Dummett’s Anti-realist Defense of Mathematical Intuitionism

GEORGE LĂZĂROIU
MĂDĂLINA COCOȘATU
MIHNEA DRUMEĂ
SOFIA BRATU

Journalism, Communication and Public Relations Department, Spiru Haret University
Public Administration Department, S.N.S.P.A
Law and Public Administration Department, Spiru Haret University – Constanța

Ion Ghica 13, Bucharest
ROMÂNIA

george.lazaroiu@spiruharet.ro, madacocosatu@yahoo.com, sofia_pub@yahoo.com
www.spiruharet.ro

Abstract: - On Dummett’s view, for intuitionists, one understands a mathematical statement if one is able to recognize a proof or a disproof of it. The intuitionist takes the word “proof” as covering a very restricted type of proof that he calls “canonical.” The intuitionist conforms to the principle of compositionality: the understanding of any expression can depend only on the understanding of simpler expressions, including those that compose it. The intuitionist requires each logical constant to be explained by specifying what would constitute a canonical proof of a statement of which it is the principal operator. The intuitionist is not content to abide by conventional methods of reasoning involving the logical constants.

Key-Words: - Anti-realist Defense, Mathematical Intuitionism, Logical Constants

1 Introduction
Dummett argues that falsity and truth do not have equal rights: one might specify the meaning of a statement by laying down that it was to be considered true in a certain case and false in a certain other case. The fundamental notion of the truth of a statement is the condition for an assertion of it to be correct. The content of a statement consists in what is needed to show it to be true.

2 Problem Formulation and Solutions
Phenomenalism and behaviorism attempted to reduce statements about the given subject-matter to statements of some other class. In saying what the referent is, we have to choose a particular way of saying this, a particular means of determining something as the referent. From an intuitionistic standpoint, an understanding of a mathematical statement consists in the capacity to recognize a proof of it when presented with one. Regarded as a form of constructivism, intuitionism is the most “thoroughly thought-through” presentation of idealism in modern philosophy.

The reductive forms of anti-realism (those that maintain that statements of the disputed class are translatable into statements of some other (the reductive) class) are untenable. The meaning of every sentence must be given in accordance with its composition (i.e. in terms of the meanings of its component expressions).

On Dummett’s view, for intuitionists, one understands a mathematical statement if one is able to recognize a proof or a disproof of it (its meaning consists in an effective classification of mathematical constructions into those that are proofs of it and those that are not). The understanding of some expressions might depend on the understanding of others.

The intuitionist mathematician takes the meaning of a mathematical statement to be characterized in terms of proof (the meanings of the logical constants must be characterized in terms of proof). A mathematical statement is true only if there is a proof of it. The intuitionist takes the word “proof” as
covering a very restricted type of proof that he calls “canonical.” The intuitionist conforms to the principle of compositionality: the understanding of any expression can depend only on the understanding of simpler expressions, including those that compose it. The intuitionist requires each logical constant to be explained by specifying what would constitute a canonical proof of a statement of which it is the principal operator. The intuitionist is not content to abide by conventional methods of reasoning involving the logical constants. The meanings of the intuitionistic logical constants can be explained in a very direct way, without any apparatus of semantic theory, in terms of the use made of them in practice. Intuitionistically interpreted, \( \forall t \neg K(\varphi t) \) holds good only if there is a general reason why it cannot be known at each time \( t \) that \( \varphi \) [2]

Dummett’s anti-realism is founded on the semantics of natural language (it can only be satisfactorily given in mathematics by intuitionism). The realist relies on assuming bivalence in order to provide an example of a true statement that will never be known to be true – the necessity of mathematical statements is enough to rule out our possession of a faculty of mathematical intuition conceived in analogy with perception. What mathematical objects there are within a fundamental domain of quantification is supposed to be independent of how things happen to be in the world. The notion of a model cannot serve to explain what it is to know the meaning of the expression “natural number.” Fundamental mathematical theories, such as the theory of natural numbers or the theory of real numbers, are those from which we initially derive our conceptions of different infinite cardinalities. An existential conjecture, to have definite content, requires a prior circumscription of the domain of quantification. In mathematics, if an effective procedure is available, it always remains available. Classical negation is not amenable to any proof-theoretic justification procedure based on laws that may reasonably be regarded as self-justifying.

Dummett’s anti-realist thinks that a statement is true if there is now a ground available to us for accepting it as true. The meaning of a statement that some event occurred or that some state of affairs obtained at some particular place or time has two components: what evidence would justify its assertion, and what it said. The adherent of a truth-conditional semantics thinks that a statement may be true independently of whether we have or could have any grounds for thinking it to be true. An adequate theory of meaning must account for how we understand sentences, which is given by how we use them. Although truth is central to the meaning of assertoric sentences, there is more to the meaning of sentences than their assertoric content. It is one thing saying that truth is indispensable and another saying what truth is. If meaning is to be explained in terms of truth, then statements that are neither determinately true nor false can have no meaning. Whereas a complex predicate must be viewed as a feature of a sentence, or a pattern in it, a simple first-level predicate is an isolable part of a sentence. The negation of a sentence may be asserted (or the sentence denied) just in case we have good reason to judge that it would be objectively incorrect to assert that sentence. Dummett says that a proposition, whether about the past, the future or the present, is true, timelessly, just in case someone optimally placed in time and space could have, or could have had, compelling grounds for recognizing it as true. Dummett claims that logic was for Frege principally a tool for and a prolegomenon to the study of the philosophy of mathematics.

Frege’s Begriffsschrift constituted the logical framework within which the formalization of different mathematical theories was to be effected. Frege gives an adequate account of the logic of statements involving multiple generality, and introduces variables for relations and functions. Frege introduces appropriate primitives, formulates axioms governing them, and derives the theorems of number theory. Rules of inference always belong to the logical framework, and are never peculiar to any particular mathematical theory. The truths of geometry are synthetic a priori, and therefore not reducible to truths of pure logic. There are mathematical objects which are not created by us, have not come into being nor will cease to be, but exist independently of us. In virtue of the properties of and relations between these objects our mathematical statements are true or false, independently of whether we do or can prove them. Frege wanted a framework within which all mathematical proofs might be presented and which would offer a guarantee against incorrect argumentation. Frege gives an analysis of the structure of the sentences of our language, adequate for such sentences as occur in a train of mathematical reasoning (this analysis must explain how the meaning of each sentence was determined from its internal structure). Sentences are constructed in a series of stages, should be applied to the means whereby expressions of generality are incorporated into sentences. Frege replaces the notation used to express generality in natural
language with that of quantifiers and variables. The sentence resulting from attaching a universal quantifier to a predicate is true just in case the predicate is true of everything. A logically simple expression is one which is not composed of subordinate expressions each possessing its own sense and so contributing to determining the sense of the whole. It is impossible that every word of a language should be introduced by definition (a definition presupposes a prior understanding of the words used in the defining expression). The sense of a complex expression is to be regarded as made up out of the senses of the constituent words. Most of the complex singular terms of natural language are neither terms formed by functional expressions nor ones formed with the description operator. According to Dummett, Frege constructed the first semantic theory and, by distinguishing sense from semantic value (\textit{Bedeutung}), explained how an account of meaning, as the content of linguistic understanding, should rest on a semantic theory as base. Frege defines the notion of positival class, consisting of the positive elements of an ordered group of permutations. For Frege, the notion of sense could be expressed only linguistically or symbolically. The references of the component expressions constitute their respective contributions to the determination of its truth-value. [3] Sullivan claims that, for Dummett, the concept of natural number is indefinitely extensible. Dummett draws the requirement that the domain of a mathematical theory must be specified \textit{in advance} of laying down the interpretation of its terms. Dummett need not presume that the existence and properties of mathematical objects would be, on the analogical platonist’s view, in any absolute sense contingent. Sullivan holds that Dummett has no reason to endorse the simple argument’s reduction of the notion of an intuitive model to what is settled by a given axiomatization.

The ground for regarding number as a logical notion is not one that renders it obligatory for any thinker whatever. Understanding reference to an empirical object automatically carries with it grasp of a domain of generalization to which it belongs. Generalization involving a mathematical concept requires us to settle criteria of application and identity for the concept. Sullivan maintains that any assessment of Dummett’s case against realism in mathematics must attend to that question of analogy. [9] Sullivan contends that the core of Dummett’s distinction between simple and complex predicates lies in the different theoretical roles that the two notions are required to fill. [10]

Moretti points out that Dummett’s justificationist conception of truth is a consequence of the conception of meaning that Dummett finds appropriate to us: the conditions under which we can justify the assertion of a statement constitute both that statement’s meaning (or content) and its conditions of truth (Dummett’s justificationist is not interested in analyzing a notion of truth proper to an omniscient being or one with cognitive faculties highly idealized). [6] MacFarlane argues that, for Dummett, a set of T-biconditionals or a recursive definition of “true in $L$” cannot simultaneously explain the meanings of the expressions of L and the meaning of “true in $L$” (the central convention governing the speech act of assertion is to assert only what is true). Our grip on truth comes from an understanding of its relation to assertion. MacFarlane think that there is no prospect of generalizing the Dummettian conception of truth as the aim of assertion in a way that makes assessment-relative truth intelligible. [4]

Mitchell refutes that an analog of Dummett’s argument will collapse intuitionism into strict finitism, and links the coherence of strict finitism to a view of confirmation (our actual practical abilities cannot confirm we know what would happen if we could compute impracticably vast problems). The strict finitists have to suppose that we grasp the truth conditions of sentences we cannot actually decide. Mitchell reasons that our actual abilities must be capable of confirming that we know what would be the case if actually undecidable sentences were true, contradicting the view of confirmation. [5] Dummett’s case against Platonism rests on arguments concerning the acquisition and manifestation of knowledge of meaning. [1] Tieszen observes that Dummett has interpreted and expounded upon intuitionism under the influence of Wittgensteinian views on language, meaning and cognition. Human cognition exhibits intentionality. Intuitionism holds that mathematical constructions are mental processes or objects. Constructions are, in the first instance, forms of consciousness or possible experience of a particular type (they must be understood in terms of the concept of intentionality). [11]
3 Conclusions
Shieh points out that one of the two major parts of Dummett’s defense of intuitionism is the rejection of classical in favor of intuitionistic reasoning in mathematics, given that mathematical discourse is anti-realist. Shieh investigates a fundamental question: to what view of the relation between deductive principles and meaning is anti-realism committed? The result of this investigation is a constraint on meaning theoretic assessments of logical laws. Shieh shows that a consistent anti-realist critique of classical logic cannot rely on the rejection of bivalence, and that a consistent anti-realist defense of intuitionism must begin with a radical rejection of the very conception of logical consequence that underlies realist classical logic (anti-realist intuitionism seems committed to proceeding by proof theoretic means). [8]

Murzi considers the idea that, on an intuitionistic interpretation of the conditional, there is no paradox to start with, and shows that this proposal only works if proofs are thought of as tokens (anti-realists have good reasons for thinking of proofs as types): intuitionists can either point out the intuitionistic invalidity of the inference from the claim that all truths are knowable to the insane conclusion that all truths are known, or they can outright demur from asserting the existence of forever-unknown truths. Murzi claims that if intuitionists reject strict finitism (the view that all truths are knowable by beings just like us) the prospects for either proposal look bleak. [7]

References: