The subtle sets theory (SST) to the modeling socio-economic space *

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Motto: *"It is…easy to be certain. One has only to be sufficiently vague."* *Charles S. Peirce*
* *(sub-ti-li-té ; prononcé suttilité au XVIe s. d'après PALSGRAVE, p. 113).*

Abstract: *The advantage of using the subtle sets is due to the multiple modeling possibilities supplied by the concept of sequence (or queuing) and by the new instrument called "operator of act".*
By using the subtle sets theory (SST) to the modeling socio-economic space, we can reach the experience to allowing the transformation of the "invisible" statistics, into an "apparent" statistics.

Key -Words: *the method of Boolean variables, the fundamental axioms on subtle sets, "operator of act", fuzzy sets, subtle sets.*

1 Introductory Notes on Subtle Sets

1.1 Brief Historian

During the 16th century, the concept of subtlety has been theorized and successfully applied in his work, by the mathematician Gerolamo Cardano[9]. Into the theoretical substantiation of subtlety, he said "Praevident spiritus quod mihi imminent and emphasis on diagnostic analysis and the forecast"[2].

In 1965 in reference works [14], Lotfi Askar Zadeh define the concept of "Fuzzy Sets ", which differs from the crowd, in respect Cantor[13]. Recently, in 1997, Professor Petre Osmătescu [8] continued the spirit of L. A. Zadeh, and defined the notion of subtle sets and insidious concept of subtle space, by using the idea of fiber and by the instrument called "operator of act".

2 Preliminaries

2.1 Assumptions of general studies

Assumed to be defined by *the observer O*, there are a lot of *elements* $\{e_1, e_2, .., e_n\}$, with the *characteristic* S. The elements are systems, subsystems, complex objects, etc. To be: $SYO = \{O\} \cup MO$, where MO is a lot of means of *observation* $SPO = \{e_1, e_2, ..., e_n\} \cup S$, SO is the *supporting* observed.The *global socio-economic S,* could be estimated by O, who provides a series of criteria $f_1^0, f_2^0, ..., f_k^0$. $W_{ik}^0 (k \in \overline{1, p})$ is the *event,* that consists in estimating the consequences a_{ik}^0 of the criterion f_k; the observer O attach the elements $e_i (i = \overline{1, n})$. The characteristic S_0 is as (1):

$$S_0 = \left\{ f_1^0, f_2^0, ..., f_p^0; e_1, e_2, ..., e_n; a_{ik}^0, i \in \overline{1, n}; k \in \overline{1, p} \right\}$$

p = number of the influencing factors(criteria, tests);
e_i = elements under observation; a_{ik}^0 = consequences of criterion k, estimated by O, for elements i.

Both the subtle set and the "global characteristic", will be denoted by S_0. The observer O, can be outside the element $e_{i,}$, or can be part of this. The first case is when an "invisible" statistics is developed, the second one, when an "apparent"statistics has resulted.

2.2 The Sequence Concept

A set of criteria $f_1^0, f_2^0, ..., f_p^0$ can be attached to a characteristic S_0. The criterion f_k^0, involves the *event* $W_k^0 (k \in \overline{1, p})$, that consists in estimating the consequences a_{ik}^0 of the O, for each $e_i (i \in \overline{1, n})$.

The types of consequences are:
Quantitative, where statistical determinations expressed in usual measuring units can be made;
Qualitative, where determinations can not be made, but by *evaluations.*

*T*he consequences have different size orders, this fact making impossible the comparability and the general judgments. According on the nature of phenomenon, two methods are used:

1. The method of membership degrees: e_i can be described by the maximum f_k^0 *entirely* (if a_{ik}^0 max to $i \in \overline{1,n}$), or only *partially*. If e_i has the property f_k^0 *entirely*, the membership degree μ_{ik}^0 is maximum, $\mu_{ik}^0 = 1$. If e_i has the property f_k^0, at minimum, $\mu_{ik}^0 = 0$. If a_{ik} is placed between the min. and the max. level, we have: $\mu_{ik}^0 = \frac{a_{ik}^0 - a_{k0}^{\min}}{a_{k0}^{\max} - a_{k0}^{\min}} \in [0,1]$ (2)

$a_{k0}^{\min} = \operatorname{Min}_i a_{ik}^0$ and $a_{k0}^{\max} = \operatorname{Max}_i a_{ik}^0$

$a_{ik}^0 =$ consequences of element e_i appreciated by O, according to k criterion. In case of a f_j^0 of minimum, the membership degree can be calculated as:

$$\mu_{ij}^0 = \frac{a_{j0}^{\max} - a_{ij}^0}{a_{jo}^{\max} - a_{j0}^{\min}} \in [0,1] \qquad (3)$$

2. The method of ratios: when the consequences of f_q are expressed into the number of favorable cases a_{iq}^0, from the viewpoint of S_0, for e_i. The ratio is (4):

$$r_{iq}^0 = \frac{a_{iq}^0}{a_{q0}^{\max}} \in [0,1], q \in \overline{1,p}; \quad i \in \overline{1,n} \quad a_{q0}^{\max} = \operatorname{Max}_q a_{iq}^0$$

If $a_{q0}^{\max}$ is high, the ratio r_{iq}^0 tends to probability.

3. The method of Boolean variables

The Boolean variables are denoted by δ_{ij}^0, and are calculated by the relation (5):

$$\delta_{ij}^0 = \begin{cases} 0, & \text{if element } e_i \text{ does not have property } f_j^0 \\ 1, & \text{if element } e_i \text{ has entirely property } f_j \end{cases}$$

We can define the concept of sequence.

A set of e_i, which the a-dimensional consequences μ_{ik}^0 have been attached to, included into [0,1], according to a certain criterion of evaluations f_k^0 of a characteristic S_0, by O, *is called sequence*. For the a-dimensional consequences of the μ_{ik}^0 form, according to fuzzy criterion f_k^0, we get a fuzzy sequence of O, upon S_0: $F_{k0}^{fs} = \begin{Bmatrix} e_1 & e_2 \ldots e_i \ldots e_n \\ \mu_{1k}^0 & \mu_{2k}^0 \ldots \mu_{ik}^0 \ldots \mu_{nk}^0 \end{Bmatrix}$ (6)

For the a-dimensional consequences of form r_{iq}^0, according to *frequentist* criterion f_k^0, we get a *frequentist sequence*: $F_{q0'}^{pS} = \begin{Bmatrix} e_1 & e_2 \ldots & e_i \ldots & e_n \\ r_{1q}^{0'} & r_{2q}^{0'} & r_{iq}^{0'} & r_{nq}^{0'} \end{Bmatrix}$ (7)

where $0'(0' \neq 0)$ is a set of observers expanded also to persons, which do not belong to any of the $e_i (i \in \overline{1,n})$.

With the aid of Boolean variables, estimated by (5),(6), we get a *determinist sequence* of observer O (or O'):

$F_{v0}^{bS} = \begin{Bmatrix} e_1 & e_2 \ldots & e_n \\ \mu_{1v}^0 & \mu_{2v}^0 \ldots & \mu_{nv}^0 \end{Bmatrix}$ (8) $F_{v0'}^{bS} = \begin{Bmatrix} e_1 & e_2 \ldots & e_n \\ \gamma_{1v}^{0'} & \gamma_{2v}^{0'} \ldots & \gamma_{nv}^{0'} \end{Bmatrix}$ (9)

2.3 The Subtle Set Concept [10]

According to the preliminary considerations, several versions of subtle set concept can be stated.

Definition 2.3.1 We call a subtle set S_0, a set of fuzzy and/or of frequentist/probabilistic and/or determinist (Boolean) sequences of O, upon a characteristic S_0:

$S_0 = \{ F_{k0}^{fS}, F_{q0}^{fS}, F_{vo}^{bS} \bigvee F_{vo'}^{bS}, \ldots \}$ (10)

where sign $\bigvee$ is the disjunctive or.

Definition 2.3.2 We call a subtle set without frequentist (probabilistic) sequences, with a deterministic appearance, a set $S_0^{'}$ of fuzzy sequences F_{k0}^{fS}: $S_0^{'} = \{F_{k0}^{fS}, F_{t0}^{fS}, \ldots\}$ (11)

Definition 2.3.3 We call a subtle set without frequentist (probabilistic) sequences, with a fuzzy appearance, a fuzzy set $S_{0F}^{'}$ of fuzzy sequences F_{k0}^{fS}:

$$S_{0F}^{'} = \begin{Bmatrix} F_{k0}^{fS}, F_{t0}^{fS}, \ldots\ldots \\ \chi_k^0, \ldots \chi_t^0, \ldots\ldots\ldots \end{Bmatrix} \qquad (12)$$

$\chi_k^0, \chi_t^0 \ldots =$ the membership degrees of sequences $F_{k0}^{fS}, F_{t0}^{fS}, \ldots$ on set $S_{0F}^{'}$. If: $\chi_k^0 = \chi_1^0 = \ldots = 1 : S_0^{'} = S_{0F}^{'}$ (13)

So, $S_0^{'}$ is a particular case of $S_{0F}^{'}$, when the membership degrees are equal to 1, it becomes apparently "deterministic". $S_0^{'}$ and $S_{0F}^{'}$ can take same form. Relation (10) becomes $(10^{'})$:

$$S_0^{'} = \left\{ \begin{Bmatrix} e_1, & e_2, \ldots, & e_n \\ \mu_{1k}^0, & \mu_{2k}^0, \ldots, & \mu_{nk}^0 \end{Bmatrix}, \begin{Bmatrix} e_1, & e_2, \ldots, & e_n \\ \mu_{1t}^0, & \mu_{2t}^0, \ldots, & \mu_{nt}^0 \end{Bmatrix} \ldots \right\}$$

Similarly, relation (12) becomes (12'):

$$S_{OF}^{'} = \left\{ \begin{matrix} \begin{Bmatrix} e_1, & e_2, \ldots, & e_n \\ \mu_{1k}^0, & \mu_{2k}^0, \ldots, & \mu_{nk}^0 \end{Bmatrix}, & \begin{Bmatrix} e_1, & e_2, \ldots, & e_n \\ \mu_{1t}^0, & \mu_{2t}^0, \ldots, & \mu_{nt}^0 \end{Bmatrix} & \ldots \\ \chi_k & \chi_t & \ldots \end{matrix} \right\}$$

As $F_{k0}^{fS}, F_{t0}^{fS} \ldots$ has $\chi_k^0, \chi_t^0, \ldots$, e_i has μ_{ik}^0 on sequence F_{k0}^{fS}, μ_{it}^0, on sequence F_{t0}^{fS} etc., this means that $g_{ik}^0, g_{it}^0, \ldots$, of e_i on criteria $f_k^0, f_t^0, \ldots$, and on $S_{0F}^{'}$ are given by the square root of goods $\mu_{ik}^0 \chi_k^0, \mu_{ik}^0 \chi_t^0, \ldots$: $g_{ik}^0 = \sqrt{\mu_{ik}^0 \chi_k^0}$; $g_{it}^0 = \sqrt{\mu_{it}^0 \chi_t^0}, \ldots$ (14)

$$S_{oF}^{'} = \left\{ \begin{Bmatrix} e_1, & e_2, \ldots, & e_n \\ g_{1k}^0, & g_{2k}^0, \ldots, & g_{nk}^0 \end{Bmatrix}, \begin{Bmatrix} e_1, & e_2, \ldots, & e_n \\ g_{1t}^0, & g_{2t}^0, \ldots, & g_{nt}^0 \end{Bmatrix} \ldots \right\} \qquad (12'')$$

More economically it is the matrix form, where $e_1, e_2, \ldots, e_n$ are written once.

Relations (10'), (12'') becomes:

$$S_0^{'} = \begin{Bmatrix} e_1 & e_2 & \dots & e_n \\ \mu_{1k}^0 & \mu_{2k}^0 & \dots & \mu_{nk}^0 \\ \mu_{1t}^0 & \mu_{2t}^0 & \dots & \mu_{nt}^0 \\ \dots & \dots & \dots & \dots \end{Bmatrix} (15) \qquad S_{0F}^{'} = \begin{Bmatrix} e_1 & e_2 & \dots & e_n \\ g_{1k}^0 & g_{2k}^0 & \dots & g_{nk}^0 \\ g_{1t}^0 & g_{2t}^0 & \dots & g_{nt}^0 \\ \dots & \dots & \dots & \dots \end{Bmatrix} (16)$$

The membership degrees are placed into [0,1] and considering relation (15), it follows that:

$g_{ik}^0 \le \mu_{ik}^0;\ g_{it}^0 \le \mu_{it}^0; \dots (17)$, hence: $S_{0F}^{'} \subseteq S_0^{'}$ (18)

The 3^{rd} version of subtle set definition covers a higher specificity than the 2^{nd} version of the same. The 2^{nd} version of the subtle set definition is more specific in relation to the 1^{st} one, it follows that the order of their presentation is an increasing one, in relation to specificity. According to the concrete necessities of the study, other versions of a subtle set can be also defined.

2.4 The Definition of the Subtle Space

According to (11), we can write the relation defining the subtle space of O, on the set of elements E, described

by: $S_0 = \left\{ f_1^0, f_2^0, \dots, f_p^0 \right\}$ namely as follow (19):

$$S_0 = \left\{ (e_i, \mu_{i1}^0, \mu_{i2}^0, \dots, \mu_{in}^0) / \mu_{ij}^0 : (e_i, f_j^0) \to [0,1], (e_i, f_j^0) \in E \times S_0, i \in \overline{1,n};\ j \in \overline{1,p}; n, p \in N \right\}$$

e_i are independent and f_j^0 are independent, too. According to the dependence among e_i and f_j^0, the relation (19) becomes (20):

$$S_0^d = \begin{Bmatrix} (e_i, \mu_{i1}^0, \mu_{i2}^0, \dots, \mu_{in}^0) / \mu_{ij}^0 : (e_i, f_j) \to [0,1], (e_i, f_j) \in E \times S, i \in \overline{1,n},\ j \in \overline{1,p}, \\ (\Psi_{kh}^0, T_{kh}^{S_0} \in [0,1], k, h \in I_1), (R_{uv}^{S_0} \in [0,1], u, v \in I_2), n, p \in N \end{Bmatrix}$$

I_1,I_2 = the subsets of indices that specify e_k and e_h, with influences among them, the criteria (indicators) f_u and f_v, among which a partial correlation ρ_{vu}^0 exists.

3 The Hypotheses and Axioms on SST

3.1 The Fundamental Hypotheses on SST

3.1.1 The Estimator's hypothesis

There is an estimator (expert), within the set of observers, to measuring and/or evaluating a_{ij}^0.

3.1.2 The Expert's hypothesis

There is one expert, within the set of observers, to problems related to characteristic S, especially to evaluating and establishing the normal limits of a_{ij}^0, which have correspondence in the field $\mu_{ij}^* \inf, \mu_{ij}^* \sup$, so the concepts related to expert, *do not depend upon observer.*

3.1.3 The Hypothesis of Correct Diagnosis

The expert can notice if the normal limits are kept, on the basis of analyzing the consequences a_{ij}^0, for each element e_i, according to criterion (indicator) f_j^0.

3.1.4 The Hypothesis of Ability

The set of observers O, who has the ability to describe an algorithm, that to allow to get a_{ij}^0 into normal limits, at a level $\tilde{a}_{ij}^0$, with a corresponding $\tilde{\mu}_{ij}$. In case of a single O, we suppose he meets all four requests.

3.1.5 The Incompatibility Hypothesis

If a level a_{ij} is modified, an observer can distinguish the appearance of incompatibilities among criteria (indicators).

3.2 The Fundamental Axioms on SST

3.2.1 The Axiom of Existence

There is an element e_i , within a subtle set, whose level μ_{ij}^0 attached to criterion j, is considered by O, as not belonging to the normal limits $\mu_{ij}^* \inf, \mu_{ij}^* \sup$.

3.2.2 The Membership Axiom

In order to significantly modify the level μ_{ij}^0, the element e_i *has* an incompatibility between the new level $\tilde{\mu}_{ij}^0$ and a level μ_{ih}^0, according to a criterion f_h^0.

3.2.3 The Axiom of Action

The O *makes* corrections, with a view to eliminate any incompatibility between levels $\tilde{\mu}_{ij}^0$ and μ_{ih}^0, new evaluations, hypothesis and projections for developing future actions. To make these corrections, evaluations, and hypothesis, the observer uses an *operator of act*:

$\mathbf{A_0}(\mu_{ij}^0 \notin \left[\mu_{ij} \inf, \mu_{ij} \sup \right] \Rightarrow \mathbf{Tr}$

$(\mu_{ij}^0) = \tilde{\mu}_{ij}^0; IF(\tilde{\mu}_{ij}^0, \mu_{ih}^0) = INCOMP \to COMP(\mu_{ih}^0) = \tilde{\mu}_{ih}^0, \dots)$

The O is the support of the operator of act, who, has noticed that μ_{ij}^0 is not situated into normal limits. He applied an algorithm Tr**,** in order to get the level μ_{ij}^0 to a new level: $\tilde{\mu}_{ij}^0 \in \left[\mu_{ij} \inf, \mu_{ij} \sup \right]$.He compared the new level $\tilde{\mu}_{ij}^0$ to the level μ_{ih}^0 of another f_h^0, and he obtained the result INCOMP = incompatibility. He applied the procedure COMP(μ_{ih}^0) that consists in *modifying the old level of argument,* such that, the new level $\tilde{\mu}_{ij}^0$, to be compatible with $\tilde{\mu}_{ij}^0$ *and* $\tilde{\mu}_{\text{ih}}^0 \in \left[\mu_{ih} \inf, \mu_{ih} \sup \right]$, etc.

3.2.4 The Axiom of Stopping the Operator of Act for an Element e_i

The operator of act makes use of algorithms and final procedures. After the comparison of sequences F_{k0}^{fS}, F_{h0}^{fS} was made, and the noticed incompatibilities have been eliminated, we can calculate, for each e_i, the level of the remained incompatibility degree. We consider:

$$\mu_{ik}^0 = \lambda_{ih}^0 \mu_{ih}^0 + \beta_{ih}^0, k \in \overline{1,p};\ k \ne h \qquad (21)$$

$\lambda_{ih}^0, \beta_{ih}^0$ = the coefficients of initial regression, according the which, we select the coefficients of maximum unfavorable influence:

$\lambda_{ih}^{0} = Max\left[\lambda_{ij}^{0}\right] h \neq j, j \in \overline{1,p}$

$\beta_{ih}^{0} = Max\left[\beta_{ij}^{0}\right] h \neq j, j \in \overline{1,p}$ (22)

If we change μ_{ih}^{0} into $\tilde{\mu}_{ih}^{0}$, we get a deviation ε_{ik}^{0}:

$\varepsilon_{ik}^{0} = \left|\lambda_{ih}^{0} - \tilde{\mu}_{ih}^{0} + \beta_{ih}^{0} - \mu_{ih}^{0}\right| \; k \in \overline{1,p}$ (23)

If $\varepsilon_{ik}^{0} > \varepsilon_{adm}$ we recalculate a new level $\tilde{\mu}_{ik}^{0}$:

$\tilde{\mu}_{ik}^{0} = \lambda_{ih}^{0}\tilde{\mu}_{ih} + \beta_{ih}^{0}$ (24)

If $\tilde{\mu}_{ik}^{0} \notin \left[\mu_{ik} \inf, \mu_{ik} \sup\right]$, we calculate the deviation δ_{ik}^{0}, from the normal limits:

$\delta_{ik}^{0} = \begin{Bmatrix} \tilde{\mu}_{ik}^{0} - \mu_{ik} \inf, \text{ if } \tilde{\mu}_{ik}^{0} > \mu_{ik} \sup \\ \mu_{ik} \inf - \tilde{\mu}_{ik}, \text{ if } \tilde{\mu}_{ik}^{0} < \mu_{ik} \inf \end{Bmatrix}$ (25)

Let J_{1i}^{0} be the set of indices estimating the deviations from correlation ε_{ik}^{0}, and let J_{2i}^{0} be the set of indices estimating the deviations δ_{ik}^{0} from the normal limits:

$n_{1i}^{0} = card\ J_{1i}^{0};\ n_{2i}^{0} = card\ J_{2i}^{0}$ (26)

The average incompatibility degree $\overline{g}_{inc_0}^{i}$ of i is:

$\overline{g}_{inc_0}^{i} = \frac{\sum_{k \in J_{1i}} \varepsilon_{ik}^{0} + \sum_{k \in J_{2i}} \delta_{ik}^{0}}{n_{1i}^{0} + n_{2i}^{0}}$ (27)

We can state the axiom of stopping the operator of act for e_i, if: $\overline{g}_{inc}^{i} \leq g_{inc}^{i\,adm}$ (28)

the operator of act can be stopped and if $i \neq p$, we go to element e_{i+1}.

3.2.5 The Axiom of Stopping the Operator of Act

If e_i acts to diminishing the favorable components of S of the e_i, the O will consider an incompatibility between e_i and e_j by inc(e_i,e_j).This incompatibility is measured with the *unfavorable variation of the global membership degree* of e_i on S, denoted by μ_{gj}^{S}. Before e_i, to act, e_j had the membership degree μ_{jk}^{0} on $f_k \in S$.

In order to calculate the global membership degree μ_{gj}^{S}, we can use a Cobb-Douglas production function:

$\mu_{gj}^{S_0} = \mu_{j1}^{\Lambda_1^0} \mu_{j2}^{\Lambda_2^0} ... \mu_{jk}^{\Lambda_k^0} ... \mu_{jp}^{\Lambda_p^0}$ (29)

$\Lambda_1^0, \Lambda_2^0, ..., \Lambda_p^0$ are the exponents with the property:

$\Lambda_1^0 + \Lambda_2^0 + ... + \Lambda_p^0 > 1$ (30)

If e_j has positive synergy, and $\Lambda_1^0 + \Lambda_2^0 + . + \Lambda_p^0 < 1$ (31) in case when the same element has negative synergy. Let n_{inc}^{0} be the elements number of this set. The average incompatibility of set J_{inc}^{0} is: $\overline{inc}_0 = \frac{\sum_{(i,j) \in J_{inc}} inc_o(e_i, e_j)}{n_{inc}^{0}}$ (32)

A comparison is made. The stopping axiom is:

$\overline{inc}_0 < inc_{adm}$ (33) if the operator of act stops. We try to project certain modifications, in order to get the average incompatibilities of the subtle sets.

3.2.6 The Axiom of Free Choice

Any choice of an entity belonging to a subtle set is based on the principle of the free choice. Any choice is based on evaluating the entity state that makes the decision of choosing the state of "happiness". The state of happiness is relative and depends on the criteria selected by the observing entity.

4 An issue assessment of phenomena socio-economy and the related spheres of activity [3]

4.1 Example no. 1

An illustration of the socio-economic assessment problem is the size of the enterprises e_i, $i \in \overline{1,n}$, in an industrial town A. We define a lot subtler. It considers the elements of this crowd, the n firms into the city A. S_0 main feature of these enterprises are property of being large, in terms of O ($\overline{S}_0$ is the property of being smalls).We decompose S_0 into several criteria $C_1^0, C_2^0, ..,$ and $\overline{S_0}$ breaks down into the criteria $\overline{C}_1^0, \overline{C}_2^0, ..,$etc.

An expert Exp. could help O, into the evaluation process: the criteria's are broken down into sub (on the second level of decomposition), etc. The first level of decomposition could be: $C_1^0 = N_o$, no. of employees; $C_2^0 = CA_0$ the turnover; $C_3^0 = d_0$, the u.p.v./ year demand volume (u.p.v.= production units in terms of value); $C_4^0 = CP_0$, the capacity (u.p.v. / year); $C_5^0 = CF_0$, fixed capital; $C_6^0 = CTF_0$, the quality of technology used; $C_7^0 = CM_0$,the management quality, $C_8^0 = CAL_0$, quality of production achieved; $C_9^0 = P_0$ return size (gross profit); $C_{10}^0 = CEM_0$, working ergonomic conditions (including environmental quality); $C_{11}^0 = CSM_0$, the average specific consumption of materials (value/ u.p); C_{12}^0 = CSE_0, the average specific energy consumption (expressed in u.p.v), etc. Top 10 criteria should be maximized, and the last two minimized. S_0= owned enterprises to be high, C_j^0 ($j \in \overline{1,12}$) must work towards their development: $C_1^0, C_2^0, ..., C_{10}^0$ to be maximized; C_{11}^0 and C_{12}^0 should be minimized. Feature $\overline{S_0}$ = owned enterprises to be small, the $\overline{C}_j^o$ ($j \in \overline{1,12}$) criteria work in opposite development: $\overline{C}_1^0, \overline{C}_2^0, ..., \overline{C}_{10}^0$ take minimum values of

$C_1^0, C_2^0, .., C_{10}^0$; $\overline{C}_{11}^0$ and $\overline{C}_{12}^0$ take maximum values of C_{11}^0 and C_{12}^0. O may consider the second level of decomposition of criteria, such as: C_1^0 = number N of employees can be split into: $C_{11}^0 = NM_o$, number of workers and $C_{12}^0 = NS_o$ = no. of employees. Volume d_o application can be split in sorts, where $C_1^0, C_2^0, .., C_S^0$, s = number of grades request by the market. CP_0 can be split /section (which runs the s types), etc. Similarly, it made the decomposition of the: $\overline{C}_1^0$ in $\overline{C}_{11}^0$ and $\overline{C}_{12}^0$; $\overline{d_0}$ in $\overline{d}_1^0, \overline{d}_2^0, .., \overline{d}_s^0$; $\overline{CP}_0$ in $\overline{CP}_1^0, \overline{CP}_2^0, ..., \overline{CP}_s$. These are two trees that represent a decomposition S_0 feature in several criteria, the various levels of decomposition. Another decomposition of the $\overline{S}_0$ on various levels of decomposition. Disaggregation operation starts from the top S_0 ($\overline{S}_0$) of the tree, and ends on the last level of decomposition thereof. Aggregation operation starts at the last level of decomposition. Sub for the last level (representing INDEX), the aggregate indicators "more aggregate" located on the penultimate level, etc. until it reaches the aggregation of all features S_0 ($\overline{S}_0$).

4.2 Example no. 2

An example more relevant than before, is the efficiency of $e_i, i \in \overline{1,n}$ from the city A. New crowd /subtle elements are $e_i, i \in \overline{1,n}$, as in the first instance. The new feature will be the main EF_o, estimated by the O. Antithetical feature will be ineffective $\overline{EF_0}$ as assessed by O. Crowd subtle evaluation criteria K_j^0 attached to the EF_o efficiency, will be: K_1^0, work productivity of the employees, $W_1^0 = \frac{CA_0}{N_0}$; K_2^0, profits made by an employer, $\Pi_0 = \frac{P_0}{N_0}$; K_3^0 turnover per thousand value units capital, K_4^0 quality of technology used, CTF_o; K_5^0 quality of management, CM_o; K_6^0 quality of production achieved, CAL_o; K_7^0 ergonomic conditions at work, CEM_o; K_8^0 average specific consumption of materials (u.p.v.), CSM_o; K_9^0 average specific energy consumption, CSE_o. To obtain maximum efficiency, it is necessary to maximize the first 7 criteria's and to minimize the last 2 criteria's. To calculate ineffectiveness, inefficiency is considered $\overline{K}_j^0$, $j \in \overline{1,9}$. Maximum criteria are achieved, when the first 9 criteria are minimal and the last two maximum.

4.3 Comparative analysis between the 'big' companies and the 'effective' companies

Basis of two examples, we can construct two sets of subtle way: Set S_0 subtle crowd of its business e_j, with the characteristic of being large in the city A, described by C_j^0, and the consequences a_{ij}^0, of which form is:

$$S_0 = \left\{ \begin{matrix} (e_i, C_j^0)(e_i, C_2^0), ..., (e_i, C_{12}^0) \\ a_{i1}^0 \quad a_{i2}^0 \quad ... \quad a_{i12}^0 \\ i = \overline{1,n} \end{matrix} \right\} \quad (34)$$

EF_0 subtle set theory (SST) of business characteristics, to be effective, located in the city A, described the criteria K_j^0 and the consequences b_{ij}^0, of the form :

$$EF_0 = \left\{ \begin{matrix} (e_i, K_1^0), \ (e_i, K_2^0) \ ... \ (e_i K_9^0) \\ b_{i1}^0 \quad b_{i2}^0 \quad ... \quad b_{i9}^0 \\ i = \overline{1,n} \end{matrix} \right\} \quad (35):$$

To highlight the similarity between the two sets subtle intersection, is made (36) :

$$S_0 \cap EF_0 = \left\{ \begin{matrix} (e_i, C_6^0), & (e_i, C_7^0), & (e_i, C_8^0), & (e_i, C_{10}^0), & (e_i, C_{11}^0), & (e_i, C_{12}^0) \\ a_{i6}^0 & a_{i7}^0 & a_{i8}^0 & a_{i10}^0 & a_{i11}^0 & a_{i12}^0 \end{matrix} \right\} =$$

$$= \left\{ \begin{matrix} (e_i, K_4^0), & (e_i, K_5^0), & (e_i, K_6^0), & (e_i, K_7^0), & (e_i, K_8^0), & (e_i, K_9^0) \\ b_{i4}^0 \ , & b_{i5}^0 \ , & b_{i6}^0 \ , & b_{i7}^0 \ , & b_{i8}^0 \ , & b_{i9}^0 \end{matrix} \right\}$$

The justification for this result is the identity of the six criteria for S_0 and subtle crowd EF_0, as (37):

$$C_6^0 = K_4^0; C_7^0 = K_5^0; C_8^0 = K_6^0; C_{10}^0 = K_7^0; C_{11}^0 = K_8^0; C_{12}^0 = K_9^0$$

and gender appropriate consequences (38):

$$a_{i6}^0 = b_{i4}^0; a_{i7}^0 = b_{i5}^0; a_{i8}^0 = b_{i6}^0; a_{i10}^0 = b_{i7}^0; a_{i11}^0 = b_{i8}^0; a_{i12}^0 = b_{i9}^0$$

This new crowd is a subtle set of companies, which have simultaneously two main features: to be big & to be effective, as the 6 common criteria's. Similarly, could to build the set of subtle antithetical ("small") and the subtle set $\overline{EF}_0$ ("inefficient firms").These are:

$$\overline{S}_0 = \left\{ \begin{matrix} (e_i, \overline{C}_1^0), & (e_i, \overline{C}_2^0), & ..., & (e_i, \overline{C}_{12}^0) \\ \overline{a}_{i1}^0 & ; \ \overline{a}_{i2}^0 & ; & \overline{a}_{i12}^0 \end{matrix} \right\} \quad (39)$$

$$\overline{EF}_0 = \left\{ \begin{matrix} (e_i, \overline{K}_1^0); & (e_i, \overline{K}_2^0), & ..., & (e_i, \overline{K}_9^0) \\ \overline{b}_{i1}^0 & ; \ \overline{b}_{i2}^0, & ..., & \overline{b}_{i9}^0 \end{matrix} \right\} \quad (40)$$

which for $\overline{a}_{ik}$ and $\overline{b}_{ih}$ are the antithetical consequences of $\overline{C}_k$, $\overline{K}_h$, criteria for undertaking e_j. Simmilar way with the relationship (36),can be build a new variety of subtle enterprises, whose main characteristic is to be simultaneously small and inefficient,that meaning (41):

$$\bar{S}_0 \cap \overline{EF}_0 = \begin{Bmatrix} (e_i, \bar{C}_6^0), (e_i, \bar{C}_7^0), (e_i, \bar{C}_8^0), (e_i, \bar{C}_{10}^0), (e_i, \bar{C}_{11}^0), (e_i, \bar{C}_{12}^0) \\ \bar{a}_{i6}^0, \bar{a}_{i2}^0, \bar{a}_{i8}^0, \bar{a}_{i10}^0, \bar{a}_{i11}^0, \bar{a}_{i12}^0 \end{Bmatrix} =$$

$$= \begin{Bmatrix} (e_i, \bar{K}_4^0), (e_i, \bar{K}_5^0), (e_i, \bar{K}_6^0), (e_i, \bar{K}_7^0), (e_i, \bar{K}_8^0), (e_i, \bar{K}_9^0) \\ \bar{b}_{i4}^0, \bar{b}_{i5}^0, \bar{b}_{i6}^0, \bar{b}_{i7}^0, \bar{b}_{i8}^0, \bar{b}_{i9}^0 \end{Bmatrix}$$

Using "intersection", can be build two more subtle sets:
-Subtle set of enterprises which simultaneously main features: to be small, but effective, $\bar{S}_0 \cap EF_0$;
-Subtle set of enterprises which simultaneously main features: to be large, but inefficient: $S_0 \cap \overline{EF}_0$.

Because the subtle sets $S_0, \bar{S}_0, EF, \overline{EF}_0$ are not structured and are assessed by different criteria, to performing operation "intersection", in the two different situations, is a difficult process. Thus, crowds S_0 and $\bar{S}_0$ shall be based on redundant criteria, such as demand (C_3^0) and production capacity (C_4^0), which are in a very strong correlation (except for some very short periods). The difference between the criteria's is clear, as subtle sets S_0 and $\bar{S}_0$, are based on the 12 criteria's, while subtle sets EF, respectively $\overline{EF}_0$, on 9 criteria, 6 of which for, are common criteria's. Some criteria's may occur a partial incompatibility (sometimes total incompatibilities),as example between (C_9^0) and (C_{10}^0).It is also necessary to transform $a_{ij}^0, \bar{a}_{ij}$, to the corresponding/adequate membership grades. In order to conduct economic analysis to evaluate the characteristics, it is absolutely necessary as the primary data to be prepared, that mean to eliminate the redundant information, to mitigate the incompatibilities, etc. Additionally, in order to ensure the comparability between the crowds subtle, it is necessary to bring all the crowds that we operate, on the same criteria.

5 Conclusions

The complexity of Zadeh's paradox, can be explained by SST, and is calculated assessment of the fence redundancy and incompatibility factors, the influence of the system studied.

6 Connection to meditation

There seems to be a close correlation between Fuzzy Logic and Linguistics. So much of what is inherent in Fuzzy Logic relates to the way people think and talk-in other words, their use of natural language. What influence did your early exposure to so many different languages play in shaping these attitudes? (Before Zadeh was twelve, he was having to deal with 4 different languages-Russian, Azerbaijani, Persian, English, and 3 separate scripts, Cyrillic, Arabic, Latin).

Open Problem

(SST) & Chaos theory!

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