Rotor Flux and Speed Rotating Observer Design with Stator Currents Measurements

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Abstract: This article proposes an asymptotic observer for rotor flux and rotating speed. As the induction machine belongs to the general nonlinear systems; while exploiting the local uniformity of observability of the model and we do a change of variable which transforms the model to two subsystems interconnected one a local system of coordinates pyramidal and other nonlinear. This technique of observation (observer specific to the systems locally observable interconnected with an estimator) makes it possible to rebuild variable magnetic (flux) and variable mechanics (speed) by using as information the stator currents. The adjustment of proposed observer is done with through the choice of only one gain parameter $\theta$.

Key–Words: Field-Oriented Control (FOC), asynchronous machine, cascade observer interconnected with an estimator, rotor flux, rotating speed.

1 Introduction

The Control without mechanical sensor (speed, position and load torque) became a major subject of concern. There exists in the literature several nonlinear techniques based on the machine model to determine the mechanical variables of the asynchronous machines. In general, by using the state model of the asynchronous machine, mechanical speed can be calculated starting from the tensions and of the stator currents [1] and [2]. In [1], flux is obtained by Luenberger observer of a full nature. In this case, the law of adaptation making it possible to estimate speed uses the cross product of the error of the current vector and the vector flux observed like input. Adaptive methods [3], [4] can also be used as alternative solutions for the control without sensor of the asynchronous machine. In addition, in [5], [6] and [7], two observers were used to consider the mechanical speed of the asynchronous machine : extended Kalman filter and an extended Luenberger observer. Extended Kalman filter and an observer by sliding modes were also used in [8] for esteeming the speed mechanical of the machine. The methods quoted before function correctly safe at low speeds, particularly when the stator frequency is null. It was shown, for example in [7], [5], [6] that extended Luenberger observer diverges brutally when the machine functions at low speed and that extended Kalman filter remains exploitable if the parameters of the asynchronous machine are well-known. This list of methods is obviously not exhaustive.

The rest of the paper is organized as follows:

1. Model of the asynchronous(induction) machine.
2. Study of observability.
3. Design of observer specific to the systems locally observable interconnected with an estimator.
4. Results and simulations.

2 Dynamic Model of Induction Machine

The model used is a traditional induction machine model of Park in a stator ($\alpha$, $\beta$) fixed reference frame related to the stator, given by [12] and [13]:
\[
\frac{d\Omega}{dt} = \frac{p M}{J L_r} \left( i_{s\beta} \psi_{ra} - i_{s\alpha} \psi_{rb} \right) - \frac{C_{res}}{J} \tag{1}
\]
\[
\frac{dC_{res}}{dt} = 0
\]

The parameters are defined as follows:
\[
T_r = \frac{L_r}{R_r} ; \quad \sigma = 1 - \frac{M^2}{L_r L_r}
\]
\[
K = \frac{M}{\sigma L_s L_r} ; \quad \gamma = \frac{R_s}{\sigma L_s} + \frac{R_r M^2}{\sigma L_s L_r}
\]

In these equations:
- \( L_s \): Stator inductance cyclic,
- \( L_r \): Rotor inductance cyclic,
- \( M \): Cyclic mutual inductance between stator and rotor
- \( R_s \): Stator resistance,
- \( R_r \): Rotor resistance,
- \( \sigma \): Scattering coefficient,
- \( T_r \): Time constant of the rotor dynamics,
- \( J \): Rotor inertia,
- \( C_{res} \): Resistive torque,
- \( p \): Pole pair machine.

### 3 Study of Observability

#### 3.1 Flux observation with the velocity measurement

We study here directly the induction machine, in the nonlinear case i.e. under the hypothesis variable speed. The model (1) is written:

\[
\begin{cases}
\dot{X} = F(X) + G.U \\
y = H(X)
\end{cases}
\]

Where
\[
X^T = [x_1, x_2, x_3, x_4, x_5] = [i_{s\alpha}, i_{s\beta}, \psi_{ra}, \psi_{rb}, \Omega]
\]

\[
F(X) = \begin{bmatrix}
-\gamma x_1 + \frac{K}{T_r} x_3 + p x_5 K x_4 \\
-\gamma x_2 - p K x_5 x_3 + \frac{K}{T_r} x_4 \\
\frac{M}{T_r} x_1 - \frac{1}{T_r} x_3 - p x_5 x_4 \\
\frac{M}{T_r} x_2 + p x_5 x_3 - \frac{1}{T_r} x_4 \\
\frac{p}{T_r} (x_2 x_3 - x_1 x_4) - \frac{C_{res}}{J}
\end{bmatrix}
\]

\[
G(X) = \begin{bmatrix}
\frac{1}{\sigma L_s} & 0 & 0 & 0 & 0 \\
0 & \frac{1}{\sigma L_s} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
H(X) = \begin{bmatrix}
h_1 (x) \\
h_2 (x) \\
h_3 (x) \\
h_4 (x) \\
h_5 (x)
\end{bmatrix}
\]

Following space \(Q_{\psi_r}\) generated by the outputs and their derivative is continuous in observability space \(O_{\psi_r}\) and determines a dimension space, equal to the system dimension, like us let us see it. Thus it is in this case useless to introduce derivative of a higher order see [14].

\[
Q_{\psi_r}(X) = \begin{bmatrix}
h_1 (x) \\
h_2 (x) \\
h_3 (x)
\end{bmatrix}
\]

\[
Q_{\psi_r}(X) = \begin{bmatrix}
x_1 \\
x_2 \\
x_3
\end{bmatrix}
\]

\[
\begin{bmatrix}
\frac{1}{\sigma L_s} & 0 & 0 & 0 & 0 \\
0 & \frac{1}{\sigma L_s} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
\frac{1}{\sigma L_s} & 0 & 0 & 0 & 0 \\
0 & \frac{1}{\sigma L_s} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

The Jacobian of \(Q_{\psi_r}\) compared to the state makes it possible to characterize the observation within the rank condition:

\[
O_{\psi_r} = \frac{dQ_{\psi_r}}{dX} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & -\gamma & p x_5 K & -p K x_4
\end{bmatrix}
\]

The determinant of this matrix is worth:

\[
\det (O_{\psi_r}) = \det \left( \frac{dQ_{\psi_r}}{dX} \right) = \left( \frac{K}{T_r} \right)^2 + (p x_5 K)^2
\]

The rank of the matrix \(O_{\psi_r}\) is thus with the system order and this independently of the speed.

**Result 1** (see [15]):

The system is uniformly locally observable.

#### 3.2 Observability condition under constant velocity

The checking of observability is obtained starting from the calculation of the determinant of the matrix of associated observability (the rank condition). The characteristic of observability obtained is then local.

That is to say two in the following way generated spaces \(Q_1\) and \(Q_2\) (cf.[14]):

\[
Q_1 = \begin{bmatrix}
\frac{1}{\sigma L_s} & 0 & 0 & 0 & 0 \\
0 & \frac{1}{\sigma L_s} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

\[
Q_2 = \begin{bmatrix}
\frac{1}{\sigma L_s} & 0 & 0 & 0 & 0 \\
0 & \frac{1}{\sigma L_s} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

Following space \(Q_{\psi_r}\) generated by the outputs and their derivative is continuous in observability space \(O_{\psi_r}\) and determines a dimension space, equal to the system dimension, like us let us see it. Thus it is in this case useless to introduce derivative of a higher order see [14].
The associated matrices of observability are:

\[ O_1 = \frac{\partial}{\partial X} (Q_1) ; \quad O_2 = \frac{\partial}{\partial X} (Q_2) \]

The respective determinants are then:

\[ \det (O_1) = -pK^3 [\dot{x}_3] \left( \frac{1}{T_r^2} + p^2 \dot{x}_5^2 \right) \quad (9) \]

\[ \det (O_2) = pK^3 [\dot{x}_1] \left( \frac{1}{T_r^2} + p^2 \dot{x}_5^2 \right) \quad (10) \]

There is thus a local singularity for certain points of operations which are related to the choice of the observability matrix carried out. Nevertheless whatever this choice, the point \( \dot{x}_3 = \dot{x}_4 = 0 \) seems a physical singularity of the system.

result (see [14])

- The rank of \( \det (O_1) \) and \( \det (O_2) \) is the input independent.
- All entered \( U \) the system makes observable locally on space: \( \Theta = \{ x : \dot{x}_3 + \dot{x}_4 \neq 0, \dot{x}_5 = 0 \} \).

### 3.3 Observability condition under varying velocity

Space \( Q_3 \) is generated way according to [14]:

\[ Q_3 = \begin{bmatrix} h_1 (X) \\ h_2 (X) \\ L_1 + u_g h_1 (X) \\ L_1 + u_g h_2 (X) \\ L_1'^2 + u_g h_1 (X) \\ L_1'^2 + u_g h_2 (X) \end{bmatrix} \]

The matrix of associated observability is then:

\[ O_3 = \frac{\partial}{\partial X} (Q_3) \]

The determinant of this matrix is:

\[ O_3 = -\frac{K^4p^2}{J T_r} x_3 \left( \frac{p^2 M}{J L_r} x_1 x_3 - \frac{x_3}{T_r} + p^2 \dot{x}_5^2 T_r \right) - \frac{M}{J T_r} x_2 x_3^2 + \frac{x_3^2}{T_r} + p x_3 \left( \frac{p^2 M}{J T_r^2} x_3 x_3^3 \right) + \frac{M}{T_r^2} + \frac{p^2 \dot{x}_5^2 M}{J} \}

\[ \quad + \frac{K^4p^2}{J T_r} \left( -p \left( \frac{p M}{J L_r} x_2 x_3 \right) - \frac{C_{res}}{J} \right) + \frac{1}{x_3 T_r} \left( \frac{M}{L_r} x_2 + p x_3 x_5 \right) \left( 1 + p^2 \dot{x}_5^2 T_r^2 \right) \]

The literal evaluation of the determinant of the matrix \( O_3 \) does not bring any exploitable information. It is possible to study the determinant under the hypothesis:

- Constant flux \( (\dot{x}_3 = \dot{x}_4 = 0) \).
- Flux oriented \( x_3 = \psi_{r\alpha} = C_{d\epsilon} \) and \( x_4 = \psi_{r\beta} = 0 \).

Under this simplifying hypothesis, we can reduce the determinant to the expression:

\[ \det (O_3) = - \frac{K^4p^3}{J T_r} x_3^2 \left( 1 + p^2 \dot{x}_5^2 T_r^2 \right) \quad (12) \]

Within the framework of the assumption carried out, \( th2 \) is endentiquement null \( (th2 = \dot{x}_4 = 0) \), thus we obtains:

\[ \det (O_3) = - \frac{K^4p^3}{J T_r} x_3^2 \dot{x}_5 \quad (13) \]

This expression is null for:

\[ x_3 = 0 \] , this case is interest little since the hypothesis of \( \dot{x}_4 = \psi_{r\beta} = 0 \) exemption a null flux in the rotor.

\[ \dot{x}_5 = 0 \] , this case means that speed is constant (steady state).

**Result 2 (cf.[15])**

In the two preceding analyses of observability §3.2 and §3.3, if speed is supposed to be constant, the rank condition is not satisfied if flux is constant.

### 4 Synthesis and Analysis of Rotor Flux and Speed Rotating Observer Design with Stator Currents Measurements

In this part we are interested in the work developed in [9] and [10], who relate to the synthesis of high gain (11) observer for the systems uniformly locally observable.
According to paragraph §3.1 the system (2) is uniformly observable. Then, it is possible to carry out the change of variable according to:

\[ z_1 = \Phi(x) = \begin{bmatrix} h_1 \\ h_2 \\ \mathcal{L}_{f+v_{\omega}}(h_1) \\ \mathcal{L}_{f+v_{\omega}}(h_2) \end{bmatrix} \]

It is equivalent to:

\[
\begin{align*}
    z_{11} &= i_{\sigma} \\
    z_{12} &= i_{s\beta} \\
    z_{13} &= -\gamma i_{\sigma} + \frac{K}{T_r} \psi_\alpha + pK\Omega \psi_\beta \\
    z_{14} &= -\gamma i_{s\beta} - pK\Omega \psi_\alpha + \frac{K}{T_r} \psi_\beta
\end{align*}
\]  

(14)

Set \( z_2 = \Omega; \) under these changes, the model (1) takes the following form:

\[
\begin{align*}
    \dot{z}_{11} &= z_{13} + \frac{1}{\sigma \omega_{\sigma}} v_{\sigma} \\
    \dot{z}_{12} &= z_{14} + \frac{1}{\omega_{s}} v_{s} \\
    \dot{z}_{13} &= \left( \frac{K}{T_r} \frac{M}{T_r} - \frac{1}{T_r} \right) z_{11} + p\omega_2 \left( \frac{K M}{T_r} - \gamma \right) z_{12} \\
    &\quad - \left( \frac{1}{T_r} + \frac{1}{\tau} \right) z_{13} - p\omega_2 z_{14} \\
    \dot{z}_{14} &= -p\omega_2 \left( \frac{K M}{T_r} - \gamma \right) z_{11} + \left( \frac{K}{T_r} \frac{M}{T_r} - \frac{1}{T_r} \right) z_{12} \\
    &\quad + p\omega_2 \left( \frac{1}{T_r} + \frac{1}{\tau} \right) z_{13} - \gamma \frac{1}{T_r} z_{14} \\
    \dot{z}_2 &= -\frac{p M}{\mathcal{L}_{\omega_{\sigma}}} K \left( \frac{1}{T_r} + p^2 \frac{z^2}{T_r} \right) \\
    &\quad \{z_{12} (z_{13} - \gamma z_{11} + p\omega_2 T_r (z_{14} + \gamma z_{12})) \\
    &\quad + z_{11} (z_{14} + \gamma z_{12} + p\omega_2 T_r (z_{13} + \gamma z_{11})) \} \\
    &\quad - \frac{C_{\omega_{\alpha}}}{J} \frac{1}{T_r} z_{i} \quad i = 1, 2, 3, 4
\end{align*}
\]

or in the matrix form:

\[
\begin{bmatrix}
    \dot{z}_{11} \\
    \dot{z}_{12} \\
    \dot{z}_{13} \\
    \dot{z}_{14}
\end{bmatrix} =
\begin{bmatrix}
    0 & 0 & 1 & 0 \\
    0 & 0 & 0 & 1 \\
    0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
    z_{11} \\
    z_{12} \\
    z_{13} \\
    z_{14}
\end{bmatrix}
\begin{bmatrix}
    \frac{1}{\sigma \omega_{\sigma}} v_{\sigma} \\
    \frac{1}{\omega_{s}} v_{s} \\
    \left( \frac{K}{T_r} \frac{M}{T_r} - \frac{1}{T_r} \right) z_{11} + p\omega_2 \left( \frac{K M}{T_r} - \gamma \right) z_{12} \\
    - \left( \frac{1}{T_r} + \frac{1}{\tau} \right) z_{13} - p\omega_2 z_{14} \\
    \left( \frac{K M}{T_r} - \gamma \right) z_{11} + \left( \frac{K}{T_r} \frac{M}{T_r} - \frac{1}{T_r} \right) z_{12} \\
    -p\omega_2 \left( \frac{1}{T_r} + \frac{1}{\tau} \right) z_{13} - \gamma \frac{1}{T_r} z_{14} \\
    -\frac{p M}{\mathcal{L}_{\omega_{\sigma}}} K \left( \frac{1}{T_r} + p^2 \frac{z^2}{T_r} \right) \\
    &\quad \{z_{12} (z_{13} - \gamma z_{11} + p\omega_2 T_r (z_{14} + \gamma z_{12})) \\
    &\quad + z_{11} (z_{14} + \gamma z_{12} + p\omega_2 T_r (z_{13} + \gamma z_{11})) \} \\
    &\quad - \frac{C_{\omega_{\alpha}}}{J} \frac{1}{T_r} z_{i} \quad i = 1, 2, 3, 4
\end{bmatrix}
\]

(15)

Set:

\[
\begin{align*}
    * \quad z &= \begin{bmatrix} z_{11} \\ z_{12} \end{bmatrix} , \quad z_{i} = \begin{bmatrix} z_{i1} \\ z_{i2} \end{bmatrix} \\
    \text{and} \quad z_{\psi} &= \begin{bmatrix} z_{13} \\ z_{14} \end{bmatrix} \\
    \mathcal{A}_1 &= \begin{bmatrix} 0_{2,2} & I \\ 0_{2,2} & 0_{2,2} \end{bmatrix} ; \quad \text{with} \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \\
    \mathcal{A}_2 &= 0.
\end{align*}
\]

* \( \varphi_1 (v_s, z_{1,2}) = \begin{bmatrix} \varphi_1 (v_s, z_{2,1}) \\ \varphi_2 (v_s, z_{2,2}, z_{i}) \end{bmatrix} \)

with

\[
\begin{align*}
    \varphi_1 (v_s, z_{2,1}) &= \begin{bmatrix} \frac{1}{\sigma \omega_{\sigma}} v_{\sigma} \\ \frac{1}{\sigma \omega_{\sigma}} v_{s} \end{bmatrix} \\
    \varphi_2 (v_s, z_{2,2}, z_{i}) &= \begin{bmatrix} z_{13} \\ z_{14} \end{bmatrix}
\end{align*}
\]

and

\[
\begin{align*}
    * \quad y &= C z \quad \text{with} \quad C = \begin{bmatrix} I & 0_{2,2} \end{bmatrix}.
\end{align*}
\]

* Hypothesis 3

\( \mathcal{H}_1 : \varphi_1 \text{ is global LIPSCHITZ compared to } z_i, \text{i.e.:} \)

\[
\| \varphi_1 (z_i) - \varphi_1 (\hat{z}_i) \| \leq l_i \| z_i - \hat{z}_i \| \quad \forall z_i, \hat{z}_i \in \mathbb{R}^k
\]

\( l_i > 0 \) : Constant of Lipschitz.
$\mathcal{H}_2 : \varphi_2 \text{ is global LIPSCHITZ compared to } z_2, \text{ i.e. }$

$$\|\varphi_2(z_2) - \varphi_2(\hat{z}_2)\| \leq l_2 \|z_2 - \hat{z}_2\| \forall z_2, \hat{z}_2 \in \mathbb{R}^n,$$

$l_2 > 0 : \text{Constant of LIPSCHITZ.}$

This Hypothesis can be omitted as soon as the state $(z_1, z_2)$ lies in a compact set (one can indeed, as in [9], prolongate nonlinearities $\varphi_1, \varphi_2$ outside a compact set to make them global Lipschitz). Moreover, considering the following subsystem:

$$\begin{cases}
\dot{z}_1 &= A_1 z_1 + \varphi_1(v_s, z_1, z_2) \\
y &= C z_1 \tag{18}
\end{cases}$$

Hypothesis 3 - together with the structure of $A_1, C$, $\varphi_1(v_s, z_1, z_2)$ ensures the existence of a high gain observer for (18) of the following form [9] and [11]:

$$\begin{cases}
\dot{\hat{z}}_1 &= A_1 \hat{z}_1 + \varphi(v_s, \hat{z}_1, \hat{z}_2) - S_0^{-1} C^T C (\hat{z}_1 - z_1) \\
0 &= \theta S_0 + A_1^T S_0 + S_0 A_1 - C^T C \tag{19}
\end{cases}$$

Hypothesis 4 :

The pair $(C, A_1)$ is detectable i.e. there is a parameter $\theta$ such as:

$$\theta > 2 L \frac{\lambda_{\text{max}}(S_0)}{\lambda_{\text{min}}(S_0)} \tag{20}$$

Where:

$$S_0 = S_0^T = \begin{bmatrix}
\theta^{-1} \mathbf{1} & -\theta^{-2} \mathbf{1} \\
-\theta^{-2} \mathbf{1} & 2 \theta^{-3} \mathbf{1}
\end{bmatrix} ; \text{ with } \theta > 0$$

is single solution of the ALGEBRAIC EQUATION OF LYAPUNOV following:

$$\theta S_0 + A_1^T S_0 + S_0 A_1 = C^T C \tag{21}$$

This THAU like condition [17] ensures that states of the following subsystem:

$$\dot{\hat{z}}_2 = A_2 \hat{z}_2 + \varphi_2(v_s, z_1, z_2) \tag{22}$$

can be estimated by an uncorrected copy of its dynamics as follows:

$$\dot{\hat{z}}_2 = A_2 \hat{z}_2 + \varphi_2(v_s, \hat{z}_1, \hat{z}_2) \tag{23}$$

Now we state the following:

**Theorem 5** :

If hypothesis 3 and 4, then the following system:

$$\begin{cases}
0 &= \theta S_0 + A_1^T S_0 + S_0 A_1 - C^T C \\
\dot{\hat{z}}_1 &= A_1 \hat{z}_1 + \varphi(v_s, \hat{z}_1, \hat{z}_2) - S_0^{-1} C^T C (\hat{z}_1 - z_1) \\
\dot{\hat{z}}_2 &= A_2 \hat{z}_2 + \varphi_2(v_s, \hat{z}_1, \hat{z}_2) \tag{24}
\end{cases}$$

is an asymptotic observer for system (17). Moreover, the rate of decay of the estimation error can be chosen to be as fast as the one imposed by sub-observer (23) of subsystem (22) (this is what we call partial correction).

**Proof** : The proof can be achieved by choosing appropriate candidate LYAPUNOV functions $V_1$ and $V_2$ in order.

Take $\theta > 1$ and set $e_1 = \hat{z}_1 - z_1$ and $e_2 = \hat{z}_2 - z_2$.

Then we get:

$$\begin{cases}
\dot{e}_1 &= \left( A_1 - S_0^{-1} C^T C \right) e_1 + \varphi_1(\hat{z}, v_s) - \varphi_1(z, v_s) \\
\dot{e}_2 &= A_2 e_2 + \varphi_2(v_s, \hat{z}_1, \hat{z}_2) - \varphi_2(v_s, z_1, z_2) \tag{25}
\end{cases}$$

Now, and defined $V_1$ and $V_2$ by:

$$V(e_1) = e_1^T S_0 e_1 \tag{26}$$

$$V(e_2) = e_2^T S_0 e_2 \tag{27}$$

For $V_1$: Its derivative is:

$$\begin{align*}
\dot{V}(e_1) &= e_1^T S_0 e_1 + e_1^T S_0 e_1 \\
&= e_1^T \left( A_1 - S_0^{-1} C^T C \right)^T e_1 + \varphi_1(\hat{z}, v_s) - \varphi_1(z, v_s) \\
&+ e_1^T S_0 \left\{ \left( A_1 - S_0^{-1} C^T C \right) e_1 + \varphi_1(\hat{z}, v_s) - \varphi_1(z, v_s) \right\} \\
&\leq -e_1^T \left( \theta S_0 + C^T C \right) e_1 + 2 e_1^T S_0 \left\{ \varphi_1(\hat{z}, v_s) - \varphi_1(z, v_s) \right\} \\
&\leq -e_1^T \theta S_0 e_1 + 2 e_1^T S_0 \left\{ \varphi_1(\hat{z}, v_s) - \varphi_1(z, v_s) \right\} \tag{28}
\end{align*}$$

let us suppose:

$$\varphi_1(\hat{z}, v_s) - \varphi_1(z, v_s) = \tilde{\varphi}$$

Where

$$\left| e_1^T S_0 \left\{ \varphi_1(\hat{z}, v_s) - \varphi_1(z, v_s) \right\} \right| \leq \left| e_1^T \right| \left| S_0 \right| \left| \tilde{\varphi} \right| \leq \left| e_1^T \right| \left| S_0 \right| \left| \theta \right| \left| e_1 \right| \tag{29}$$

more we have:

$$\lambda_{\text{min}}(S_0) \left| e_1 \right|^2 \leq e_1^T S_0 e_1 \leq \lambda_{\text{max}}(S_0) \left| e_1 \right|^2 \tag{30}$$

Then:

$$\dot{V}(e_1) \leq -\theta e_1^T S_0 e_1 + 2 L \frac{\lambda_{\text{max}}(S_0)}{\lambda_{\text{min}}(S_0)} e_1^T S_0 e_1$$

$$\leq - \left( \theta - 2 L \frac{\lambda_{\text{max}}(S_0)}{\lambda_{\text{min}}(S_0)} \right) V(e_1) \tag{31}$$
For \( V_2 \): in the same way, we have:

\[
\dot{V} = e_2^T e_2 + e_2^T e_2
\]
\[
= \{ A_2 e_2 + \tilde{\varphi}_2 \}^T e_2 + e_2^T \{ A_2 e_2 + \tilde{\varphi}_2 \}
\]
\[
= e_2^T \{ A_2 + A_2^T \} e_2 + 2e_2^T \tilde{\varphi}_2
\]
\[
= e_2^T \{ A_2 + A_2^T \} e_2 + 2e_2^T \tilde{\varphi}_2
\]
\[
= \mu \| e_2 \|^2 + 2\| e_2 \| \| \tilde{\varphi}_2 \|
\]
\[
= \mu \| e_2 \|^2 + 2\| e_2 \|^2 (32)
\]

where \( \mu \) is a selected positive constant to satisfy the condition of LIPSCHITZ.

\[
\mu = \max \left( 2\gamma, \frac{2}{T_r} \right) (33)
\]

By using the matric relation:

\[ \| e_2 \|^2 = e_2^T e_2 \]

Now, we rewrite the preceding expression of \( V_2 \), we obtains:

\[ \dot{V}_2 \leq (\mu + 2\| e_2 \|) V_2 \] (34)

This expression \( V \) can be rewritten in the following more compact form:

\[ \dot{V} \leq -\pi_1 V_1 - \pi_2 V_2 \] (35)

where:

\[ \pi_1 = 2 \left( \frac{\lambda_{\min}(Q)}{\lambda_{\max}(P)} - 1 \right) \] (36)

\[ \pi_2 = -(\mu + 2\| e_2 \|) \] (37)

Finally, while taking:

\[ \kappa = \max(\pi_1, \pi_2) \] (38)

it follows that:

\[ \dot{V} \leq -\kappa (V_1 + V_2) \leq -\kappa V \] (39)

4.1 Observer in the initial coordinates

The \( \hat{x} \) estimate of state \( x \) of (17); is obtained by:

\[ \hat{x} = \Phi^{-1}(z) \] (40)

The fact that \( z = \Phi(x) \) implies \( \frac{dz}{dt} = \frac{\partial \Phi(x)}{\partial x} \frac{dx}{dt} \) is another method of synthesis the observer (24) expressed in \( x \) :

\[ \dot{\hat{x}} = F(\hat{x}) + G.U - \left( \frac{\partial \Phi(x)}{\partial x} \right)^{-1} S_\theta^{-1} C^T C (\hat{x} - x) \] (41)

Or in the panaromic form:

\[
\begin{bmatrix}
\dot{\hat{i}}_{\alpha} \\
\dot{\hat{i}}_{\beta} \\
\dot{\psi}_{\alpha} \\
\dot{\psi}_{\beta}
\end{bmatrix}
= 
\begin{bmatrix}
\gamma \frac{\gamma e_\kappa}{\gamma e_\kappa + (p\Omega K)^2} & 0 & 0 & \frac{\gamma e_\kappa}{\gamma e_\kappa + (p\Omega K)^2} \\
0 & -\gamma & -p\Omega & \frac{\gamma e_\kappa}{\gamma e_\kappa + (p\Omega K)^2} \\
M & 0 & -\frac{1}{T_r} & -p\Omega \\
0 & M & 0 & -\frac{1}{T_r}
\end{bmatrix}
\begin{bmatrix}
\hat{i}_{\alpha} \\
\hat{i}_{\beta} \\
\psi_{\alpha} \\
\psi_{\beta}
\end{bmatrix}
+ 
\begin{bmatrix}
\frac{M}{\gamma e_\kappa} & 0 & 0 & 0 \\
0 & \frac{1}{\gamma e_\kappa} & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
\nu_{\alpha} \\
n_{\beta}
\end{bmatrix}
\]

\[
\Omega = \frac{p}{J} M \left( \hat{i}_{\beta} \psi_{\alpha} - \hat{i}_{\alpha} \psi_{\beta} \right) - C_{res} \frac{\gamma}{J}
\]

The adjustment of this observer is done with through the choice of only one parameter \( \theta \).

5 Results and Simulations

The state vector of the machine is initialized in a stopped state \( \begin{bmatrix} i_{\alpha} & i_{\beta} & \psi_{\alpha} & \psi_{\beta} & \Omega \end{bmatrix}^T = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T \); whereas that of the observer to the functional state \( \begin{bmatrix} \hat{i}_{\alpha} & \hat{i}_{\beta} & \hat{\psi}_{\alpha} & \hat{\psi}_{\beta} & \hat{\Omega} \end{bmatrix}^T = \begin{bmatrix} 0.2 & 0.2 & 1 & 1 & 0 \end{bmatrix}^T \), and the results are given for the machine of which a direct starting, i.e. a resistive torque null \( (C_{res} = 0) \) and these characteristics are given in the table (Tab.1):
Table 1: Parameters of the induction machine. (cf. [18])

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Notation</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Poles pairs</td>
<td>$p$</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>Stator resistance</td>
<td>$R_s$</td>
<td>9.65</td>
<td>$\Omega$</td>
</tr>
<tr>
<td>Rotor resistance</td>
<td>$R_r$</td>
<td>4.3047</td>
<td>$\Omega$</td>
</tr>
<tr>
<td>Stator inductance</td>
<td>$L_s$</td>
<td>0.4718</td>
<td>$H$</td>
</tr>
<tr>
<td>Rotor inductance</td>
<td>$L_r$</td>
<td>0.4718</td>
<td>$H$</td>
</tr>
<tr>
<td>Mutual Inductance</td>
<td>$M$</td>
<td>0.4475</td>
<td>$H$</td>
</tr>
<tr>
<td>Rotor inertia</td>
<td>$J$</td>
<td>0.0293</td>
<td>Kg.m$^2$</td>
</tr>
<tr>
<td>Resistive torque</td>
<td>$C_{res}$</td>
<td>0</td>
<td>N.m</td>
</tr>
</tbody>
</table>

The diagram SIMULINK of the observer is given by the figure (Fig.1).

Figure 1: observer specific to the systems locally observable interconnected with an estimator.

The adjustment of the dynamics of the observer is adjusted by the parameters of gain $\theta$; this is arbitrarily selected. Simulations (Fig.2), (Fig.3), (Fig.4) and (Fig.5) present the pace of flux real and observed, error of flux, the pace speed real and observed and error of rotating speed with the observer in open-loop ($\theta = 0$) and in loop-closed ($\theta = 600$) respectively.

Figure 2: Flux simulated and observed in open-loop and close-loop.

Figure 3: Flux error in open-loop and close-loop.

Figure 4: Speed simulated and observed in open-loop and close-loop.

Figure 5: Speed error in open-loop and close-loop.

6 Conclusion

observer specific to the systems locally observable interconnected with an estimator is conceived for observed flux rotor and rotating speed of the asynchronous machine by using like only information the stator currents.

Since the model of the asynchronous machine belongs to the general systems and the observer (19) of [9] and [10] is specific to systems uniformly locally
observable. We exploited the local uniformity of observation model of the asynchronous machine and by making a change of variables where the model of machine can adapt the structure of this observer. We notices an error of convergence of the parameters observed due to the difference of the dynamics of the process and observer. It is noted that the results are satisfactory.

References:


