Links between mathematical models and musical texture in Xenakis's Achorripsis

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Abstract: Exploring the transitional field between a mathematical model and the resulting musical texture is a path to enhance the listening experience and stimulate the creative imagination. In the case of Xenakis Achorripsis, issues as the considered parametrization of the sound, the characteristics of the ‘stochastic texture’, the way that specific numerical choices in the frame of the stochastic models affect the musical texture in detail, are examined to highlight the trajectory from the mathematical model to the musical result and lead to a better understanding of the composer’s creative procedure. Matlab codes for the calculations needed in the construction of the work are offered for further research.

Key-Words: - algorithmic composition, stochastic music

1 Definition of sound units in Xenakis’s stochastic works

One of the fundamental procedures of Xenakis’s compositional work is the definition of sound units. Those sound units are represented by 4-dimensional vectors of the form (c,h,g,u), where c: timbre (instruments family), h: pitch, g: intensity (dynamics) and u: duration.

This choice is an important aspect of the structural concept of Xenakis’s works for many reasons: the physical - scientific approach of music as a sound phenomenon (through an approximation of the physical reality), the parametrization of musical sound as a set of elementary properties (connected to Xenakis’s concept of creation ‘ex nihilo’), the non-temporal character of this parametrization (like a momentary “fingerprint” of a sound, considered as an instantaneous occurrence) – creator of a notion of ‘sound grains’ evolving in time (continuously or discontinuously), are some of those reasons and approaches that could (or perhaps should) affect the listening experience of his music.

2 The stochastic texture

Stochastic music was for Xenakis the solution to the impass where serialism was driven. The idea was to replace the interdependence of the sonic elements in the polyphonic texture of serial music, where he found that complexity was preventing the discerrnation of the voice – leading, with sound masses of totally independent sonic elements, where the statistical mean of movement is the basic structural tool, offered for macroscopic perception.

Most important characteristics of the stochastic musical texture are the non-symmetry of the structure, the non-causality due to not operating with any of the known rules of linear combinations and superpositions, the effect of a sonic mass made of independent elements, where the statistical mean of movement is the basic structural tool, offered for macroscopic perception.

Most important characteristics of the stochastic musical texture are the non-symmetry of the structure, the non-causality due to not operating with any of the known rules of linear combinations and superpositions, the effect of a sonic mass made of independent elements, whose behaviour is controlled with the notion of probability and perceived from a statistical point of view. The non-symmetry, the non-causality and the statistical character are more relevant to the ‘natural’ behaviour (Xenakis often referred to the rain, to the crowds, to cicadas e.t.c.) – a fact based also on the latest scientific results.
3 Over the sonification in Achorripsis

GENERAL ORGANISATION

3.1 Timbre
Defined timbral classes:
I) Flute: Piccolo, Eb Clarinet, Bass Clarinet
II) Oboe: Oboe, Bassoon, Contrabassoon
III) String glissando: Violin, Cello, Double Bass
IV) Percussion: Xylophone, Wood Blocks, Bass Drum
V) String pizz.: Violin, Cello, Double Bass
VI) Brass: 2 Trumpets, Trombone
VI) String arco: Violin, Cello, Double Bass

3.2 Density
Clouds of sounds
Suppose we have a given set of punctual sounds defined in an intensity/height space occurring in a time interval. This group of several randomly distributed notes forms a “note cloud”.

There are two main characteristics of a cloud of notes: density and pitch.

Cloud density
There can be an arbitrary number of clouds, each with its own density ($\mu$), in a composition. We can choose the value $\mu_0$ as an average value for the density of clouds. The density is measured in number of notes per second.

Given the mean surface density of this cloud, the Poisson distribution law provides the probability of having a certain density in a certain region of the intensity/height space:

$$P_\mu = \frac{\mu_0^\mu}{\mu!} e^{-\mu_0}$$

where $\mu_0$ is the mean density and $\mu$ is a certain density.

Explanation of the construction of the density matrix of Achorripsis

In “Achorripsis”, the structure of the work is based on a matrix, whose rows represent the timbral classes and whose columns are duration fragments in the time field. The steps of the construction of this matrix are:
(a) A priori decision of the total duration of the musical work (7 minutes)
(b) Definition of column length: 15 sec. (~ 6.5 measures in which the time signature is 2/2 with half note = MM 52.) This leads to a matrix with 28 (= 7 min./15 sec.) columns. Multiplied with the number of defined timbral classes (7), this leads to a matrix with 196 (=28*7) cells.

(c) A priori choice of a value of average density for the global structure ($\mu_0 = 0.60$)
(d) Application of the Poisson distribution law for integer values of $\mu$ (in “Achorripsis” it is $\mu = 0 : 4$, because for $\mu = 5$ the probability gets zero). This provides the possibility of zero, single, double, triple and quadruple “events”, which must be multiplied by the number of cells (196) to calculate the number of events in each category.

(e) Distributing the calculated numbers of events for each category (zero, simple, double, triple, quadruple) in the 28 cells. This requires another application of the Poisson distribution law, using the mean of “presence” of this specific category in the global structure, calculated from the first application of the Poisson distribution - i.e. for the simple event in “Achorripsis” we get $P_1 = 65$ from the first Poisson, hence the mean presence of the single event in the 28 columns is 65/28 cells = 2.32. The second application of the Poisson law, for $\mu_0 = 2.32$, provides the number of columns where there are 0,1,2,3,4,5,6 occurrences of the single event respectively. Observe: 3 columns with 0 single events, 6 with 1, 8 with 2, 5 with 3, 3 with 4, 2 with 5, 1 column with 6 single events.

This is repeated respectively for the other categories of events.

(g) Definition of the mean number $\delta$ of sound occurrences in a simple event. Xenakis’s choice for “Achorripsis”:
$\lambda = 5$ sounds/measure 26MM for the single event. The numbers in the cages of Achorripsis’s matrix are calculated appropriately according to this mean number.

3.3 Duration

The time (metric) is considered to be a straight line, on which the variations of other components are marked using points. The distance between two points represents the duration.

We consider a set of points on a given segment. The question is: given a set of points on a segment, which is the maximum number of segments of a given length?

The note length is determined by a random generation process which obeys the following exponential distribution:

$$P_x = \delta \cdot e^{-\delta x} dx$$

where $\delta$ is the linear density of points and $x$ is the length of a random segment.

This probabilistic formula gives the probabilities for all the possible lengths when we know the position of a set of points randomly positioned.

One can observe that a larger value will produce on average shorter notes and a smaller density will produce longer notes.
To obtain the probability that the outcome is between two values (say \( t_0 \) and \( t_1 \)), the density function must be integrated, in this case giving:

\[
P(x) = \int_{t_0}^{t_1} \delta \cdot e^{-\delta x} \, dx
\]

The value of \( \delta \) obviously affects both the density of the section and the duration of the events, which is only natural, as the duration of the sound events (time interval between successive occurrences) is an interpretation of the density.

In the example of “Achorripsis”, time intervals between successive events, calculated with the exponential distribution using the appropriate \( \delta \) are divided in five classes (in augmenting succession):

(a) \( \leq 12/52 \) sec (division of the measure in 10 quintuplet eighth notes)
(b) \( \leq 15/52 \) sec (division of the measure in 8 eighth notes)
(c) \( \leq 20/52 \) sec (division of the measure in 6 triplet quarter notes)
(d) \( \leq 30/52 \) sec (division of the measure in 4 quarter notes)
(e) \( \leq 60/52 \) sec (division of the measure in 2 half notes)

‘Duration.m’ is a matlab code for the calculation of durations, given the value of \( \delta \):

function duration()
\[
x = 0:.1:1;
dx=0.0805;
\]

%for the first column of Achorripsis matrix, \( \delta = 3.5 \):
%change the value of delta appropriately;
\[
delta=3.5;
deltax=0:.45:3.5;
\]

for i=1:10
\[
column3(i)=exp(-\delta*x(i));
column4(i)=\delta*column3(i);
column5(i)=column4(i)*dx;
column6(i) = column5(i)* (\delta *6.5-1); %the
\]
end

%open the file with write permission
fid = fopen(‘C:\duration.txt’, ‘w’);

for i=1:10
\[
fprintf(fid, ‘%6.2f %12.8f %12.8f %12.8f %12.8f %12.8f’
x(i), deltax(i), column3(i), column4(i), column5(i), column6(i));
\]
end
fclose(fid);

% view the contents of the file
%type (‘duration.txt’)

For \( \delta = 2.5 \) (cell I – \( \kappa' \) of Acorripsis matrix), the duration matrix is:

The last column returns the number of notes in every duration category (these numbers are used rounded to the closest integer). Compare the sum of those numbers in this matrix with the next two matrices: it is the smaller of all three duration matrices presented (due to the lowest value of density \( \delta \)).

An excerpt of this section of the score (only the timbral class I):

Duration matrix for \( \delta = 4.5 \) (cell III – \( \iota' \) of Achorripsis matrix). Observe the exponential distribution of the numbers notes in the considered categories of durations; the majority of notes seems clearly concentrated in the categories of smaller durations, as the value of \( \delta \) increases:
An excerpt of the score at the cell $\text{III} - \text{I}_3'$:

Duration matrix for $\delta = 20$ (cell $\text{II} - \text{κε'}$ of Achorripsis matrix):

There can be clearly noticed an impressive increase of the sum of notes (241, where the sum of the same column was 12 in the first duration matrix with $\delta = 2.5$), as also of the number of small durations (208 durations $\leq$ eight of quintuplet, where in the first matrix the respective value was 3). Notice the dense presence of small durations in this excerpt of the score at the cell $\text{II} - \text{κε'}$:

3.4 Pitch

The second main characteristic of a note cloud is its pitch. The pitches of the notes in the cloud are determined by a starting pitch along with the intervals between each pair. The intervals are determined according to the following linear probability distribution:

$$\theta (\gamma) d\gamma = \frac{2}{a} \left( 1 - \frac{\gamma}{a} \right) d\gamma$$

where $a$ is the maximum interval value specified by the composer. This law gives the probability that one segment (interval of intensity, melodic, etc) $s$, which is interior in a segment of length $a$, to have a length between $0 \leq \gamma \leq a$.

One can notice the use of an additional simple random variable equivalent to the flipping of a coin. This is used to determine whether the interval is rising or falling. The maximum interval limit helps reduce the production of sequences that are unnatural sounding or difficult to play.

interval.m is a Matlab code for the calculation of pitch changes in ‘Achorripsis’:

```matlab
function intervals()
    % a = 18; % approx. 80/4.5 (80 semitones divided in unities of 4.5 semitones)
    a = 18; % changing every 6.5 measures in Achorripsis
    delta = 3.5; % changing every 6.5 measures in Achorripsis
    for j=1:18
        P(j) = (2/a) * (1-(j-1)/a)*delta;
        L(j) = P(j)*delta*6.5;
    end
end
```

% open the file with write permission
fid = fopen('C: \intervals.txt', 'w');
for j=1:18
```
fprintf(fid, '%6.2f %12.8f %12.8f
', j-1, P(j), L(j));
end
fclose(fid);

Pitch matrix for δ = 4.5 (cell III – Ιζ'). The last column of this matrix is the probability multiplied to 6.5 * δ, in order to distribute the calculated probabilities of each category of intervals to the desired number of events (controled by δ), in the space of 6.5 measures of every cell.

Score excerpt of cell III – Ιζ':


The category j =1 manifests in this excerpt a greater frequency (which is justified by the predicted frequency value).

3.5 Speed

Xenakis considers a categorisation of sound to granular v.s. continuous sounds. Granular sounds are not evoluting in time, in contrast to continuous, whose evolution can be of less or more complexity (continuous sounds are a much bigger set than granular ones, which actually could be considered as a particular case of continuous sounds of a very small duration.) Among these continuous sounds, the ones considered here are the glissandi with a uniform behaviour (speed).

We are going to make an homogenity hypothesis which will lead us to a mathematical formula for the distribution of speeds.

The formula for the speeds is determined according to the following normal distribution formula:

\[ f(v) = \frac{1}{a\sqrt{2\pi}} e^{-\frac{v^2}{2a^2}} \]

where a is the aggregate temperature. The name of this parameter comes from the kinetic gas theory. From a statistic point of view, the above formula defines the standard deviation.

The function \( f(v) \) gives the probability of the occurrence of the v speed, the constant which defines the “temperature” of this sound atmosphere.

Speed.m is a matlab code for the calculation of the speed matrix, using the appropriate value of δ according to the cell of interest:

function speed()
    v = 0:1:10;
    alfa=3.88;
    for i=1:10
        lambda1(i)=v(i)/alfa;
        delta = 6.5; %this is the value of density delta,
        varying in each cell of the row III (string glissandi)
        fi(i)=erf(lambda1(i));
    end
    for i=1:9
        Plambda(i)=fi(i+1)-fi(i);
        column5(i)=Plambda(i)*delta*6.5;%Here I
        substituted 29 (which was 6.5 * delta for delta = 4.5,
        but now
        column6(i)=(v(i)+v(i+1))/2; %here I added the
        average of the first 2 which should also be
        calculated column6(i)=(v(i)+v(i+1))/2;
    end

% open the file with write permission
fid = fopen('C: \speed.txt', 'w');
for i=1:9
    fprintf(fid, '%6.2f %12.8f %12.8f %12.8f %12.8f
');
end
fclose(fid);
For $\delta = 6.5$ (cell III – $\mathcal{I}_3$) the speed matrix comes as follows:

and the second is for smaller $\delta = 4.5$ – observe that there are fewer parts of string div. dedicated to the timbre ‘string glissando’, hence fewer glissando sounds, a result of the smaller value of density):

4. Conclusion

The parametrization of the sound, as much as the choice of the used mathematical (stochastic) models cause a large impact on the macroscopic formation of the musical result encountered in Achorripsis. In the microstructure, the choice of the numerical values applied in the frame of the used stochastic models reveals enlightening details about the sculpting of the musical texture.

References: