Mathematic model and technical solution for surveillance and autonomous monitoring system based on endurance unmanned aerial vehicle

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Abstract: - The paper purpose is to present some aspects regarding the calculus model and technical solutions for autonomous system based on endurance UAV (unmanned aerial vehicle), used to local observations, surveillance and monitoring interest area. The calculus methodology allows a numerical simulation of UAV evolution in bad atmospheric conditions using regular guidance structure. The UAV model which will be presented has six DOF (degrees of freedom), and autonomous control system. A simple but robust guidance system will be evaluated for different fly conditions and the results will be presented. The flight parameters and guidance will be analyzed. The paper is inspired by national project SAMO (Autonomous Aerial Monitoring System for Interest Areas of Great Endurance).

Key-Words: - UAV, simulation, mathematical model, guidance, endurance, surveillance, monitoring

NOMENCLATURE
α - Attack angle (tangent definition);
β - Sideslip angle (tangent definition);
δ_a - Aileron deflection;
δ_e - Elevator deflection;
δ_e_0 The balance deflection angle for the elevator;
δ_r - Rudder deflection;
δ_T - Thrust command;
ψ - Azimuth angle;
θ - Inclination angle;
φ - Bank angle;
ρ - Air density;
Ω - Body angular velocity;
A, B, C, E - Inertia moments;
C_f, C_m, C_n - Aerodynamic coefficients of force in the mobile frame;
C_t, C_m, C_n - Aerodynamic coefficients of momentum in the mobile frame;
C_x, C_y, C_z - Thrust coefficients in the mobile frame;
C_{tx}, C_{ty}, C_{tz} - Thrust momentum coefficients in the mobile frame;
F_0 = \rho \frac{V^2}{2} - Reference aerodynamic force;
H_o = F_o l - Reference aerodynamic couple;
T_0 - Reference thrust force;
H_o^T = T_0 l - Reference couple thrust;
1 Introduction

The paper aims to evaluate the modelling and simulation of the performances of an UAV with its original design as shown in Fig. 1, in different atmospheric conditions.

Fig. 1 UAV Endurance configuration

The UAV designed will be capable of assuring a great length of video monitoring (8 hours) on an interest area, on a preprogrammed path, or guided, during the mission. The subject approached is a great interest not just in the perspective of commercial and civil applications, such as infrastructure monitoring, search and rescue missions, traffic control, but also in military applications. For achieving this objective, there have been established two major research directions. The first direction consists in designing and achieving of the carrier platform-UAV, of great endurance, capable of transporting equipment required for commanding the aircraft, for communications, data acquisition and data processing. The second major research direction synthesizes and implements the platform’s automated command system for tracking the default trajectories. It is taken into account the attainment of a flexible infrastructure for the command system which will test the alternative algorithms used for the guidance and control of the platform.

2 General movement equations

As shown in the papers [2] and [3] the UAV’s dynamic equations are the projection equations of the force, equations that are achieved from the impulse theorem, written in the Earth’s frame and the momentum equations, which come from the kinetic momentum theorem, equations written in the mobile frame. Therefore, the force equations in the ground frame are:

\[
\begin{bmatrix}
  \dot{V}_{xp} \\
  \dot{V}_{yp} \\
  \dot{V}_{zp}
\end{bmatrix} = \frac{1}{m} \mathbf{B}_p \begin{bmatrix}
  C_x^A \\
  C_y^A \\
  C_z^A
\end{bmatrix} + T_0 \begin{bmatrix}
  C_x^R \\
  0 \\
  0
\end{bmatrix} + \begin{bmatrix}
  0 \\
  0 \\
  -g
\end{bmatrix},
\]

(1)

where the matrix \( \mathbf{B}_p \) is defined using the Euler’s angles:

\[
\mathbf{B}_p = [b_{1,i}].
\]

(2)

with:

\[
\begin{align*}
  b_{1,1} &= \cos \psi \cos \theta \\
  b_{1,2} &= \sin \phi \sin \theta \cos \psi - \sin \psi \sin \phi \\
  b_{1,3} &= \sin \phi \sin \psi + \cos \phi \sin \theta \cos \psi \\
  b_{2,1} &= -\sin \psi \cos \phi \\
  b_{2,2} &= -\cos \phi \cos \psi - \sin \psi \sin \phi \\
  b_{2,3} &= \cos \psi \sin \phi - \sin \theta \sin \psi \sin \phi \\
  b_{3,1} &= \sin \theta; b_{3,2} = -\cos \theta \sin \phi \\
  b_{3,3} &= -\cos \phi \cos \theta
\end{align*}
\]

The momentum equations around the centre of the mass of the UAV, written in the mobile frame are:

\[
\begin{bmatrix}
  \dot{p} \\
  \dot{q} \\
  \dot{r}
\end{bmatrix} = \mathbf{J}^{-1} \begin{bmatrix}
  C_x^A \\
  C_y^A \\
  C_z^A
\end{bmatrix} + \mathbf{J}^{-1} \begin{bmatrix}
  (B - C)qr + Epq \\
  (C - A)rp + E(r^2 - p^2) \\
  (A - B)pq - Eqr
\end{bmatrix},
\]

(3)

where the inverse matrix for the inertia momentum is given by:

\[
\mathbf{J}^{-1} = \frac{1}{AC - E^2} \begin{bmatrix}
  C & 0 & E \\
  0 & (AC - E^2)/B & 0 \\
  E & 0 & A
\end{bmatrix}.
\]

(4)

The kinematical equations are additional equations, which allow us to obtain the linear coordinates in the inertial frame:

\[
\begin{bmatrix}
  \dot{x}_p \\
  \dot{y}_p \\
  \dot{z}_p
\end{bmatrix} = \begin{bmatrix}
  V_{xp} \\
  V_{yp} \\
  V_{zp}
\end{bmatrix},
\]

(5)
and Euler’s angle when the rotation velocity components are known:

\[
\begin{bmatrix}
\phi \\
\dot{\theta} \\
\psi \\
\end{bmatrix} = W_A \begin{bmatrix}
p \\
q \\
r \\
\end{bmatrix},
\]

where

\[
W_A = \begin{bmatrix}
1 & \sin \phi \tan \theta & \cos \phi \tan \theta \\
0 & \cos \phi & -\sin \phi \\
0 & \sin \phi \sec \theta & \cos \phi \sec \theta \\
\end{bmatrix}.
\]

Supplementary, we have mass equation which describes mass UAV’s modification during the fly:

\[
m = -C_{wp} T
\]

where \( C_{wp} \) is specific fuel consumption.

We define the aerodynamic coefficients in the mobile frame:

\[
C_x^A = \frac{X^A}{F_0}; C_y^A = \frac{Y^A}{F_0}; C_z^A = \frac{Z^A}{F_0};
\]

\[
C_l^A = \frac{L^A}{H_0}; C_m^A = \frac{M^A}{H_0}; C_n^A = \frac{N^A}{H_0}.
\]

Similarly, if we consider the thrust \( T \) and the nominal thrust as reference \( T_0 \), we can define axial thrust coefficient:

\[
C_x^T = \frac{T}{T_0}
\]

3 Guidance command

Resuming [4], the guidance commands for UAV flight are:

\[
u_u = u_b - u_y \sin \theta;
\]

\[
u_e = u_{eb} + u_z \cos \phi \cos \theta + u_0 \cos \phi + u_y \sin \phi \cos \theta;
\]

\[
u_r = -u_z \sin \phi \cos \theta - u_0 \sin \phi + u_y \cos \phi \cos \theta.
\]

\[
u_T = u_{Th} + k_u^a u
\]

where the main control signals are:

\[
u_b = k_u^b \tilde{\phi} - k_u^b \tilde{\phi}; \quad u_0 = k_u^b \tilde{\theta} - k_u^b \tilde{\theta};
\]

\[
u_y = k_y^u \tilde{\psi} - k_y^u \tilde{\psi}; \quad u_z = k_z^b h_z + k_z^b \lambda_z + k_I z_I;
\]

The relative parameters \( \tilde{\phi}; \tilde{\psi}; \tilde{\theta} \) are given by:

\[
\tilde{\phi} = \phi - \phi; \quad \tilde{\psi} = \psi - \psi; \quad \tilde{\theta} = \theta - \theta;
\]

\[
\lambda_z = -V_z \psi; \quad h_z = z_{pd} - z_p; \quad \tilde{u} = u_D - u
\]

where

\[
\phi_D, \theta_D; \quad \psi_D; \quad z_{pd}; \quad u_D
\]

are the input reference values.
4.1.2 Mechanical data
Mass characteristics of the model are: \( m_i = 70 \text{ kg} \); Corresponding to initial mass, we have: Centre of mass position: \( x_{cm} = 1.3 \text{ m} \).
Inertial moments: \( A = 10 \text{ kgm}^2 \); \( B = 20 \text{ kgm}^2 \);
\( C = 30 \text{ kgm}^2 \text{ E} = 0.5 \text{ kgm}^2 \).

4.1.3 Aerodynamic data
For the configuration from figure 4, aircraft type, considering a Taylor series expanding around the origin, taking into account the parity of the terms, we obtain the following polynomial form of the aerodynamic coefficients in a body frame of the aircraft:
\[
\begin{align*}
C_x^A &= a_1 + a_2 \alpha^2 + a_3 \beta^2 + a_4 \delta_x^2 + a_5 \delta_n^2 + a_6 \delta_r^2 + \\
& + a_7 \alpha + a_8 \delta_x + a_9 \alpha^3 + a_{10} \alpha^4
\end{align*}
\]
\[
\begin{align*}
C_y^A &= b_{12} \beta + b_{22} \delta_x + b_{32} \delta_{n} + b_{42} \hat{\alpha} + b_{52} \hat{\beta} + b_{62} \hat{\delta}_x + \\
& + b_{72} \hat{\delta}_{n} + b_{82} \hat{\delta}_r
\end{align*}
\]
\[
\begin{align*}
C_z^A &= c_0 + c_1 \beta + c_2 \delta_x + c_3 \delta_n + c_4 \delta_r
\end{align*}
\]
\[
\begin{align*}
C_m^A &= d_0 + d_1 \alpha + d_2 \delta_x + d_3 \delta_{n} + d_4 \hat{\alpha} + \\
& + d_5 \hat{\beta} + d_6 \hat{\delta}_x + d_7 \hat{\delta}_{n} + d_8 \hat{\delta}_r
\end{align*}
\]
\[
\begin{align*}
C_n^A &= e_{12} \beta + e_{22} \delta_x + e_{32} \delta_{n} + e_{42} \hat{\alpha} + e_{52} \hat{\beta} + e_{62} \hat{\delta}_x + \\
& + e_{72} \hat{\delta}_{n} + e_{82} \hat{\delta}_r
\end{align*}
\]
where the coefficients \( a_1, a_{21} \ldots \) generally are depending on Mach number and, by definition [8]
\[
\alpha = -\arctan(v/u), \quad \beta = \arctan(w/u).
\] (18)

In addition, the following notations are used for the non-dimensional angular velocities:
\[
\dot{\alpha} = a \dot{l}/V; \quad \dot{\beta} = b \dot{l}/V; \quad \dot{r} = r \dot{l}/V;
\] (19)
and for the non dimensional and non stationary attack and sideslip angles:
\[
\hat{\alpha} = \hat{a} \dot{l}/V; \quad \hat{\beta} = \hat{b} \dot{l}/V.
\] (20)

Where \( l \) is reference length – body length.

In our case, for low subsonic flow, the coefficients \( a_1, a_{21} \ldots a_{10} \) are practically constant, having the following values:
\[
\begin{align*}
a_1 &= -0.53; \quad a_{21} = 4.7; \quad a_{22} = -0.83; \quad a_6 = -6.5; \\
a_7 &= -19.6; \quad a_8 = -6.5; \quad a_9 = -2.5; \quad a_{10} = 6.7; \\
a_{11} &= -32.1; \quad a_{12} = 963.5; \quad b_{12} = 14.1; \quad b_{42} = 4.5; \\
b_{52} &= -6.5; \quad b_6 = 1.83; \quad b_{62} = 0; \quad b_{10} = -7.57; \\
b_0 &= -1.34; \quad b_{11} = -99.6; \quad b_{41} = -1.8; \quad b_{51} = 6.47; \\
b_{91} &= 0.0; \quad b_{13} = 35.2; \quad b_{14} = 952.0; \quad c_1 = -26.43; \\
c_5 &= -0.2; \quad c_6 = 1.62; \quad c_{13} = 0.054; \quad c_7 = -0.21; \\
d_0 &= 0.179; \quad d_{11} = -5.29; \quad d_{41} = -11.6; \quad d_{51} = 6.63; \quad d_{91} = 0.0; \quad d_{12} = -3.63; \quad d_{42} = -5.5; \\
d_{52} &= 6.63; \quad d_6 = -0.7; \quad d_{92} = 0.0; \quad d_{10} = 6.3.
\end{align*}
\] (21)

4.1.4 Thrust
The propeller thrust is determined by the relation:
\[
T = T_0 C_s^T
\] (22)
Where \( T_0 \) is the nominal value at ground, a fix point, and \( C_s^T \) and axial gas -dynamic coefficient.
Fashioning experimental results indicated in work [9] we obtain the following approximate relation:
\[
C_s^T = f_1(M) f_2(z_p) f_3(\delta_T)
\] (23)
where the influence of the main parameters was separated:
Mach number \( M \):
\[
f_1(M) = 1 - 3.0 M
\] (24)
Altitude \( z_p \):
\[
f_2(z_p) = 1 - 9.1 \cdot 10^{-5} z_p
\] (25)
Thrust command \( \delta_T \):
\[
f_3(\delta_T) = -0.677 + 1.677 \delta_T
\] (26)

Thrust command is limited between: \( 0.5 < \delta_T < 1 \). It is obvious that for the null velocity (fixed point flight) at ground level with maximum command the thrust takes the nominal value: \( T_0 = 563[N] \).
Similarly, we can obtain specific fuel consumption, by using relation:
\[
C_{sp} = C_{sp0} g_1(M) g_2(z_p) g_3(\delta_T)
\] (27)
where the Mach dependence is:
\[
g_1(M) = 1.026 - 0.133 M + 0.966 M^2
\] (28)
Altitude dependence:
\[
g_2(z_p) = 1 - 8. \cdot 10^{-6} z_p
\] (29)
Thrust command influence:
\[
g_3(\delta_T) = 1 + 0.3(1 - \delta_T)^2
\] (30)
The nominal specific consume at ground level with maximum command, corresponding with \( T_0 \) has the value:
\[
C_{sp0} = 1.59 \cdot 10^{-7} [\text{Kg/N/s}]
\] (31)

Using specific fuel consumption, we can evaluate fuel debit that coincides with mass variation, as we see in the equation (8).

4.1.5 Guidance parameters
For flight control system we use the following guidance gains:
\( k_u^u = 2.2; k_u^b = 1.69; k_u^c = 0.237; k_u^d = 0.637; \)
\( k_y^0 = 0.322; k_y^y = 1.679; k_y^v = 0.0762; \)
\( k_y^h = 0.0511; k_y^g = 0.061; k_f = 0.0064. \)

(32)

4.2 Calculus algorithm
The calculus algorithm consists in multi-step method Adams' predictor-corrector with variable step integration method: [1] [11]. Absolute numerical error was 1.e-12, and relative error was 1.e-10.

4.3 Calculus test case
We will consider as a calculus test the situation when the UAV takes-off, makes a rectangular path with four turns maintaining velocity and altitude flight, followed by a descending phase. During the flight, after the second turn, it attends a turbulent zone. Crossing the turbulent zone, UAV uses the guidance command system, in order to maintain flight parameters. Turbulence zone was design according with work [5]. Flight parameters are typical for surveillance activity: altitude \( z_{pd} = 200 \text{ m} \), and axial velocity \( u_d = 44 \text{ m/s} \).

4.4 Results
In Fig. 3 we showed the flight-path diagram, the test situation when the UAV made a rectangular path with four turns. Fig. 3 shows the velocity diagram during the test flight described above.
In Fig. 7 and 8 are show the attitude angles: bank angle and azimuth angle, during the flight.

In all diagrams, except flight –path diagram and azimuth diagram on can observe the influence of turbulence zone on flight parameters.

5 Conclusions

The conclusions are structured in two points as the following.

Guidance scheme: A first conclusion regarding the guiding scheme consists in the fact that the UAV will have a simple but robust command structure which is capable to lead the UAV on desired flight-path in different atmospheric conditions. From the diagram, previously presented, one can observe that the flight parameters after the turbulence zone come back to the normal values. From this point of view we must define a number of evolutions that will make several guiding structures, which will be dedicate to each kind of evolutions. This part of the command structure will be able to evolve in the same time with the development of the project, when the experimental results will be available.

Technical solution: Regarding the adopted, solution using unusual tail having two consoles arranged in the shape of the letter "V" inverted, instead of regular vertical and horizontal tail, we can obtain a better stability and at the same time a better control without increasing the deflection attach angle due to upstream wing.

References: