“Diathlassis”, for flute solo: a composition based on an application of the mathematical model of cusp catastrophe

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Abstract: - “Diathlassis” for flute solo is a music composition based on an application of a mathematical model from catastrophe theory: the cusp catastrophe. The initial inspiration for this piece came from the optical phenomenon of refraction (translated in greek: ‘diathlassis’). The exploration of the cusp manifold and, particularly, of its characteristic fold, through lines vertical to the plane of the two control variables, is interpreted by a ‘refracting melody’, due to the mapping of the state variable on the musical parameter of pitch. Timbral experiments and the corresponding notational findings are the result of the mapping of one control variable to the musical timbre.

Key-Words: - catastrophe theory, cusp catastrophe, refraction, algorithmic composition.

1 Introduction
Physical phenomena as a trigger of artistic inspiration is an idea that has always been present in the history of art. The approach of these phenomena from a scientific aspect has been a widely spread notion among composers during the last decades. “Diathlassis” for solo flute is a music project inspired by the optical phenomenon of ‘refraction’ (translated in greek: ‘diathlassis’), materialised through the application of a mathematical model of catastrophe theory: the cusp catastrophe.

The mapping used in this implementation is the result of an aesthetic evaluation of the expressive potential that appears in the natural (physical) reality of refraction and in the cited mathematical model, a choice made mainly through the filter of artistic intuition.

2 The construction of the work.

2.1 The cusp catastrophe model
The mathematical formula that describes the catastrophic manifold of the cusp model is:
$$t^3 + Yt + X = 0 \ (1),$$
where X and Y are the control variables and t is the state variable. This manifold is a folded surface, and the fold’s projection to the (X,Y)– plane indicates the area of the (X,Y) where more than one – and up to three - equilibrium states are possible: the bifurcation set of the cusp catastrophe model (see graph.1).

Graph.1: the cusp catastrophe manifold and the projection of the cusp to the (X,Y) - plane.

As (X,Y) comes ‘inside’ the bifurcation set, there are three distinct values of t for which (X,Y,t) belongs to the cusp manifold. On the ‘branches’ of the bifurcation set these possible distinct values are only two (one simple and one double) and on the cusp point (vertex of the curved triangle) there is only one (triple). The bifurcation set of the cusp catastrophe model is the main area of interest.

The formula (1), for several values of the variable t, leads to a set of equations of straight lines, whose directional coefficients are equal to -1/t. The union of all these straight lines form the catastrophe manifold; their projections on the (X,Y)-plane are the “cusp lines”, whose intersections form the bifurcation set (compare graph.2 to the projected bifurcation in graph.1).
Graph.2: The cusp lines intersect to form the bifurcation set.

2.2 The structural link with the optical phenomenon of refraction (“diathlassis”).

2.2.1 The structural link
The structure of the musical piece “Diathlassis” for solo flute is based on the idea of exploring the cusp manifold’s area of interest – the fold - following trajectories vertical to the (X, Y)-plane, passing through one of the branches of the bifurcation set. Every one of these straight lines is interrupted by the ‘catastrophic leap’ that occurs in the specific position, whose measure varies depending on this position (smaller when closer to the cusp point, bigger as distance from the cusp point increases). The idea of musically exploring the fold of the cusp catastrophe manifold is here associated with following these straight lines vertically to the (X,Y)g-plane through a branch of the bifurcation set.

2.2.2 The “triggering” variable.
A variable t* is used in this mapping as a “trigger”, to cause the cited trajectories vertically to the (X,Y)-plane. For t* = 0, by solving the system of the equations (1) and (2) the result is X = 0. Substituting this value of X in (1) leads to the formula Y = ±t² (3). This represents a continuous trajectory with two branches, like shown in the graph.4:

For all t*≠0, the corresponding graphs present discontinuous trajectories, hence a gap that varies, as mentioned, correspondingly to the distance form the cusp point. For every specific value of the trigger t*, a couple of coordinates (X*,Y*) on a branch of the bifurcation set can be calculated by solving the system of the equations (1) and (2). By substituting the found values of X* and Y* to the formula (1), there can be found the second value t*’ of t that applies at this edge of the fold. Hence, a straight line coming vertically to the (X,Y) – plane through (X*, Y*, t*) performs a catastrophic leap to the point (X*, Y*, t*’) – or vice versa.

3. The mapping to musical parameters.
The work is built in a 4-dimensional space (X, Y, t, t*). The calculations needed were all made in the programming environment of Matlab 7.2 (The Mathworks, Natick, MA)

3.1 The 4-dimensional space of the work.
The triggering variable t* is mapped to the 12-tone. A 12-tone series is used in a macroscopic aspect, creating 12 sections in the piece; every section
beginning with the trigger-note. The state variable t of the cusp model is mapped to the musical parameter of pitch. This results to a melodic discontinuousness, correspondingly to the measure of the catastrophic leap calculated in every case. The first control variable X is associated with the rhythmic structure of the piece. It is used as a factor of rhythmic fragmentation. The second control variable Y is mapped to the timbre, which is considered to evolve continuously from pure air to normal flutistic sound, passing through all the intermediate stages and from normal sound to an amalgam of normal sound with singing notes, using several stages of dynamical balance between the two combined sounds.

3.2 Issues of musical notation emerging from the specific needs of the parametrization. As mentioned about the mapping of the control variable Y to the musical parameter of timbre, intermediate stages of aeolian sounds, “aeolian mixes”, considered as a mix of normal sound with air noise, are used in the piece and presented in a scale, gradually transforming from breathy aeolian to normal sound. Upgoing/downgoing/horizontal dashed lines over the notes, when they exist in the score, indicate the gradual increase/decrease/preservation – respectively – of the portion of normal sound in the timbral mix (upgoing line from air tone to normal sound and vice versa). When no line is present, the change is not gradual, it is done non-continuously. The intermediate stages are indicated with: (a) the number 0 for the beathiest aeolian (b) the fractions 1/5, 2/5, 3/5, 4/5, for the next four stages of this timbral scale and (c) the number 1 when reaching normal sound. Aeolian sounds are indicated with rectangular shaped noteheads. (See graph.4)

Graph 4: notation for the “aeolian mixes”.

A similar notation is used to indicate the proportions of normal sound and singing note in the “singing amalgams”: relative dynamics of singing notes in comparison to played note is controlled in three stages; (a) singing is dynamically lower than played note – indicated with the number 3; dashed lines under the notes (as are also placed the numbers in this case) indicate possible continuous transitions from one stage to another. Singing notes are indicated with diamond shaped noteheads. (See graph 5).

Graph 5: notation for “singing amalgams”

3.3 The effect of the mapping on the resulting musical texture. For the 11 non-zero values of the triggering variable t* - mapped to the 12-tone – there are resulting eleven corresponding intervals through which the melodic line is respectively refracted. The table 1 shows the 12 triggers and the corresponding (catastrophic) intervals of refraction. The 12th trigger (fa#) corresponds to t*=0 and leads to a continuous trajectory.

Table 1: Triggers and corresponding intervals.

A modal melody is exposed in the first section of the work, representing the continuous bifurcated trajectory that corresponds to t*=0. The formula (3) defines the evolution of this melody; as t (pitch) is unfolded, Y (timbre) is affected. The result is a continuous melody passing through several timbral situations (see graph.6):
Graph. 6: The beginning of the first section: a continuous melody.

In the next 11 sections, similar melodic lines are refracted to another pitch, advancing by a smaller or bigger interval, according to the measure of the catastrophic leap. In every different case, different manipulations are made to enact the encumbering of the continuous melodic movement that is forced to become discontinuous. Examples of such manipulations: (a) persisting on the ‘obstacle’, using repeated notes, bending notes, fluttertongue, until finally managing to get to the other side of the fold. (See graph. 7)

Graph. 7: persisting on an obstacle, until the melody’s forced passage through the discontinuity.

(b) forcing on the ‘dead end’ until the sound dissolves to multiphonics, through which is made the passage to the other side of the fold. (See graph.8)

Graph. 8: forcing the discontinuity through multiphonic sounds.

(c) dichotomy of the melody and parallel trajectories on both sides of the fold, using multiphonic sounds. (See graph. 9).

Graph. 9: dichotomy to parallel trajectories.

Every section displays a texture of a uniform character, as the values of X (rythmic factor) and Y (timbre) remain stable, while only t (pitch) evolutes. Every change of rythmic and timbral elements indicates the opening of the next section. (See example of section change in graph.10).

Graph. 10: Texture change (rythmic, timbral) after the discontinuous interval indicates new section beginning.

4 Conclusion

The work “Diathlassis” for solo flute, inspired by the optical phenomenon of refraction, is an application of the cusp catastrophe model. During the process of this composition, the cusp catastrophe model itself has been a second pole of inspiration, through the particularities encountered in the construction of the mapping, a process that lead to timbral experiments, notational findings and, hopefully, interesting expressive strategies. The flutist Katerina Zenz, whose contribution to the refinement of the notation and of the special timbral techniques was trully valuable, made the world premiere of the work in May 2009, in Munich, Germany.

References: