The Functional Tonal Musical System and its Mathematical Acoustical Basis

DAN BUCIU
Composition Department
National University of Music, București
Str. Știrbei Vodă nr. 33, Sector 1, București 010102
ROMANIA
dan.buciu@unmb.ro

Abstract: - the tonal musical system is based on a clear acoustic foundation – the component sounds follow each other in the order of a perfect fifth; the chords that sustain these tonal functions are built on a mathematical base (Fibonacci series).

Key-Words: - harmony, acoustics, mathematics.

1. The Functional Tonal Musical System and its Mathematical Acoustical Basis
Approaching such an interdisciplinary subject, the most logical starting point would be by first defining the fields in which the theoretical proof will manifest itself with music on the one hand and physics/mathematics on the other. In order not to unnecessarily lengthen this study we start by enunciating in a simple and clear manner-instead of certain definition that would enviably take us into other areas (aesthetics, philosophy, etc.)-that music is an art but physics and mathematics are sciences. Music and mathematics/physics are two distinct areas in human activities that tend not to intersect because of a very simple rationale: human nature and its extraordinary complexity is built on two fundamental ideals: 1) the sensory element of emotions and feelings by which the human being communicates with its surroundings and other human beings, as well as enriching its own existence by developing important feelings, approaches, and emotional states; 2) the rational element, calculus, intellectual speculation, the element that gave lift to human beings to rise to the top of the biological pyramid, the element by which knows and governs in both the micro as well as the macro, everything that surrounds human beings. The two areas that will remain in our discussion (artistic-music and scientific-physics and mathematics) will provoke two fundamental human components representing, ideally, the sensory and the rational elements.

Complex existence, yet still indivisible, man cannot separate these two components which, in spite of their apparent incompatibility, will inevitably coexist; in other words, the coexistence of subjectivity and objectivity, of strict individualism versus collective being, which further implies an abstract average of everything that humans do, feel, think, act, etcetera. One might say that the way Homo Sapiens knew how to combine these two fundamental elements (in a different manner!) and that savoir faire or “life sciences,” is the capacity of going through biological existence at a level reclaimed by its own special status in the biological world. This essential aspect will be followed not only at the level of daily existence, but also on a higher level in the lofty manifestations of human spirit and feelings (sciences and arts). A propos our original subject, we could quote the famous scientist H. von Helmholtz, “I always felt attracted by the mysterious connection between math and music-the application of the most abstract and logical science at the level of sound, the physical and physiological basis of music, the most immaterial art, the most vapor and tender, the one that makes us experience the most incalculable feelings hard to define.”

The aforementioned quote can represent a departure point for our theoretical journey.

1 Paraphrasing Urnă, Dem., Acustică şi muzică, Editura științifică și enciclopedică București, 1982, pag. 5.
If we avoid to define music (Helmholz already did it admirably) and consider music simply as an “art”, our study of the meaning of tonality requires explanation: “By tonality – largely speaking – one understands the phenomena of gravitation and convergence of sounds, in a musical composition, towards an audio center called tonic.” The same author (in the same book at page 282) then makes the important remark: “The tonality is a specific concept of creation, elaborated and emulated by the high musical art” (authors note the west European music after the sixteenth century).

Attempting a short historical overview of the genesis of tonality, one needs to account for the work involving the ancient medieval Gregorian modes and the diatonic heptatonic modes present in the area of official catholic music as well as secular music. The music based on the aforementioned modal structures utilized these modes not only as musical scales, as simple musical audio “materials” but also established the element of audio melodic functionalism separated into two large steps: the finale (vox finalis) and the ténor.

The Renaissance era brought forth a spectacular development of secular music, which inherently implied the renouncement of the influence of the Catholic Church, namely it’s artistic obligation. The melody followed a more expressive path, unrestricted, and evolving polyphonic elements into a more simple line, in the shape of vertical blocks, yielding more and more clear harmony. Even though the basis will remain modal, the harmonic function will start to include more of a vertical nature alongside it’s melodic aspect. A new type of functionalism arrived, namely the chordal functionalism, where the function’s quality will be given not by the melodic line but by the chord’s harmonic function. The consequences are clear: harmonic functionalism will no longer be tied to only one solution, but will have at its disposal any number of solutions (see footnote 4) and the sounding result will be realized with a three or four note chord and not on one lonely pitch. The harmonic essence of the tonal phenomena is in the result itself; Constantin Rîpă confirms: “Tonality can only be thought of as harmonic.” The father of the functional tonal harmonic system, Jean Philippe Rameau, found himself to be “blinded” by this way of musical thought. Rameau was unable to find an alternative, being completely opposed to other types of musical cultures (for example, more traditional ones, strictly monophonic). „Music divided into harmony and melody, but we will demonstrate that the latter is rather an element of the former and that the knowledge of harmony is enough for a complete understanding of all the properties of music”.

From the above affirmations and confirmed by the quoted opinions mentioned, the result is the foundation of the principle of functionalism, and within the function itself we will find the chord.

What is functionalism? An essential component in the “equation of tonality,” it is the element that defines the tonality and the key. It can be easily defined in this way: a chord on a certain scale degree of a tonal scale has a well established role as part of the tonal mechanism. From where did the establishment of the tonal functionalism begin, who chose the tonal scale, and who “invented” tonality?

There is only one answer: the common practice spread throughout a large period of time, a period in which tonal scales were selected out of the medieval modes and the three tonal functions were born. A more specific explanation can be found in acoustics. Starting from heptatonic diatonic material, one can imagine five theoretical possibilities of building the

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1 Although there could be different points of view! We could talk about Igor Stravinsky’s opinion “Music is an act of human mind which puts order in the world of sound” (paraphrasing Giuleanu, Victor, Principii fundamentale în teoria muzicii, Editura Muzicală, Bucureşti, 1975, pag. 14) which rather sends the music in a scientific zone rather than the artistic.

2 Giuleanu, Victor, Principii fundamentale în teoria muzicii, Editura Muzicală, Bucureşti, 1975, pag. 284

3 As far as the authentic modes are concerned: protus, deuterus, tritus and tetrardus, these scale degrees, I and V (paraphrasing Rîpă, Constantin, Teoria superioară a muzicii, vol I – Sisteme tonale, Editura Media Muzica, Cluj Napoca, 2001, pag. 170-171).

4 In this manner, different functions of the melody will be able to propose different harmonic functions of, for example, the heptatonic mode; the harmonic chords will each support different functions, in this case, I when it is root, VI when it is third, IV when it is the fifth scale degree, or II when it is the seventh. As a melodic pitch, it can also appear on each of the other harmonic underpinnings (VII, V, or III)!  


tonal scale which contain the functional trinity of tonality: subdominant – tonic – dominant. A demonstration will be done by starting with the succession of the seven diatonic pitches, each a perfect fifth apart, the so-called natural order of sounds. These pitches, presented in this order of fifths, will represent “the ascending system of perfect fifths, the source of Pitagora’s scale.”

Fig. 1

Theoretically speaking, there are five ways of building a tonality which has the three functions of: SD – T – D; these would be:

Table 1

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<tr>
<th>SD</th>
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<tr>
<td>1</td>
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<td>2</td>
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<td>3</td>
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<tr>
<td>5</td>
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The question arises: how did we get from two melodic functions (vox finalis and ténor) to three tonal harmonic functions. The answer seems to be rather simple and it explains the role of each chord of the three scale degrees (also known as the principle scale degrees): IV, I, V.

Acoustically, the seven natural pitches generate each other. The starting point is Fa (F), which generates Do, which in turn generates Sol, etcetera. On first impression, Fa (the fourth scale degree) is the most important. But with a more profound analysis we come to the conclusion that Do is actually the main character; it is flanked by Fa and Sol, which apply pressure from two different directions. Fa, by generating Do, will attract it. The same relationship will be born out of the Do-Sol relationship. The balance of the tonic Do will rest amidst the mutual annihilation of the two opposed forces that act upon it.

Fig. 2

In order to obtain “Tonal Functionalism,” it is necessary to build the chords on their respective steps so that the three functions, subdominant (fourth scale degree chord), tonic (first scale degree chord), and dominant (fifth scale degree chord) will gain the necessary harmonic cohesiveness. In the major tonality, the three chords are major.

Starting from an acoustic premise – the pitch which is found a perfect fifth higher was generated by its preceding pitch and is dependent on it, having the tendency of pulling back to the generating element – the tonic chord is attracted by the subdominant chord and the dominant chord by the tonic chord. So far as this relationship (D – T) is concerned, one can notice the fact of the dominant as a scale degree and by being a chord and not just a mere pitch; it contains the leading tone of the key (the chord’s third, seventh scale degree) which, at a half-step distance from the tonic, is strongly attracted to the tonic and will resolve melodically, Si – Do. One should also take into consideration the substantial force of the dominant, which is not alone, having a potential of at least five chords, with each chord built on the second, fourth, third, and seventh scale degrees, being a possible dominant for the chord residing on the preceding step (in the order of fifths!).

It is very interesting to note that in spite of the entire chordal system getting farther away from the tonic center, it still remains strictly “closed” (!) by the powerful relationship established between the furthermore acoustic elements from the Tonic, VII (Leading tone), and I (tonic!) with which establishes a

1 In the natural harmonic sequence, the perfect fifth is the first interval that separates two distinct sounds (between the second and third harmonic) because the octave (created between the first and second harmonic) does not produce anything but the fundamental sound (the first harmonic) in a higher register (an octave higher – the second harmonic).

2 According to Urmă, Dem., Acustică și muzică. Editura științifică și enciclopedică, București, 1982, fig. 63, pag. 130.

3 For examples, we will use the diatonic material of the C Major/A Minor scale (depending on case).
strong communication, the half-step (this time, a subjective element, strictly musical, as opposed to the objective acoustic relationship of the perfect fifth.) The leading tone in minor scales, “suggested” yet missed (all of them!) by the medieval system is an essential element in the functional tonal system. Originally from the Aeolian mode but bearing in mind the functional tonal model of the Major (Do Major, former Do Ionian), the making of the minor key had to accept an essential and absolutely necessary modification: the alteration of a raised half-step on the seventh scale degree, thus obtaining the necessary leading tone to make the fifth scale degree chord (dominant) major.

Fig. 3

As seen from figure 3, the presentation of the material of the La harmonic minor scale (with raised leading tone) leads to the five potential chords for dominant (supported by D2 and...D5, the leading tone) but also leads to the depressive quality of the subdominant (supported by SD3 and SD4, SD2 being replaced by D5!). It is no accident that the musical ethos expressed by the minor key supports rather tense, expressive, less balanced, and depressed musical areas; this aspect will be preferred by the acoustical build of this type of key (with a subdominant potential more powerful than the major key, found in a superior, dominant zone).\(^1\)

Finally, we have objective acoustical support on which the musical concept called “tonality” is based, the acoustic element that explains the functional aspect and the interdependence of the tonal functional relationships.

A propos, the fact that in our tonal system, the relationships between functions (represented by the principle chords or their secondary substitutes) will be fundamentally different on the basis of the acoustics (the relationship of the fifth). The main dependence of the tone found a fifth above the preceding one will make it possible for the tone found a fifth above to have a privileged relationship, qualified as authentic, based on the interval of the fifth down (V – I), as opposed to the opposite less convincing plagal relationship (IV – I, perfect fifth up).

The relationships between chordal functions will in fact create the musical discourse, being essential in the configuration of the tonal schematic of the sound.\(^2\) “The term (author’s note harmonic relationships) implies not only the fact that a chord is followed by another but also that the succession is controlled and orderly”.\(^3\) Starting from the musical sense of each function, the tonal harmony will decide, logically, the paths between the functions, the favorable corridors, the mandatory articulations (cadential formulas) and the seldom taken more difficult paths. At the very foundation of this impressive monument, a climax of musical thought will be the acoustic element as a principle decisive factor.

The functional tonal system (as it was affirmed many times and was confirmed not only by composer’s practice but also by numerous scholars’ opinions) has as a basis the concept/idea of chord. In a traditional tonal conception (but also in the modal-renaissance arena) the chord is a sound formed of at least three different pitches, disposed, in the root position a third apart from each other. In making a reference to the musical role of this audio “product” called a chord, Marțian Negrea offers an interesting opinion: “…the sound has a force that activates in two directions: one horizontal that produces the scale and the melody, and the other vertical that produces the chord, and thusly, the harmony.”\(^4\) This noted scholar quotes the words of Arnold Schoenberg, which, in his Treaty of Harmony, ed. III, pag. 26, second paragraph, declares the following: “If the scale is an imitation of the sound in a horizontal form, the chord is an imitation of the sound in a vertical form” and then “the scale is analysis and the chord is the synthesis of the sound.”\(^5\) Trying to classify these types of chords (of three notes) we can say that the triads formed in a given key will be differentiated “by the intervals from which they are made of, in their sound quality.” There are four types of triads, classified according to the nature of the

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1 The labeling of the minor key as “sad” and the major key as “optimistic and exuberant” must be avoided due to the fact that most major historical composers knew how to surpass, when necessary, these objective acoustic barriers, creating very tense major areas or calm, elegiac, contemplative minor areas, which even sometimes did not lack in humor or joy.

2 By logically extending this concept, the relationships of the third and second will also be authentic (third down, second up) or plagal (third up, second down).


4 Negrea, Marțian, Tratat de armonie, Editura Muzicală, București, 1958, pag. 8-9.

5 Negrea, Marțian, Tratat de armonie, Editura Muzicală, București, 1958, pag. 9.
intervals formed between the fundamental and the other two notes.”¹ These four types of triads² did not all appear at once in general musical practice. Moreover, the types used³ in the archaic forms of polyphony appeared in first and/or second inversion.⁴ The intervals of such chords can lead our thoughts towards Fibonacci’s series⁵, based on component numbers in this series, the major chords (in first inversion) and minor chords (in second inversion) will be formed.

Fig. 4
Based on this predicament, one can observe the coherent organization of the main chords from the tonal system, based on this numeric series.

Fig. 5
From the above example, one can make a logical deduction that the organization of these vertical structures (the chords, to name them tonally⁶) will be executed mostly by using the numerical elements from the Fibonacci series. In the case of the great composer Béla Bartók, we are certain to find a basis in the Fibonacci series within which Bartók created his musical system. This Fibonacci basis influenced the organization of his tonal chordal structure, building a language which has the modal underpinning coexist with a logical implication of traditional tonal elements, adopted together into the new musical requirements found in modal thinking. The nucleus of his famous a⁷ chord comes from superimposing a major chord on top of a minor chord.

Fig. 6
In looking back to “traditional functional tonality” (the type that dominated western European art music from the XVI to the XIX centuries) one can say that the basic elements that define it – functionalism, harmonic relationships, chords – establish close relationships with the scientific areas of physics (acoustics) and mathematics, sciences that either validate (the acoustics) or from which they borrow elements of organization (mathematics). Hugo Riemann’s opinion “takes from the tradition of physics (Jacques Handschin), and goes through to Rameau the concept that the tonality is fundamental in acoustics.”⁸ We took liberty in using the expression “traditional functional tonality” (it could also be called “functional harmony”) because the term tonality, invented by F. J. Fétis in 1844⁹ does not cover other systems which can have connections with the traditional tonal system. “The diversity of the historical and ethical conditions generated many <kinds of tonalities>>.¹⁰ It is not the author’s intentions to open up a Pandora’s box by entering this territory. It is only the wish to point out the fact that the term of tonality can be used for many other systems and it’s definition (at least when it is referred to the European musical thinking of the Baroque, Classical, and Romantic eras) can be different in the views of different well known

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² Major, minor, diminished, and augmented.
³ The major and minor triads.
⁴ Chordal mixtures found in old forms of the gymelus or the faux-bourdon.
⁵ The “measuring” unit 1=1 half-step; thus, second = M2 (major second), third = m3 (minor third), fifth = P4 (perfect fourth).
⁶ There can be other different types of chords, utilized in modern and contemporary music, which imply different constructions and different sonorities.
⁷ Named by the musicologist Ernö Lendvai, the one that “deciphered” for the first time the mechanism of the Bartókian language.
⁹ According to Carl Dahlhaus, this will define it as “a collection of necessary relationships, successive or simultaneous notes of a scale,” see Dahlhaus, Carl, La tonalité armonique Étude des origines, traduit de l’allemand par Anne-Emmanuelle Cenlemans, Pierre Mardaga, éditeur Liège 1993, pag. 7.
musicologists that analyze the same phenomenon (see Fétis-Reimann).
The short conclusions that we wish to draw point to the “natural” element that the tonal system implies. Having acoustic support and using sound material (chords), the tonal system will constitute the logic of a numeric series (Fibonacci) which implies specific proportions (golden section). The tonality will justify its “naturalness” being able to articulate a clear expression, logical yet sensitive. Not by accident, tonality will prolong its existence in the twentieth and twenty-first centuries in symphonic music, chamber music, opera, or choral as well as the areas of jazz and pop. Even though tonality is not any longer the “unique” musical system that is used, it continues to have major importance in the context of the music of today. It is thought that these acoustic-mathematical foundations are not foreign to the impact which tonality has had on music, and thus, on human psyche. This problem stays open; it is the duty of new musicological research to point out new connections that musical art in general and the tonal system in particular have in common with the realm of science.

2. Conclusion

The functional tonal musical system proves to be built and to function based on an acoustic foundation (the natural order of sound). The chords that create the tonal functions have at the very foundation numeric proportions deducted from the Fibonacci series.

References:


1 The tonal system can be found (partially, together with other systems as neomodal, atonal, dodecaphonic etc.) in the modern and contemporary periods in the closed „formula” to Romantic period of the nineteenth century (Rachmaninov, Elgar) or new „formulas”, neotonal (Prokofiev, Hindemith, Richard Strauss) or in different other musics (jazz, pop etc.).