On the Determination of the Uncertainty in Environmental Noise Measurements

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Abstract: -In the paper is presented an effective computer code developed for real-time evaluation of confidence intervals associated to equivalent levels measurements in environmental noise measurements. The code has been developed using the application of bootstrap theory [1] and uses as input the equivalent level $L_{eq}$ or the quantile levels $L_q$ data obtained by sound level meter measurement. The code, developed using the Mathematica 4.0 programming language is valid for independent and identically distributed data obtained by environmental noise measurements (i.e. in many cases of traffic noise), allowing to obtain directly, once the confidence level probability is chosen, the upper (sup) and lower (inf) limits of the confidence interval by analyzing the n elements of the data set acquired. In this paper, furthermore, is shown the theoretical basis of the proposed algorithm.

Key-Words: - Environmental Noise Measurements, Leq, Uncertainty, Bootstrap Method.

1 Introduction

It is well known that environmental noise levels can vary over a wide range as a result of the diversity of site conditions and activities occurring during field measurements. Environmental noise very often occurs in the form of randomly fluctuating sound signals. To quantitatively describe this phenomenon, noise indices such as equivalent pressure level $L_{eq}$ and quantile levels $L_{q}$ are widely used and are expressed in decibels relative to a reference pressure of 20 $\mu$Pa. In practice when performing a measurement with a sound level meter, the acoustic sound pressure level $p(t)$ is transformed into a discrete set of equivalent levels $L_{st}(t_i)$ gived by $L_{st}(t_i)=20 \log_{10} \left( \frac{p(t_i)}{p_0} \right)$, where $p(t_i)$ denotes the time average of the absolute value of $p(t)$ in the interval $[t_i, t_i+\tau]$ and $p_0$ the reference pressure. Generally, an experiment designed to determine the value of a parameter $L_{true}$ will do this by applying an appropriate transformation to the measured data set $D_0$. The obtained value for the parameter $L_{0}$ for this data set will probably differ from the true one due to the effects of the errors throughout the experiment chain and in the physical phenomenon under study. In most physical experiments there will be a random component affecting the data set so that even repeating it under identical stationary conditions, which can be viewed as re-extracting from the distribution describing the physical measurement $D$, different data set realizations $D_i$ will be formed.

A number of authors have already made significant contributions in this field. Craven and Kerry [2], for example, recommends the uncertainty budget method for estimating uncertainties in environmental noise measurements. The separate uncertainties associated with each of the variables affecting the measured noise level are added together to derive a combined overall uncertainty. Because of limited time and resources, each component of the overall uncertainty must normally be estimated based on scientific judgment or practical experience rather than be determined from the results of a large set of repeated observations. Craven’s approach follows the recommendations of the “Guide to the expression of uncertainty in measurements (GUM ISO Publication 6461-3:1995) which states that uncertainty estimations could be obtained based either on a professional judgment or on real data. Indeed other authors have also adopted this methodology to estimate uncertainties in noise prediction methods.

On the other hand Farrely and Brambilla [1] introduced a methodology to calculate the uncertainty based on bootstrap method. This method was proposed by Bradley Efron in 1979, with the purpose to calculate the standard error of a parameter of a population. In few years, this procedure has had a rapid evolution and now is one of the most used technique of re-sampling of data set.

2 Problem Formulation

Because of the difficulty in the application of the norm ISO IEC 13005 for the determination of the uncertainty in environmental measurements, a different approach has been used by using the bootstrap method.

In the present paper is described the method and the way to apply it to the measurement of the environmental noise, with the purpose to evaluate of the confidence limits of the measure of specific parameters like the quantile levels, $L_q$ and the sound pressure equivalent levels, $L_{eq}$.

It needs nevertheless to underline that the method is valid for independent and identically distributed data set, but it
can simply be applied to the treatment of dependent data set.

2.1 The bootstrap method
The bootstrap is used to say with how much accuracy a certain statistic \( s(x) \) calculated on data observed represent the corresponding quantity reported to the whole population. The problems of statistic inference involve the respect of some aspects of the distribution function of the probability \( F \) of the population under observation, on the base of the empirical distribution function \( \hat{F} \). The \( \hat{F} \) is an enough statistic of the \( F \); this means that all the informations concerning \( F \) contained in \( x \) are also contained in \( \hat{F} \). The bootstrap is a direct application of such principle, known as plug-in principle; the hypothesis is that the data \( x \) has been produced through a casual sampling on the base of the distribution \( F \).

In the paper we’ll indicate with the symbol \( \sim \) the quantities obtained by the observed data.

If the champion is formed from \( k \) independent data, the idea is to extract from it by using simple sampling with repetition many champions of \( k \) observations, to the purpose to find the probability that the measure falls inside a predetermined intervals.

The bootstrap sample is the native sample in which, because of the extraction with repetition, some data are repeated and others, to maintain the same number of observations, are absent. Each of these laces of \( k \) observations can contain two or more identical values, with the obvious exclusion of other values that are inside the original sample. They are called bootstrap samples and each of them allows to get an evaluation of the desired statistic.

In order to give a methodology it is possible to state the following steps:

Starting from an observed dataset \( x=(x_1, x_2, \ldots, x_k) \), obtained in random way from a population with unknown probability distribution function \( F \), we need to estimate the \( \theta=t(F) \) parameter. With this purpose we calculate an probability distribution function \( F \), we need to estimate the population of \( n \) laces of \( k \) data.

The bootstrap method allows to estimate the standard error of \( \hat{\theta} \), in order to obtain information on its accuracy. By using a random sampling with repetition, we extract from \( x \), \( k \) data making \( n \) bootstrap samples, each of them formed of \( k \) elements. In this way we can associate to each element of the observed sample a probability of \( 1/k \) to be extract and then the empiric \( \hat{F} \) function is a discrete distribution. The random sampling procedure guarantees the independence of data. In these way it is possible to construct a virtual population of \( n \) laces of \( k \) data.

The bootstrap assessment \( se_F(\hat{\theta}) \) (the standard error of the statistic) is a plug-in value that uses the function of empirical distribution \( \hat{F} \) instead of \( F \), that is not known. In particular the bootstrap assessment of \( se_F(\hat{\theta}) \) is defined as \( se_F(\hat{\theta}^*) \). The quantity \( se_F(\hat{\theta}^*) \) is defined as the ideal bootstrap value of the standard error of \( \hat{\theta} \).

In conclusion after calculating the bootstrap replications of \( \hat{\theta} \) (the \( \hat{\theta}^* \)) we calculate the standard error of \( \hat{\theta} \) by calculating the empirical standard deviation of the replications of the \( n \) bootstrap data set:

\[
\hat{se}_n = \sqrt{\frac{1}{n-1} \sum_{b=1}^{n} (\hat{\theta}^*(b) - \hat{\theta}^*)^2}
\]

Where

\[
\hat{\theta}^*(b) = \frac{\sum_{i=1}^{n} \hat{\theta}^*(b)}{n}
\]

By increasing the generated bootstrap samplings number \( n \), the empirical standard deviation stabilizes to a limit value:

\[
limit_{n \to \infty} \hat{se}_n = se_F(\hat{\theta}^*)
\]

That represents the bootstrap assessment of \( se_F(\hat{\theta}) \).

The ideal bootstrap value \( se_F(\hat{\theta}^*) \) and its approximation \( \hat{se}_n \) are called many times non parametric bootstrap assessments.

In order to determinate how many bootstrap replications are necessary to consider to obtain an accurate value of the standard error we consider the coefficient of variation of \( \hat{se}_n \) defined by:

\[
CV(\hat{se}_n) = \sqrt{\frac{\text{var}(\hat{se}_n)}{\hat{se}_n^2}} + \frac{E(\hat{\Delta}) + 2}{4n}
\]

In which \( \hat{\Delta} \) is \( \hat{m}_4 / \hat{m}_2^2 - 3 \) and \( \hat{m}_i \) is de \( i \)-th moment of the bootstrap distribution of \( s(x^*) \).
The bias value is defined as the difference between the expected value of the estimator $\hat{\theta} = s(x)$ and the quantity to estimate $\theta$.

$$bias_p = bias_p(\hat{\theta}, \theta) = E_p(s(x)) - \ell(F)$$

and

$$bias = E[s(x*)] - \ell(\hat{F})$$

For the calculation possible to approximate the expected value $E_p(s(x*))$ with the mean value $\hat{\theta}^*(\cdot)$

$$E_p(s(x*)) = \hat{\theta}^*(\cdot) = \frac{1}{n} \sum_{l=1}^{n} s(x^*_l)$$

The bootstrap assessment of the $bias_n$ based on the $n$ replications is:

$$bias_n = \hat{\theta}^*(\cdot) - \ell($$

In order to give a better approximation of $bias_p$ in this study the re-sampling methods has been adopted [4].

This method can be applied when $\hat{\theta}$ is the plug-in assessment $\ell(\hat{F})$ of $\theta = t(F)$. With this purpose we define a re-sampling vector $P^* = (P1^*, P2^*, \ldots, Pk^*)$ with components defined as $Pj^* = \#\{x_l^* = x_j\}/k$ with $j=1, 2, \ldots, k$. A bootstrap replication $\hat{\theta}^* = s(x^*)$ can be guess as a function of the re-sampling vectors it is possible to calculate their mean value:

$$\overline{P^*} = \frac{1}{n} \sum_{l=1}^{n} P^*_l$$

The best bootstrap assessment of the bias is:

$$bias_n = \hat{\theta}^*(\cdot) - T(\overline{P^*})$$

And the corrected estimator is:

$$\overline{\theta} = \hat{\theta} - bias$$

In which $bias$ can be set equal to $bias_n = \hat{\theta}^*(\cdot) - \hat{\theta}$ and then:

$$\overline{\theta} = 2\hat{\theta} - \hat{\theta}^*(\cdot)$$

### 2.2 The confidence interval

Often the standard error is used to give a confidence interval of the considered $\theta$ parameter. Given an estimator and valued the standard error, the confidence interval for $\theta$ is:

$$\hat{\theta} \pm z^{(\alpha)} \cdot \hat{s}e$$

In which $z^{(\alpha)}$ is obtained by the normal distribution.

For the confidence interval calculation several method are used like the bootstrap-t, the BCA (Bias Corrected and accelerated), the ABC (Approximate Bootstrap Confidence).

In the developed computer code the bootstrap method has been used. By using this method it is possible to remove the hypothesis of normal distribution of data.

After generating the $n$ bootstrap samplings $x1^*, x2^*, \ldots, xn^*$, for each one the $z^*[b]$ value is calcolo as:

$$z*[b] = \frac{\hat{\theta}*[b] - \hat{\theta}}{\hat{s}e*[b]}$$

In which $\hat{s}e*[b]$ is the assessment of the standard error of $\hat{\theta}*[b]$ for the bootstrap sample $xb^*$. The $\alpha$-th percentile of $z^*[b]$ is estimate from the value $\hat{z}^{(\alpha)}$ t.c.

$$\{z*[b] \leq \hat{z}^{(\alpha)}\}/n = \alpha$$

In conclusion the confidence interval obtained from the bootstrap-t method is:

$$[\hat{\theta} - T^{(\alpha/2)} \hat{s}e, \hat{\theta} + T^{(\alpha/2)} \hat{s}e(\theta)]$$

This method is particularly effective when as estimator we choose a position parameter as the mean value, or the median or the percentile levels.

### 3 The application of the bootstrap method to the environmental noise measuring

More than few difficulties are been found in the application of the norm CEI 13005, with the purpose to express the uncertainty connected to the measurement of environmental noise. First of all it necessary to observe that phonometric measurements are effected mainly in the open space, where environmental conditions are extremely varying in the space and in the time and in frequencies of the acoustic signal. Insofar the uncertainty connected to the attenuation of the sound during the propagation in external environment is not well definable and therefore is not simple to give an assessment of it.

For these reasons a different approach based on the bootstrap method applied to the measured acoustic levels is proposed and a computer code has been developed in Mathematica 4.0 programming language. The code allows the automatic calculation of the uncertainty related to a certain observation period.

The basic consideration is that the environmental noise is made of several independent signals generated by many acoustic sources. Due to the uncorrelated nature of these sources a statistical representation of it is possible. The noise can be considered as a multidimensional aleatory variable. For the acoustical characterization of the phenomenon, once the time constant desired slow, fast or Impulse) parameters as continuous equivalent level Leq and percentile levels $Lq$, referred to a time interval $T = n \tau$, can be used.
Pointing out with \( D \), which there is a certain probability to have the “true” value (upper limit) \( e \ L(p)_{\text{inf}} \) (lower limit) which delimit a range in which there is a certain probability to have the “true” value \( L_{\text{true}} \).

Pointing out with \( D \), the measured data set it is possible to evaluate both the \( L_{\text{eq}} \) and the percentile levels \( L_{qi} \). The value resulting from the measurement is \( L_0 \) will be, in general, different from the corresponding \( L_{\text{true}} \) due to the errors associated to the measurement process. Each \( D_i \) set data can be viewed as a realization of the acoustic aleatory phenomenon; for this reason it is possible consider \( L \) as a aleatory variable and \( L_0 \) as an extracted value from a set of possible data for \( L \). The distribution of \( L_{eq} \) is an assessment of \( L_{true} \).

If \( D \) has a finite variance ad its data are independent, for the central limit theorem, the distribution of their mean value approaches to a normal distribution with the increasing of the number of elements of \( D_0 \). This simplify the determination of the confidence limits for \( L_{eq} \) by using the standard error. On the other hand the procedure for the \( L_{qi} \) determination in general does not converge rapidly to a normal distribution and, for this reason, it is necessary to apply the a Monte Carlo simulation process in order to generate a number \( m \) of data \( D_i \) sufficient to extrapolate the respective \( L \).

The sound level meter measure the sound levels (\( L_{eq} \) or \( L_{qi} \)), the measured data give the time series \( D_0 \), from which it is possible to obtain the desired \( L_0 \). The \( k \) data of the \( D_0 \) series are assumed as assessment of the distribution required from the Monte-Carlo method for the generation of the \( n \) data of \( D \), each one constituted of \( k \) elements.(Fig.2)

The final purpose of the proposed computer code is to calculate to these levels two confidence limits \( L(p)_{\text{sup}} \) (upper limit) e \( L(p)_{\text{inf}} \) (lower limit) which delimit a range in which the \( h \)-th (the greatest or the lowest, \( L(p)_{\text{sup}} \) or \( L(p)_{\text{inf}} \)) element in the \( L \) data set. The \( h \) value is determined from the probability associated to the confidence level:

\[
h = \frac{1}{2} p \cdot n
\]

Finally the confidence interval amplitude is calculated as

\[
DL(p)=L(p)_{\text{sup}} - L(p)_{\text{inf}}
\]

### 3.1 An application example

The developed computer code read the short equivalent levels \( L_0 \) valuated on a time interval of 1/8 sec, from an input text data \( D_0 \) of \( k \) dimension and first calculates the equivalent level \( L_{eq} \) and the quantile levels required \( L_{qi} \):

\[
L_{eq} = 10 \log \left( \frac{1}{k} \sum_{j=1}^{k} 10^{L_{eqj} / 10} \right)
\]

Starting from the \( D_0 \) data set with a repeated simple sampling it generates \( n \) array \( (D_0^1, D_0^2, \ldots, D_0^n) \) each one of \( k \) elements and it calculates \( k \) \( L_{eq} \) and \( L_{qi} \).

On the basis of bootstrap repetitions of the \( k \) \( L_{eq} \) and of the \( k \) \( L_{qi} \) it calculates the mean value and the middle quadratic discard and subsequently the best bias value of the \( L_{eq} \) in the following manner; starting from the \( D_0 \) data set and from the bootstrapped \( nxk \) matrices of the short \( L_{eq} \) are generate the resampling vectors for the calculation of the \( L_{eq} \) corresponding to each string, by using

\[
L_{eq} = 10 \log \left( \frac{1}{k} \sum_{j=1}^{k} \sum_{i=1}^{n} 10^{L_{eqij} / 10} \right)
\]

In which \( \text{Leq} \) represents the vector of the \( n \) equivalent levels calculated for each string, \( \text{V} \) is the \( nxk \) matrix of the resampling vectors and \( v \) is the vector defined in the following manner:

\[
v_j = 10^{L_{eqj} / 10}
\]

with \( L_{eq} \) the measured values.

The \( \text{Leq} \) best bias is calculated by substracting from the mean values of the vector components of \( \text{Leq} \) the 10\( \log (\text{V} \cdot v) \) value.

For example we consider a sample of 32 elements obtained from a real environmental measuring made by the Department of Mechanical Engineering of the University of Salerno [6].

Dataset = \{ 5.32, 70.15, 71.97, 68.98, 64.38, 67.59, 71.82, 71.87, 62.91, 70.16, 72.88, 70.86, 70.95, 76.79, 72.27, 71.75, 71.71, 74.83, 68.62, 71.88, 72.46, 62.99, 70.74, 69.45, 68.3, 57.08, 68.83, 73.45, 71.88, 73.98, 73.48, 76.79, 72.27 \}.

We calculate:

\( L_{eq} = 71.2 \text{ dB} \)
\( L_{50} = 74.8 \text{ dB} \)
\( L_{90} = 63.0 \text{ dB} \)
After the code generates 25 bootstrap samplings and from each one it calculates the $L_{eq}$, $L_{5}$, and $L_{90}$:

{71.51, 70.28, 70.63, 71.09, 71.58, 71.17, 71.74, 70.94, 70.96, 71.41, 70.94, 69.11, 71.68, 70.51, 71.97, 70.89, 71.66, 70.57, 71.47, 71.04, 71.07, 71.75, 71.37, 71.50, 70.46}

$L_{5}$:

{76.79, 72.88, 73.48, 74.83, 74.83, 73.98, 76.79, 76.79, 73.45, 74.83, 74.83, 72.27, 76.79, 73.48, 76.79, 74.83, 74.83, 73.45, 74.83, 76.79, 73.98, 76.79, 74.83}

$L_{90}$:

{57.08, 65.32, 64.38, 62.99, 62.91, 62.99, 62.99, 64.38, 64.38, 62.91, 62.99, 62.91, 62.99, 67.59, 57.08, 62.99, 62.91, 57.08, 57.08, 65.32, 67.59, 64.38, 62.99, 57.08}.

We obtain:

<table>
<thead>
<tr>
<th>Index</th>
<th>Mean quadratic discard</th>
<th>Mean value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{eq}$</td>
<td>0.61</td>
<td>71.09</td>
</tr>
<tr>
<td>$L_{5}$</td>
<td>1.46</td>
<td>74.93</td>
</tr>
<tr>
<td>$L_{90}$</td>
<td>3.10</td>
<td>62.57</td>
</tr>
</tbody>
</table>

Table 1: Mean quadratic discards and mean values of the dataset.

and

<table>
<thead>
<tr>
<th>Index</th>
<th>Bias</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{eq}$</td>
<td>-0.097</td>
</tr>
<tr>
<td>$L_{5}$</td>
<td>0.106</td>
</tr>
<tr>
<td>$L_{90}$</td>
<td>-0.421</td>
</tr>
</tbody>
</table>

Table 2: Bias of the dataset.

For the best bias estimation the re-sampling $L_{eq}$ vector and the re-sampling $\bar{V}_{j} = \frac{1}{n} \sum_{i=1}^{n} V_{ij}$ vector has been calculated:

$\bar{V} = \{0.0375, 0.02, 0.03, 0.03375, 0.03125, 0.035, 0.02, 0.025, 0.035, 0.035, 0.03875, 0.02875, 0.035, 0.0325, 0.04375, 0.03375, 0.03125, 0.02875, 0.03375, 0.03125, 0.0325, 0.02625, 0.03125, 0.02875, 0.035, 0.02875, 0.035, 0.065, 0.02125, 0.025, 0.035\}$.

The best bootstrap bias is -0.0034.

With the purpose to obtain the confidence limits it is now estimates the probability distribution associated to the generic analyzed parameter $L_{o}$ by using the bootstrap-t method as discussed in chapter 3.

In the case of the showed example the confidence intervals are for $L_{eq}$ -2.10 and 1.24; for $L_{5}$ -3.42 e 1.58; and for $L_{90}$ -1.9 and 1.9.

4 Conclusion

In this paper is an effective computer code developed for real-time evaluation of confidence intervals associated to equivalent levels measurements in environmental noise measurements presented. The code has been developed using the application of bootstrap theory and uses as input the equivalent level $L_{eq}$ or the quantile levels $L_{qi}$ data obtained by sound level meter measurement. The code, developed using the Mathematica 4.0 programming language, is valid in all cases in which it is necessary to calculate the limits of the confidence interval in a environmental noise measurements data set of independent and identically distributed data.

A numerical example provide to show the obtained results operating on a representative short data set. Of course the code allows to operate automatically on a numerous data set obtained from a long time measurement operation, and allows to give judgment of the quality of the environmental investigation without apply the classical uncertainty theory.

Acknowledgements

This work was spring up from the work made by ing. Evelyn Messone in the Degree Dissertation titled “Determinazione dell’Incertezza in Misure di Rumore Ambientale utilizzando il Metodo Bootstrap: un’applicazione al caso di Traffico Veicolare” (in Italian, June 2008).

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