Optimizing anthropomorphic form’s of the flat feet modular walking robots MERO

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Abstract: - The walking robots represent a special category of robots, characterized by having the power source and technological equipments embarked on the platform which can be transported on unarranged, horizontal and rough terrain. The performance of walking robots is closely related to the adopted gait and forms of the feet. Some robot foot soles have curved front and rear ends, when the foot soles land on or are lifted off the terrain, they are in linear contact with the terrain. Therefore, the pressure applied to the foot sole on terrain is transmitted through successively varying contact generative and the forces are transmitted smoothly from the terrain to the foot sole for a stable gait. The paper relates some particularities and the optimum design possibilities of the foot soles shape.

Key-Words: - Modular Walking Robot, Walk, Gait, Leg, Foot Sole

1. Introduction

The feet of the walking robots in general, the modular walking robots especially (Fig.1),[4],[5] must be built so that the robots are able to move with smooth and quick gait. [6] If the fact soles were not shaped to fit with the terrain surface, then the foot would not be able to apply necessary driving forces and the gait results to be not uniformly. In a simplified form, the leg of a modular walking robot is built by three members (Fig. 2), [4] namely thigh (1), shank (2) and foot (3). All of the joint axes are parallel with the support plane of the land.

Fig. 1 Scheme of the modular walking robot MERO computer graphics [5]

Fig. 2 Kinematics scheme of the an anthropomorphous leg

The modular robot foot soles have curved front and rear ends, corresponding to the toes tip and to the heel respectively.

If the position of the axis of the pair A is defined with respect to the fixed coordinate axes system fastened on the support plane, the leg mechanism has a degree of freedom in the support phase and three degree of freedom in the transfer phase.

Therefore, the angles \(\phi_1\) and \(\phi_2\) (Fig. 3) and the distance \(S\), which
defines the positions of the leg elements, cannot be calculated only in term of the coordinates \( x_A, Y_A \).

In other words, an unknown must be specified irrespective of the coordinates of the center of the pair \( A \). In consequence, the foot (3) always may step on the land with the flat surface of the sole.

The module body may be moved with respect to the terrain without the changing of foot (3) position.

This walking possibility is not similar with human walking and may be achieved only if the velocity and acceleration of the robot body is small.

In general, the foot can be supported on the land both with the flat surface and the curved front and rear ends.

The plane surface of the sole and the cylindrical surface of the front end are tangent along of the rear ends.

The plane surface of the sole and the cylindrical surface of the front end are tangent along of the generatrix \( R \) (Fig. 4).

In the plane of motion, the position of the generatrix \( R \) with respect to the mobile coordinate axes system is given by the coordinate’s \( x_{3R}, y_{3R} \).

The size of the flat surface of the foot, i.e. the position of the generatrix \( R \), is determined in terms of allowable pressure on the terrain.

The curve directories of the cylindrical surface of the front end is defined by the parametrical equations: \( x_3 = x_3(\lambda), y_3 = y_3(\lambda) \), with respect to the mobile coordinate axes system attached to this element.

The generatrix in which the plane surface of the foot is tangent with the cylindrical surface of the front end is positioned by the parameter \( \lambda_0 \): \( x_{3R} = x_3(\lambda_0), y_{3R} = y_3(\lambda_0) \).

2. Kinematics Analysis of the Leg Mechanism

In the support phase, when the flat surface of the foot is in contact with the terrain (Fig. 3.a), the analysis equations are:

\[
\begin{align*}
X_A + AB \cos \varphi_1 + BC \cos \varphi_2 - S &= 0; \\
Y_A + AB \sin \varphi_1 + BC \sin \varphi_2 - x_{3R} &= 0. \\
\end{align*}
\]

The system is indeterminate because contains three unknowns, namely \( \varphi_1, \varphi_2 \) and \( S \). In order to solve it, the value of an unknown must be imposed, for example the angle \( \varphi_1 \). It is considered as known the position of the pair axis \( A \). In this hypothesis, the solutions of the system (1) are:

\[
\begin{align*}
\varphi_2 &= \arccos \frac{\sqrt{BC^2 - (x_{3R} - Y_A - AB \sin \varphi_1)^2}}{BC} \\
\varphi_2 &= \arcsin \frac{x_{3R} - Y_A - AB \sin \varphi_1}{BC} \\
S &= \sqrt{BC^2 - (x_{3R} - Y_A - AB \sin \varphi_1)^2}. \\
\end{align*}
\]

The coordinates of the tangent point \( R \) have the expressions:

\[
X_R = X_A + \sqrt{BC^2 - (x_{3R} - Y_A - AB \sin \varphi_1)^2} + y_{3R}, \\
Y_R = 0.
\]

In the end of the support phase, when the contact of the foot with the land is made along the generatrix which passes through the \( P \) point (Fig. 4.b), the analysis equations are:

\[
\begin{align*}
Y_A + AB \sin \varphi_1 + BC \sin \varphi_2 + CP \sin (\varphi_1 + u) &= Y_P; \\
X_A + AB \cos \varphi_1 + BC \cos \varphi_2 + CP \cos (\varphi_1 + u) &= X_P; \\
\end{align*}
\]

where : \( u = \arctan \frac{y_3(\lambda_P)}{x_3(\lambda_P)} \),

\[
CP = \sqrt{x_3^2(\lambda_P) + y_3^2(\lambda_P)};
\]

\( \lambda_P \) is the value of the parameter \( \lambda \) corresponding to the generator passing through the point \( P \) of curve directories;

\[
X_P = X_R + \int_{\lambda_0}^{\lambda_P} \left( \frac{dx_3}{d\lambda} \right)^2 + \left( \frac{dy_3}{d\lambda} \right)^2 \ d\lambda,
\]

because it is assumed that the foot sole do not slip on the land surface.
The equations (2) are solved with respect to the unknown’s \( \phi_2, \phi_1 \) and \( \lambda \), in terms of the coordinates \( X_A, Y_A \) of the center of the couple \( A \) and the \( \lambda \) and the amount imposed angle \( \phi_1 \).

By differentiation with respect to the time of the equations (2), result the velocity transmission functions:

\[
\begin{align*}
\frac{dY_A}{dt} & + AB \cos \phi_1 \frac{d\phi_1}{dt} + BC \cos \phi_2 \frac{d\phi_2}{dt} + CP \cos(\phi_1 + u) (d\lambda) \frac{d\lambda}{dt} \\
\frac{d\phi_3}{dt} + \frac{du}{dt} \frac{d\lambda}{dt} + \frac{dCP}{dt} \frac{d\lambda}{dt} & = 0;
\end{align*}
\]

\[
AB \sin \phi_1 \frac{d\phi_1}{dt} + BC \sin \phi_2 \frac{d\phi_2}{dt} + CP \sin(\phi_1 + u) (d\lambda) \frac{d\lambda}{dt} \
\] (3)

which are simultaneously solved with respect to the unknowns \( \frac{d\phi_2}{dt}, \frac{d\phi_3}{dt} \) and \( \frac{d\lambda}{dt} \), where:

\[
\begin{align*}
\frac{du}{dt} & = \frac{dx_3}{dt} (\lambda) \frac{d\lambda}{dt} - \frac{dx_3}{dt} \frac{d\lambda}{dt} \frac{d\lambda}{dt} \\
\frac{dCP}{dt} & = \frac{x_2(\lambda) \frac{d\lambda}{dt} + x_3(\lambda) \frac{d\lambda}{dt}}{\frac{d\lambda}{dt}} \\
\frac{dX_P}{d\lambda} & = \frac{d}{d\lambda} \left[ \int_{\phi_0}^{\phi_1} \left( \frac{dx_3}{dt} \frac{d\lambda}{dt} \right)^2 + \left( \frac{dy_3}{dt} \frac{d\lambda}{dt} \right)^2 \right] \frac{d\lambda}{dt}.
\end{align*}
\]

Further on, by differentiation the equations (3) result the acceleration transmission functions:

\[
\begin{align*}
\frac{d^2Y_A}{dt^2} + AB & \left[ \cos \phi_2 \frac{d^2\phi_2}{dt^2} - \sin \phi_2 \left( \frac{d\phi_2}{dt} \right)^2 \right] + BC \\
& \left[ \cos \phi_1 \frac{d^2\phi_1}{dt^2} - \sin \phi_1 \left( \frac{d\phi_1}{dt} \right)^2 \right] + CP \cos(\phi_1 + u) \\
\frac{d^2\phi_3}{dt^2} + \frac{d^2u}{dt^2} \left( \frac{d\lambda}{dt} \right)^2 & + \frac{du}{dt} \frac{d^2\lambda}{dt^2} \frac{d\lambda}{dt} \\
& \left( \frac{d\phi_3}{dt} + \frac{d\lambda}{dt} \frac{d\lambda}{dt} \right)^2 + CP \sin(\phi_1 + u)
\end{align*}
\]
\[
\frac{dy_2}{d\lambda} x_3(\lambda) - \frac{dx_2}{d\lambda} y_3(\lambda) + \frac{d^2\lambda}{dt^2} Q = \left(\frac{dx_3}{d\lambda}\right)^2 - \left(\frac{dy_3}{d\lambda}\right)^2.
\]

### 3. Forces Distribution in the Leg Mechanism

The goal of the forces analysis in the leg mechanism is the determination of the conditions of the static stability of the feet and of the fully robot. [8] The leg mechanism is plane, and the reaction forces from the pairs are within the motion plane.

The pressure on the contact surface or generatrix is assumed to be equally distributed. From the equilibrium equations of the forces which act on the leg mechanism elements (Fig. 4), the reaction forces from pairs A, B and C and the modulus and the origin of the normal reaction N are calculated.

If the position of the origin of normal reaction force N is outside of the support surface, the foot overturns. To avoid this phenomenon it is necessary that the origin of the normal reaction force fill a certain position, defined by the distance \(d\). In this case, a driving moment \(M_{01}\) in the pair \(A\), applied between the body \((0)\) and the thigh \((1)\) is added. This moment is the sixth unknown quantity of the forces distribution problem.

Taking into consideration the particularities of the contact between terrain and foot, the leg mechanism is analyzed in two steps:
- the first: it is solved the equations (4), which define the equilibrium of the forces acting on the elements (1) and (2),
- the second: it is solved the equations (5), which express the equilibrium of the forces acting on the foot (3).

The particularities consist in the fact that the foot (3) is supported or rolled without sliding on terrain. As a result, the reaction force acting to the foot (3) has two components, namely \(N\) along the normal on the support plane and \(F\) holds in the support plane. The rolled without sliding of the front or rear end of the foot is done if \(T < \mu N\) only, where \(\mu\) is the frictional coefficient between foot and terrain.

The forces analysis is made in two situations.

1. The foot is supported with his flat surface on the terrain (Fig. 4a).

Fig. 4 Forces distribution in the leg mechanism

The equations of the forces equilibrium which act on the links (1) and (2) are:
\[
\begin{align*}
Q_x + F_{1xx} & - R_{1xx} + R_{01xx} = 0; \\
Q_y + F_{1yy} & - m_1 g - R_{1yy} + R_{01yy} = 0; \\
M_{01} + (F_{1xx} - m_1 g)(X_{G1} - X_0) - F_{1xx} (Y_{G1} - Y_0) + R_{01x}(Y_0 - Y_0) + R_{01x}(X_0 - X_1) + M_{01} = 0; \\
F_{2xx} + R_{1xx} + R_{2xx} = 0; \\
F_{2yy} - m_2 g + R_{1yy} + R_{2yy} = 0; \\
(F_{2xx} - m_2 g)(X_{G2} - X_0) - F_{2xx} (Y_{G2} - Y_0) + R_{2xx}(X_0 - X_0) - R_{2xx}(Y_0 - Y_0) + M_{2} = 0,
\end{align*}
\]

where \(M_{01} = 0\).

\[
\bar{Q} = Q_x \bar{i} + Q_y \bar{j}
\]

is the direct acting load on the leg in the center of the pair \(A\):
\[
\begin{align*}
F_{1ij} = & -m_1 \frac{d^2 X_{Gj}}{dt^2}, \\
F_{2ij} = & -m_2 \frac{d^2 Y_{Gj}}{dt^2}, \\
M_{ij} = & -I_{Gj} \frac{d^2 \varphi_j}{dt^2}; j = 1, 3,
\end{align*}
\]

\[
\begin{align*}
\frac{d^2 X_{G1}}{dt^2} = & \frac{d^2 X_A}{dt^2} - (x_{1G1} \sin \varphi_1 + y_{1G1} \cos \varphi_1) \frac{d^2 \varphi_1}{dt^2} \\
& - (x_{1G1} \cos \varphi_1 - y_{1G1} \sin \varphi_1) \left(\frac{d\varphi_1}{dt}\right)^2;
\end{align*}
\]
The equations (4) are simultaneously solved with respect to the unknowns $N, T$ and $d$. If $T > \mu N$, the foot slipped on the terrain. In this case, the input moment $M_0 \neq 0$ must be applied to the thigh (1).

The magnitude of this moment is calculated by solving of the equations (4), where $R_{32x} < \mu N$. The sets of equations (4) and (5) are solved iteratively, until the difference between two successive iterations decreases under a certain limit.

2. The foot is supported with his front end on the terrain (Fig. 4.b).

The foot (3) may be in this position if a driving moment is applied in pair $C$, between links (2) and (3). The reaction forces from pair $A$ and $B$ are the solutions of the equations (6):

$$Q_x + F_{ix} - R_{12x} + R_{01x} = 0; \quad Q_y + F_{iy} - m_1 g - R_{12y} + R_{01y} = 0; \quad M_{01} + (F_{iy} - m_1 g)(X_G1 - X_A) - F_{ix} (Y_G1 - Y_A) + R_{01x} (Y_A - Y_B) + R_{01y} (X_A - X_B) + M_{12} = 0; \quad F_{12x} + R_{12x} + R_{32x} = 0; \quad F_{12y} - m_2 g + R_{12y} + R_{32y} = 0; \quad (6)$$

The unknowns of these equations are $R_{01x}, R_{01y}, R_{32x}$ and $R_{32y}$.

Equilibrium of the forces which act on the foot (3) is expressed by equations (7):

$$T = R_{32x} + F_{32x} = 0; \quad N - R_{32y} - m_3 g + F_{32y} = 0; \quad M_{32} + M_{33} + N (X_F - X_C) + T Y_C + (F_{32y} - m_3 g)(X_G3 - X_C) - F_{32x}(Y_G3 - Y_C) = 0, \quad (7)$$

Solutions of these equations are $N, T$ and $M_{32}$. If $T > \mu N$, the foot slipped on the terrain and the robot overturns.

4. Optimum Design of the Foot

The bottom surface of the foot of a walking robot may have various shapes. These surfaces differ by the size of the flat surface and the forms of the front and rear cylindrical surfaces. The most adequate form of the bottom surface of the foot, i.e. the expressions of the directories of the front and rear cylindrical surfaces, is determined by optimization of some parameters. The objective [8] function, which is minimized in the optimization process may be expressed:

- maximum angular velocity: \( \frac{d\phi_1}{dt}, \frac{d\phi_2}{dt} \) or \( \frac{d\phi_3}{dt} \);
- maximum angular acceleration: \( \frac{d^2\phi_1}{dt^2}, \frac{d^2\phi_2}{dt^2} \) or \( \frac{d^2\phi_3}{dt^2} \);
- the maximum driving forces or moments etc.

The design variables with respect to the objective function is minimized are:
- the lengths $AB$ and $BC$ of the links (1) and (2),
- the coordinate $x_{31x}, y_{31y}$ of the point $R$;
- the coefficients from the equations of the curve directories of the surfaces of the front and rear ends.

The minimization of the objective function is performed in the presence of constrains which expressed:
- the directory curves of the front and rear ends are tangent to the flat surface of the foot sole,
- the ordinates $y_{3R}$ and $y_{1T}$ of the points $R$ and $T$ respectively in which the directory curves are tangent to the flat surfaces are limit by the minimum flat surface of the foot.

5. Example

The parametrical equations of the curve directories of the front end cylindrical surface are assumed as polynomial ones: $y_3 = \lambda; \quad x_3 = \sum_{i=1}^{6} c_i \lambda^{i-1}$. Because the curve directories must pass through the point $R$ defined by the coordinates $x_{3R}, y_{3R}$, results: $c_1 = x_{3R} = c_2y_{3R} - c_3y_{3R} - c_4y_3^2 - c_5y_3^3 - c_6y_3^4$.

In the point $R$, the curve directories (is) are tangent to the bottom flat surface of the sole:

$$\frac{d y_3}{dx_3} \bigg|_{y=y_{3R}} = 0.$$ Whence it is results

$$c_2 = -2c_3y_{3R} - 3c_4y_3^2 - 4c_5y_3^3 - 5c_6y_3^4.$$  

The goal of the optimization problem is to calculate the coefficient of the polynomial directory and the dimensions of the leg mechanism links. The function with respect to which the leg design is optimized, i.e. the objective function, expressed the maximum value of the angular acceleration $\frac{d^2 \phi_1}{dt^2}$, $\frac{d^2 \phi_2}{dt^2}$ and $\frac{d^2 \phi_3}{dt^2}$, in terms of the coefficients of the polynomial directory and the dimensions of the leg mechanism.

The optimum shape of the front end of the foot is shown in fig. 4, curve (1). [1], [8] This polynomial curve has the following coefficients:

$c_1 = 0.4140775, \quad c_2 = 0.8095373, \quad c_3 = -5.841186, \quad c_4 = 0, \quad c_5 = 0.09742488, \quad c_6 = 0.$

The coordinate of the point $R$ with respect to the mobile axes system $x_1Cy_3$ are:

$x_{3R} = 0.4421264, \quad y_{3R} = 0.06929674.$

6. Conclusion

Low speed and high energy expenses characterize the walking robots. Usually, the increase of robot speed can be achieved by a very simple solution, namely increase of capability of the drives.

A rational solution is to use an analogy of the robot leg architecture with human leg. Of course, the complexity of the human leg is very large, so that many simplifications must be made. The use of an adequate surface shape of the rigid foot sole of the leg has the effect a betterment of the gait.

Because the leg is used for module of walking robot, the analysis and improvement of the proposal hardware architecture will be continued.

References