

# Mamdani Fuzzy Logic Controller with Mobile Agents for Matching

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*Abstract:* - The task of a standard fuzzy logic controller is to find a crisp control action from the fuzzy rule-base and from a set of crisp inputs. In this paper we propose an extension of this type of reasoning using Mobile Agents which works with crisp data, intervals and/or linguistic terms as inputs and with various matching methods. For any matching one obtain a crisp value as output using Slave Agents. Finally, these outputs are combined by Master Agent to obtain the overall crisp output action of the system.

*Key-Words:* - Fuzzy logic, Fuzzy Controller, Mobile Agents, Matching techniques, t-norm, OWA operator, Visual Studio .Net technologies.

## 1 Introduction

Distributed Artificial Intelligence systems can be defined as cooperative systems where a set of agents act together to solve a given problem. These agents are often heterogeneous (e.g., in Decision Support System, the interaction takes place between a human and an artificial problem solver).

Agent-based systems technologies in the sense of distributed computing is an area of emerging interest in the domain of complex systems design. The agent-based paradigm can be seen as a real enhancement of the object oriented paradigm where objects become autonomous, proactive, and perceptive with respect to their environments.

The concept of mobile agent has been defined in [1], [2] and [3]. They are autonomous objects that can migrate from node to node on behalf of the user who have executed them and make use of the databases or computation resources from computers connected by the network. In order for a mobile agent to be able to migrate, there must be a virtual place, the so-called mobile agent system, that supports mobility.

The mobile agents technology promotes applications made up of network-aware entities (agents) capable of changing their execution environment by transferring them-selves while executing (migration).

In the general field of agent-based systems we can identify two major diverging directions, agent theory and industrial applications. On the one hand there is considerable work on formalization of multi-agent systems, e.g., commitments, capabilities, know-how. Advanced logics which capture essential properties of agents like concurrency, time dependent behavior

or inconsistent states are under development. Formal specification techniques like Z are applied to formally capture those systems. On the other hand, there exists a number of successful implementations dedicated to tackle real-world problems like flight control, manufacturing resource allocation, and information retrieval in large databases. Significant research and development into mobile agency has been conducted in recent years, and there are many mobile agent architectures available today. The AgentLink project [4] maintains a list of ongoing projects with regard to any kind of agent-related topics and also maintains a list of available agent toolkits. Nevertheless, several issues still need to be faced to make the mobile agent technology widely accepted: secure and efficient execution supports, standardization, appropriate programming languages and coordination models.

In this paper a new approach regarding Mamdani Fuzzy Logic Controller with Mobile Agents is presented. We have introduced the Mobile Agents in order to get better results than the traditional Mamdani Fuzzy Logic Controller. We implemented and tested this approach using Visual Studio .Net(C#) technologies.

## 2 Structure of a Fuzzy Controller

The database of a rule-based system may contain imprecisions which appear in the description of the rules given by the expert. Because such an inference cannot be made by the methods which use classical two valued logic or many valued logic, Zadeh [5] extends the traditional Modus Ponens rule in order to work with fuzzy sets, obtaining the Generalized

Modus Ponens (GMP) rule. The investigation of inference processes in the fuzzy if-then rules was the subject of many other papers.

An important part of fuzzy reasoning is represented by Fuzzy Logic Control (FLC), derived from control theory based on mathematical models of the open-loop process to be controlled. Fuzzy control provides a formal methodology for representing, manipulating and implementing human's heuristic knowledge about how to control a system. In a fuzzy logic controller, the expert knowledge are of the form

**IF** (a set of conditions are satisfied)  
**THEN** (a set of consequences are inferred)

where the antecedents and the consequences of the rules are associated with fuzzy concepts (linguistic terms). In the terminology of FLC systems, a rule in which the antecedent is a condition and the consequent is a control action for the system is called control rule. The task of a FLC system is to find a crisp control action from the fuzzy rule-base and from the actual crisp inputs. Because the inputs and the outputs of fuzzy rule-based systems are fuzzy sets, we have to fuzzify the crisp inputs and to defuzzify the fuzzy outputs. A standard FLC system consists from four parts, as in the Figure 1.

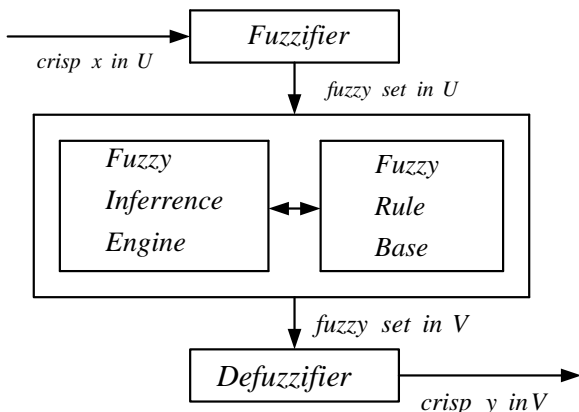


Fig. 1 Fuzzy Logic Controller

A fuzzification operator transforms crisp data into fuzzy sets. For instance,  $x_0 \in U$  is fuzzified into  $\bar{x}_0$

$$\mu_{\bar{x}_0}(x) = \begin{cases} 1 & \text{if } x = x_0 \\ 0 & \text{if } x \neq x_0 \end{cases}$$

The procedure used by Fuzzy Inference Engine in order to obtain a fuzzy output consists of the following steps:

- 1) find the firing level of each rule,
- 2) find the output of each rule,

3) aggregate the individual rules outputs, in order to obtain the overall action of control system.

The fuzzy control action  $C$  inferred from the fuzzy control system is transformed into a crisp control action:

$$z_0 = \text{defuzzifier}(C),$$

where *defuzzifier* is a defuzzification operator. The most used defuzzification operators, for a discrete fuzzy set  $C$  having the universe of discourse  $V$  are:

- Center-of-Gravity:

$$z_0 = \left( \frac{\sum_{j=1}^N z_j \mu_C(z_j)}{\sum_{j=1}^N \mu_C(z_j)} \right)$$

- Middle-of-Maxima: the defuzzified value is defined as mean of all values of the universe of discourse, having maximal membership grades

$$z_0 = \left( \frac{\sum_{j=1}^N z_j}{N} \right)$$

- Max-Criterion: this method chooses an arbitrary value, from the set of maximizing elements of  $C$ ,

$$z_0 \in \left\{ z / \mu_C(z) = \max_{v \in V} \mu_C(v) \right\},$$

where  $\mu_C$  is the membership function of  $C$  and  $Z = \{z_1, \dots, z_N\}$  is a set of elements from the universe  $V$ .

The most known FLC systems are: Mamdani, Tsukamoto, Sugeno and Larsen which work with crisp data as inputs. An extension of the Mamdani model in order to work with crisp data, intervals and/or linguistic terms as inputs and various implications is presented in [6].

In this paper we propose a fuzzy reasoning system characterized by:

- the linguistic terms (or values), that are represented by trapezoidal fuzzy numbers
- the inputs, which can be crisp data, intervals and/or linguistic terms
- various types of matching
- Lukasiewicz implication, which is used to represent the rules
- the crisp control action of a rule, computed by Middle-of-Maxima method
- a set of mobile agents of slave type, which compute the control action using the discrete Center-of-Gravity, for every type of matching
- the overall crisp control action, computed by a mobile agent master type using the Ordered Weighted Averaging (OWA) operator.

The following definitions will be used in the next sections.

**Definition 1** A function  $T : [0,1]^2 \rightarrow [0,1]$  is a  $t$ -norm iff it is commutative, associative, nondecreasing and  $T(x,1)=1, \forall x \in [0,1]$ .

The  $t$ -norms are used to compute the firing levels of the rules.

**Definition 2.** An OWA operator of dimension  $n$  is a mapping  $F : R^n \rightarrow R$  that has an associated  $n$  vector  $w = (w_1, \dots, w_n)^t$  such as

$$w_i \in [0,1], 1 \leq i \leq n, \sum_{i=1}^n w_i = 1$$

The aggregation operator of the values  $\{a_1, \dots, a_n\}$  is

$$F(a_1, a_2, \dots, a_n) = \sum_{j=1}^n w_j b_j$$

where  $b_j$  is the  $j$ -th largest element from  $\{a_1, \dots, a_n\}$ .

### 3 The Proposed Model

#### 3.1 Preliminaries

The system architecture that includes the related components is described below and can be visualized in Figure 2, that presents the conceptual view of our proposed model.

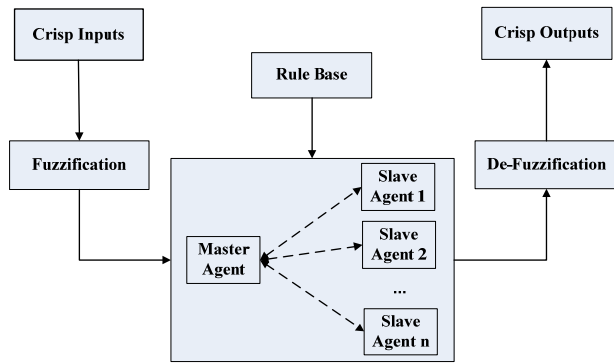


Fig. 2 Architecture of the Mamdani Fuzzy Logic Controller with Mobile Agents

The Master Agent (MA) is the agent that coordinates the process, divides the tasks and sends to slave agents (SA). An SA calculates a crisp value for a matching method and MA combine these outputs in order to obtain the overall crisp output action of the system.

The linguistic variables are represented as fuzzy sets having the membership functions given by

trapezoidal fuzzy numbers. Such a number  $N = (\underline{m}, \bar{m}, \alpha, \beta)$  is defined as

$$\mu_N(x) = \begin{cases} 0, & \text{for } x < \underline{m} - \alpha \\ \frac{x - \underline{m} + \alpha}{\alpha}, & \text{for } x \in [\underline{m} - \alpha, \underline{m}] \\ 1, & \text{for } x \in [\underline{m}, \bar{m}] \\ \frac{\bar{m} + \beta - x}{\beta}, & \text{for } x \in [\bar{m}, \bar{m} + \beta] \\ 0, & \text{for } x > \bar{m} + \beta \end{cases}$$

Generally, a rule is represented by a fuzzy implication. Let  $X$  and  $Y$  be two variables whose domains are  $U$  and  $V$ , respectively. The rule

if  $X$  is  $A$  then  $Y$  is  $B$

is represented as a conditional possibility distribution ([7])

$$\pi_{Y/X}(v,u) = \mu_A(u) \rightarrow \mu_B(v), \forall u \in U, \forall v \in V$$

where  $\rightarrow$  is an implication operator and  $\mu_A$  and  $\mu_B$  are the membership functions of the fuzzy sets  $A$  and  $B$ , respectively. We work with Lukasiewicz implication  $I_L(x,y) = \min(1, 1-x+y)$  because from a set of 10 the most important implications,  $I_L$  is the single which verifies 13 important properties [8]. It is sufficiently to use rules with a single consequent because a rule with multiple consequent can be treated as a set of such rules.

A rule is characterized by

- a set of linguistic variable  $A$ , having as domain an interval  $I_A = [a_A, b_A]$
- $n_A$  linguistic values  $A_1, A_2, \dots, A_{n_A}$  for each linguistic variable  $A$
- membership function for each value  $A_i$  is  $\mu_{A_i}^0(x)$ , where  $i \in \{1, 2, \dots, n_A\}$  and  $x \in I_A$ .

According to the structure presented in the Figure 1, the following steps are necessary in order to work with a FLC system.

#### 3.2 Fuzzification

For an interval input  $[a, b]$  with  $a_A \leq a < b \leq b_A$ , the membership function is

$$\mu_{[a,b]}(x) = \begin{cases} 1 & \text{if } x \in [a, b] \\ 0 & \text{otherwise.} \end{cases}$$

If the input is a crisp value  $x_0$  then the firing level is  $\mu_{A_i}^0(x_0)$ .

For a linguistic value  $A_i$ , the firing level  $\mu_{A_i}$  generated by an input value  $B \in \{[a, b], A_i'\}$ , where  $A_i'$  is an input linguistic value, can be computed using the following types of matching.

**M1) Height intersection:**

$$\mu_{A_i} = \max\{\min\{\mu_{A_i}^0(x), \mu_B(x)\}\}$$

**M2) Ratio 1:**

$$\mu_{A_i} = \frac{\text{Pr}_X\{\min\{\mu_{A_i}^0(x), \mu_B(x)\}\}}{m_{A_i} - \underline{m}_{A_i} + \alpha_{A_i} + \beta_{A_i}}$$

where  $\text{Pr}_X$  is the projection on  $OX$  axis and the denominator is the length of  $A_i$ 's support;

**M3) Ratio 2.**

$$\mu_{A_i} = \frac{\int_{a_{A_i}}^{b_{A_i}} \min\{\mu_{A_i}^0(x), \mu_B(x)\} dx}{\int_{a_{A_i}}^{b_{A_i}} \mu_{A_i}^0(x) dx}$$

which is the area of  $A_i \cap B$  divided by area defined by  $\mu_{A_i}^0$ ;

**M4) Ratio 3.**  $\mu_{A_i}$  is the area of  $A_i \cap B$  divided by area of  $A_i \cup B$ .

Another possibilities for computing the firing level can be derived from similarity measures. We consider the discrete expression of fuzzy sets, using a set of  $N$  points, and we use the previous notations. It results [9]:

$$\text{M5) } \mu_{A_i} = \frac{\sum_k \mu_{A_i}(x_k) * \mu_B(x_k)}{\sqrt{\sum_k \mu_{A_i}^2(x_k)} \sqrt{\sum_k \mu_B^2(x_k)}};$$

$$\text{M6) } \mu_{A_i} = 1 - \frac{\sqrt{\sum_k (\mu_{A_i}(x_k) - \mu_B(x_k))^2}}{\max_k\{\mu_{A_i}(x_k), \mu_B(x_k)\}};$$

$$\text{M7) } \mu_{A_i} = 1 - \frac{2 \sum_k (\mu_{A_i}(x_k) - \mu_B(x_k))^2}{Z_{A_i} + Z_B}$$

where  $Z_C = \sum_k (2\mu_C(x_k) - 1)^2$ .

### 3.3. Fuzzy inference

We consider a set of fuzzy control rules

$$R_i : \text{if } X_1 \text{ is } A_i^1 \text{ and } \dots \text{ and } X_n \text{ is } A_i^n \\ \text{then } Y \text{ is } C_i$$

where the variables  $X_j, j \in \{1, 2, \dots, n\}$  and  $Y$  have the domains  $U_j$  and  $V$ , respectively. The firing level of the rule  $R_i$ , denoted by  $\alpha_i$ , is computed by

$$\alpha_i = T(\alpha_i^1, \dots, \alpha_i^n)$$

where  $T$  is a t-norm and  $\alpha_i^j$  is the firing level for  $A_i^j, j \in \{1, 2, \dots, n\}$ , generated by an input value. The causal link from  $X_1, \dots, X_n$  to  $Y$  is represented using the Lukasiewicz implication  $I_L$ . It results that the conclusion  $C'$  inferred from the rule  $R_i$  is

$$\mu_{C'_i}(v) = I_L(\alpha_i, \mu_{C_i}(v)) = \min(1 - \alpha_i + \mu_{C_i}(v), 1)$$

as it results from the following figure

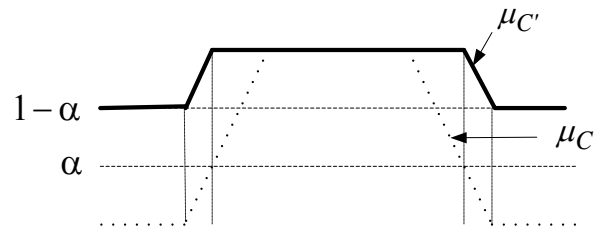


Fig. 3 Conclusion obtained with Lukasiewicz implication

### 3.4. Defuzzification

The fuzzy output  $C'$  of the rule  $R_i$  is transformed in a crisp output  $z_i$  using the Middle-of-Maxima operator. The crisp value  $z_0$  associated to a conclusion  $C'$  inferred from a rule having the firing level  $\alpha$  and the conclusion  $C$  represented by the fuzzy number  $(\underline{m}_C, \bar{m}_C, \alpha_C, \beta_C)$  is [6]:

$$z_0 = \frac{\underline{m}_C + \bar{m}_C + (1 - \alpha)(\beta_C - \alpha_C)}{2}$$

The overall crisp control action corresponding to the matching technique  $M$  is computed by the discrete Center-of-Gravity method: if the number of fired rules is  $N$  then the final control action is

$$z_0^M = \frac{\sum_{i=1}^N \alpha_i z_i}{\sum_{i=1}^N \alpha_i}$$

and the firing level for the matching  $M$  is

$$\alpha_M = \frac{1}{N} \sum_{i=1}^N \alpha_i$$

where  $\alpha_i$  is the firing level and  $z_i$  is the crisp output of the  $i$ -th rule.

## 4 An application

In order to show how the proposed system works, we consider an example inspired from [10]. We consider rules with two inputs and one output. The input variables are *quality* ( $Q$ ) and *price* ( $P$ ); the output variable is *satisfaction score* ( $S$ ). The fuzzy rule-base consist of

- R1: if  $Q$  is Poor and  $P$  is Low then  $S$  is Middle
- R2: if  $Q$  is Poor and  $P$  is Middle then  $S$  is Low
- R3: if  $Q$  is Poor and  $P$  is High then  $S$  is VeryLow
- R4: if  $Q$  is Average and  $P$  is Low then  $S$  is High
- R5: if  $Q$  is Average and  $P$  is Middle then  $S$  is Middle
- R6: if  $Q$  is Average and  $P$  is High then  $S$  is Low
- R7: if  $Q$  is Good and  $P$  is Low then  $S$  is Very High
- R8: if  $Q$  is Good and  $P$  is Middle then  $S$  is High
- R9: if  $Q$  is Good and  $P$  is High then  $S$  is Middle.

There are three linguistic values for the variable *price*

$$\{Low, Middle, High\}$$

and five linguistic values for the variable *quality*

$$\{Poor, BelowAverage, Average, AboveAverage, Good\}$$

We consider the universes of discourse  $[0, 800]$  for *price* and  $[0, 10]$  for *quality*. The membership functions corresponding to the linguistic values are represented by the following trapezoidal fuzzy numbers for the output variable:

$$Low = (0, 100, 0, 200)$$

$$Middle = (300, 500, 100, 100)$$

$$High = (700, 800, 200, 0)$$

$$Poor = (0, 1, 0, 2)$$

$$BelowAverage = (2, 3, 1, 1)$$

$$Average = (4, 6, 2, 2)$$

$$AboveAverage = (7, 8, 1, 1)$$

$$Good = (9, 10, 2, 0).$$

The *satisfaction score* has following linguistic values

$$\{VeryLow, Low, Middle, High, VeryHigh\}.$$

For the universe  $[0, 10]$  we consider the following membership functions:

$$VeryLow = (0, 1, 0, 1)$$

$$Low = (2, 3, 1, 1.5)$$

$$Middle = (4, 6, 1, 1)$$

$$High = (7, 8, 1, 2)$$

$$VeryHigh = (9, 10, 1, 0).$$

We consider a person interested to buy a computer with *price* = 400 – 600 EUR and *quality* = *Above Average*.

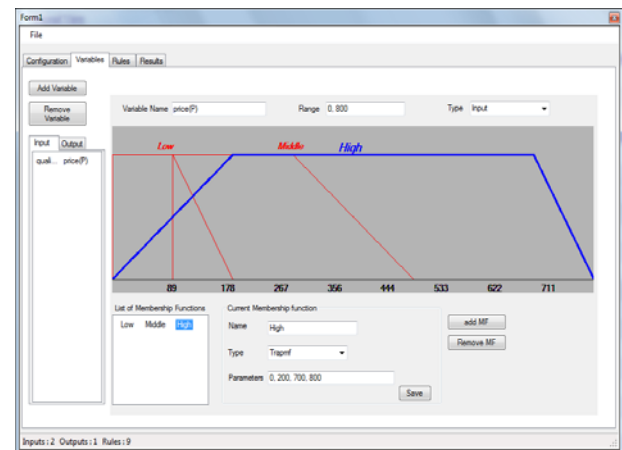


Fig. 4 System interface. Input/output variables from example

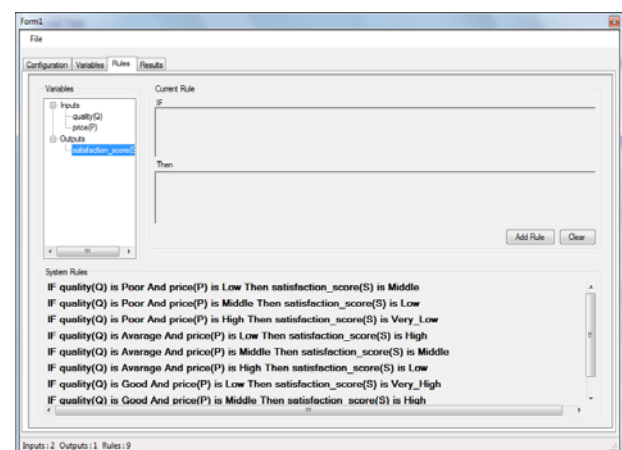


Fig. 5 System interface. Fuzzy rule base from example

For system implementation we used Visual Studio .Net(C#) technologies. The input/output variables and the fuzzy rule-base from example was

introduced using system interface as shown in Figure 4 and Figure 5.

We use the matching M1, M2 and M3 and work with t-norm Product  $t(x, y) = xy$  in order to compute the firing levels; three agents implement the reasoning corresponding to these methods. In the next sets the  $i$ -th element corresponds to the matching  $M_i$ . The positive firing levels corresponding to the linguistic values of the input variable *price* are

$$\mu_{Middle} \in \{1, 0.5, 0.5\}, \mu_{High} \in \{0.5, 1/3, 1/8\}$$

and the positive firing levels corresponding to the linguistic values of the input variable *quality* are:

$$\mu_{Average} \in \{2/3, 1/3, 1/6\}, \mu_{Good} \in \{2/3, 2/3, 1/3\}$$

The fired rules and their firing levels, computed with t-norm Product, are:

$$R_5 \text{ with fired level } \alpha_5 \in \{2/3, 1/6, 1/12\}$$

$$R_6 \text{ with } \alpha_6 \in \{1/3, 1/9, 1/48\}$$

$$R_8 \text{ with } \alpha_8 \in \{2/3, 1/3, 1/6\} \text{ and}$$

$$R_9 \text{ with } \alpha_9 \in \{1/3, 2/9, 1/24\}.$$

The fired rules give the following crisp values as output:

$$z_5 \in \{5, 5, 5\}, z_6 \in \{2.66, 2.72, 2.74\},$$

$$z_8 \in \{7.66, 7.83, 7.92\}, z_9 \in \{5, 5, 5\};$$

the overall control action for M1, M2 and M3 is

$$z_0^1 = 5.5, z_0^2 = 5.83, z_0^3 = 6.4,$$

respectively. The firing levels corresponding to matching methods are

$$\alpha^1 = 0.5, \alpha^2 = 0.208, \alpha^3 = 0.078,$$

where  $\alpha^i$  is associated with  $M_i$ . The values  $\{z_0^i\}$  and  $\{\alpha^i\}$  are transmitted to the master agent which, using OWA technique with

$$w_j = \alpha^j / (\alpha^1 + \alpha^2 + \alpha^3), a_j = z_0^j, j \in \{1, 2, 3\},$$

it calculates the overall output of system:  $z_0 = 6.159$ .

## 5 Conclusions

This paper presents a fuzzy reasoning model of Mamdani type using Mobile Agents which works with crisp data, intervals and/or linguistic terms as inputs and with various matching methods. For any matching one obtain a crisp value as output using

Slave Agents. Finally, these outputs are combined by Master Agent to obtain the overall crisp output action of the system. Because it can work not only with crisp data as inputs but, also, with interval and or linguistic terms its area of applications is very large. As it is mentioned in [10], a very important domain of its application is WEB shopping. The working with various matching methods in the same time and, moreover, the possibility to aggregate the results given by these methods offer a strong base for more accurate results of our system.

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