fuzzy Tagagi-Sugeno model is inferred as follows:

\[ \dot{x}(t) = \sum_{i=1}^{r} h_i(t)(A_i x(t) + B_i u(t)) \quad (3) \]

The global output of Takagi-Sugeno model is interpolated as follows:

\[ y(t) = \sum_{i=1}^{r} h_i(t) C_i x(t) \quad (4) \]

2.2 Fuzzy Regulators and Stability Conditions

A fuzzy regulator design via parallel distributed compensation PDC [12] and [17] offers a procedure to design a fuzzy controller from the T-S fuzzy model (3). The design of the PDC fuzzy controller shares the same fuzzy sets as the fuzzy model and the same weights \( w_i \) \((z_i(t))\) in the premise parts. The state feedback fuzzy controller is constructed via PDC as follows [18]:

Regulator rule i:

\[ \text{If } z_i(t) \text{ is } M^i_1(t) \text{ and} \ldots \text{ and } z_r(t) \text{ is } M^i_r(t) \]

\[ \text{then } u(t) = -F_i x(t) \quad \text{for } i = 1, 2, \ldots, r \]

The fuzzy control rules have a linear controller in the consequent parts and the overall fuzzy controller is represented by:

\[ u(t) = -\sum_{i=1}^{r} h_i(z(t)) F_i x(t) \quad (5) \]

The fuzzy regulator design is to determine the local feedback gains \( F_i \in \mathbb{R}^{n \times m} \) in the consequent parts. Thus, the PDC is simple and natural.

By substituting (7) into (3), the closed loop model is written as:

\[ x(t) = \sum_{i=1}^{r} \sum_{j=1}^{r} h_i(z(t)) h_j(z(t))(A_i - B_i F_j)x(t) \quad (6) \]

For the stability analysis the following Lyapunov function candidate is considered [12]:

\[ V(x(t)) = x^T P x \]

Where \( P = P^T \in \mathbb{R}^{n \times m} > 0 \) such that \( \dot{V}(x(t)) < 0 \) along every nonzero trajectory, then the dynamic system is quadratically stable.

The stability analysis result is summarized in the following theorem [17]:

**Theorem 1:** The equilibrium of the fuzzy model in closed loop (6) is asymptotically stable if there exists a common positive definite matrix \( P \) which satisfies the following Lyapunov inequalities:

\[ G_i^T P + P G_i < 0 \quad (7a) \]

\[ \left( \frac{G_{ij} + G_{ji}}{2} \right)^T P + P \left( \frac{G_{ij} + G_{ji}}{2} \right) < 0 \quad \text{for } i < j \quad (7b) \]

With \( G_{ij} = A_i - B_i F_j \)

For each \( i, j \in \{1, 2, \ldots, r\} \) excepted the pairs such that \( \forall t h_i(z(t)) h_j(z(t)) \neq 0 \) where \( r \) is the number of if-then rules.

This stability condition could be transformed into the form of linear matrix inequalities (LMI’s) [16] and yields to a sufficient condition for the exponential
stability, e.g. published in [18].

Conditions of Theorem 1 can be easily transformed into LMIs [1]. Actually, after multiplying from the left and right by \( P^{-1} \), and using \( X = P^{-1} \); \( M_i = F_i X \), the LMI conditions are:

Find \( X > 0 \) and \( M_i \) such that \( \forall i \in \{1, 2, \ldots, r\} \):

\[
X A_i^T + A_i X - B_i M_i - M_i^T B_i^T < 0 \quad i = 1, \ldots, r
\]

(8a)

\[
x (x_i^T + a_i^T) x - [B_i M_j + B_i M_i] - [B_i M_j + B_i M_i]^T < 0
\]

i < j \leq r

(8b)

3 APPLICATION: STABILIZATION OF THE SYNCHRONOUS MACHINE

3.1 The Synchronous Machine and its Mathematical Model

This section presents briefly the dynamics model of proposed synchronous machine with amortisor adopted in this work. The mathematical description of the synchronous machine is obtained if a certain transformation of variables is performed. Park's transformation consist to transform all stator quantities from phase a, b and c into equivalent d-q axis new variables.

The details of there above equation and its parameters can be found in [22]. The electromagnetic dynamic model of the synchronous machine with amortisor in d-q reference frame can be described as:

\[
x(t) = f(x(t)) + g(x(t)) \cdot u(t)
\]

(9)

Where

\[
f(x(t)) =
\begin{bmatrix}
x_1(t) \\
x_2(t) \\
x_3(t) \\
x_4(t)
\end{bmatrix}
\]

\[
g(x(t)) =
\begin{bmatrix}
a_1 \\
a_2 \\
a_3 \\
a_4
\end{bmatrix}
\]

Augmented with an output vector:

\[
y(t) = C \cdot x(t) \quad \text{with} \quad C = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 \end{bmatrix}^T
\]

The entire symbols for a nonlinear model used in this paper are given in Appendix A.

The synchronous machine model parameters are given in Table 1.

The order of the nonlinear model that describes the proposed model is 5th order. The local linear models of the synchronous machine can be derived by applying the Takagi-Sugeno fuzzy technique.

B. Construction of Fuzzy Model

The TS fuzzy model represents exactly many nonlinear models on a limited interval of the state variables [23]. As commented earlier, the number of rules grow by \( 2^2 \) where \( n \) is the number of non-linearities [18], [23]. Still, it has been used successfully in real time control of an inverted pendulum, for example [24] and [25]. Thus, it is reasonable to consider this approach for the control of synchronous machine.

Note that there are four non-linearities in the non-linear dynamical model (9); \( \sin(x_1(t)) = 2 \cdot \sin(x_1(t)) \cdot \cos(x_1(t)), \)

\( x_1(t), \sin(x_1(t)) \) and \( x_1(t). \cos(x_1(t)). \) Thus \( n = 4 \) indicating \( e = 2^4 = 16 \) rules are required. However, with some compromise the number of rules can be reduced to 2 while maintaining model [21], [23] and [26].

First, we can rewrite two of the nonlinear terms in \( \sin(x_1(t)) \) and \( \cos(x_1(t)) \) as:

\[
\frac{\sin(\theta_1)}{\theta_1} = \frac{\theta_0_1 \sin(\theta_0_1) - \theta_0_1 \sin(\theta_0_1) \sin(\theta_0_1)}{\theta_1} + \frac{\theta_0_1(\theta_1 - \sin(\theta_1)) \sin(\theta_1)}{\theta_1} \frac{\theta_0_1}{\theta_1}
\]

And

\[
\cos(\theta_1) = \cos(\theta_0_1) \cdot \cos(\theta_0_1) + \frac{1 - \cos(\theta_0_1)}{1 - \cos(\theta_0_1)} \cos(\theta_0_1)
\]

Whose membership functions are bounded in the range, \( \theta_1 \in [\theta_0_1, \theta_0_1] \) for \( \theta_0_1 \in [0, \frac{\pi}{2}] \) implying:

\[
\frac{\theta_0_1 \sin(\theta_0_1) - \theta_0_1 \sin(\theta_0_1)}{\theta_0_1 \cos(\theta_0_1) - \cos(\theta_0_1)} \cdot \cos(\theta_0_1) - \cos(\theta_0_1) + \frac{1 - \cos(\theta_0_1)}{1 - \cos(\theta_0_1)} \cos(\theta_0_1)
\]

(2,4%) Therefore the transformation on \( \cos(\theta_0_1) \) can be eliminated with little compromise and the fuzzy model order reduced to 2 or 4 rules. Then, the final fuzzy model is described by only two rules; the premise vector is defined by:

\[
z(t) = (z_1(t), z_2(t)), \ \text{with} \ z_1(t) = \theta_1(t) \ \text{and} \ z_2(t) = x_1(t)
\]

For a premise terms, define \( z_1(t) = x_1(t), \quad i = 1, 2 \).

Next, calculate the minimum and maximum values of \( z_1(t) \) under \( x_i \in [-a, a] \). They are obtained as follow:

\[
\max z_1(t) = a, \quad \min z_1(t) = -a
\]

From the maximum and minimum values, \( z_1(t) \) can be represented by:

\[
z_1(t) = \text{M}_1^1 z_1(t) \cdot a + \text{M}_1^2 z_1(t) \cdot (-a)
\]

where \( \text{M}_1^1 (z_1(t)) + \text{M}_1^2 (z_1(t)) = 1 \)

Therefore the membership functions can be calculated as:

\[
\text{M}_1^1 (z_1(t)) = \frac{z_1(t) + a}{2a} \quad ; \quad \text{M}_1^2 (z_1(t)) = \frac{a - z_1(t)}{2a}
\]

Finally, the complete fuzzy model is comprised of four rules with the following membership functions respectively,
Matrix $P$ and the controller’s matrices $F_1, F_2, F_3,$ and $F_4$.

### 4 SIMULATIONS

In this section, we present the simulation of a synchronous machine connected to an infinite bus discussed previously. The simulations results proved the performance of the PDC controller of a synchronous machine. This work is evaluated by computer simulation studies using MATLAB/SIMULINK software. The present work uses the classical results of quadratic stabilization. Other works are also available allowing the use of relaxed stabilization conditions [19], or non quadratic Lyapunov functions [20], and also other control laws [21] allowing to outperform the results of the quadratic condition of stabilization.

We emphasize that with common B matrices for the motor, only $r$ LMIs are to verify. For these simulations, the model rules are chosen for

$$ \theta_{d}(t) \in \left[-\theta_{d0}, \theta_{d0}\right] \text{ and } x_3 \in [-a, a].$$

Assume that the initial conditions \( x(0) \) of the synchronous machine: \( x(0) = (0.52, 0.00, 0.36, 0.80, 0.71)^T \).

A fuzzy controller design via PDC is to determine four feedback gains. The simulations results displays the trajectory of the state vector at startup respectively, for a values of $\theta_{d0} = \frac{\pi}{4}$ and $\theta_{d0} = \frac{\pi}{2}$.

The state feedback fuzzy controller is constructed via PDC as follows:

$$ u(t) = \sum_{i=1}^{4} h_i(z_i(t)) F_i x(t) $$

With the premise variable: \( z_i(t) = \theta_{d}(t) \) and \( z_i(t) = x_3(t).\)

The synthesis of the controller consists of finding the feedback gains of the conclusion parts $F_i$ which guaranteed the asymptotic stability of the closed loop system.

Using the LMI optimization algorithm to resolve the eqs (8a, 8b), we obtain the positive definite symmetric matrix $P$ and the controller’s matrices $F_1, F_2, F_3,$ and $F_4$. 

![Graph](image_url)
Fig. 1. The trajectories of the state vector: (a) $\theta_d$, angular position of rotor (rad), (b) $\omega_d$, Rotor angular speed (rad/s), (c) $E_d^+$, $d$-axis subtransient F.E.M, (d) $E_q^+$, $q$-axis transient F.E.M, (e) $E_q^+$, $q$-axis subtransient F.E.M. Those simulations illustrate the effect of the different values of the angle $\theta_{d0}$.

Fig. 2. The trajectories of the system response, $\omega_d$ speed Rotoric (rad/s), the simulation results show the effects of the angle $\theta_d$. However, it must be show clearly the convergence of position trajectories. It can easily see from these figures that the design method proposed here is much superior in performance than the classical method existing in the bibliography for a synchronous machine [22].

These results demonstrate and confirm the highlight effectiveness of the proposed control law, and the asymptotic stability is equivalent to the feasibility of the LMI’s.

4 CONCLUSION

In this paper, a PDC controller design based on a Takagi-Sugeno fuzzy modelling of synchronous machine infinite bus system is presented.

This approach is no novelty in nonlinear control theory and was applied in several previous works. But, it can be seen that for a very complex mathematical model, the design procedure adopted in this work was conceptually simple and natural. Also, the stability analysis and control design problems are reduced to Linear Matrix Inequality (LMI) problems. Moreover, it can be easily argued that using more recent theoretical results in terms of control performances may lead to better results. The design methodology is illustrated by using Matlab/Simulink (R2007a).

APPENDIX A

$$a_1 = \frac{-\omega_0}{2T_L} \left( \frac{1}{X_d} - \frac{1}{X_q} \right); \quad a_2 = \frac{-\omega_0}{T_L} X_q; \quad a_3 = \frac{-\omega_0}{T_L} X_d; \quad a_4 = \frac{-\omega_0}{T_L}$$

$$b_1 = \frac{-1}{T_q} \left( \frac{X_d}{X_q} - 1 \right); \quad b_2 = \frac{-X_q}{T_q} X_q; \quad b_3 = \frac{1}{T_d}$$

$$c_1 = \frac{1}{T_d} \left( \frac{X_q - X_d}{X_d} \right) \left( 1 - \frac{X_d}{X_q} \right); \quad c_2 = \frac{-1}{T_q} \left( \frac{X_q}{X_d} - X_q \right);$$
\[
c_3 = \frac{1}{T_{d0}} \left( \frac{X_d}{X_d} \frac{X_a - X_d}{X_a} \right) ; \quad c_4 = \frac{1}{T_{d0}} \\
d_1 = \frac{X_d - X_a}{X_a - X_d} + \frac{1}{T_{d0}} \left( \frac{1}{X_d} \right) ; \quad d_2 = \frac{X_d - X_a}{X_a - X_d} c_2 + \frac{1}{T_{d0}} \\
d_3 = \frac{X_d - X_a}{X_a - X_d} c_3 + \frac{1}{T_{d0}} \frac{X_d}{X_d} ; \quad d_4 = \frac{X_d - X_a}{X_a - X_d} c_4
\]

**TABLE 1. MACHINE SYNCHRONOUS DATA (CAPACITY POWER 200VA)**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Quantity</th>
<th>Value (p.u)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_d$</td>
<td>d - axis magnetic reactance</td>
<td>1.10</td>
</tr>
<tr>
<td>$X'_d$</td>
<td>d - axis transient reactance</td>
<td>0.50</td>
</tr>
<tr>
<td>$X''_d$</td>
<td>d - axis subtransient reactance</td>
<td>0.35</td>
</tr>
<tr>
<td>$X_q$</td>
<td>q - axis magnetic reactance</td>
<td>1.10</td>
</tr>
<tr>
<td>$X''_q$</td>
<td>q - axis subtransient reactance</td>
<td>0.30</td>
</tr>
<tr>
<td>$X_a$</td>
<td>field leakage reactance</td>
<td>0.19</td>
</tr>
<tr>
<td>$T_L$</td>
<td>magnetic dipole moment</td>
<td>10.00</td>
</tr>
<tr>
<td>$T'_{d0}$</td>
<td>d - axis transient open circuit time constant</td>
<td>7.00</td>
</tr>
<tr>
<td>$T''_{d0}$</td>
<td>d - axis subtransient open circuit time constant</td>
<td>0.07</td>
</tr>
<tr>
<td>$T'_{q0}$</td>
<td>q - axis subtransient open circuit time constant</td>
<td>0.18</td>
</tr>
<tr>
<td>$\omega_0$</td>
<td>synchronous rotor angular speed</td>
<td>100. $\pi$</td>
</tr>
</tbody>
</table>

References:


