Considering Intersection Performance in Road Network Flow Optimization Using User Equilibrium Approach

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Abstract: - Intersections are important components of a road network. Generally, the road network flow optimization algorithms consider the roadway link information in assigning and optimizing traffic flow in a road network. Intersection capacity and level of service are not considered in these models. The focus of this paper is to integrate intersection performance information with road network flow optimization algorithm. In this study, the user equilibrium model is used to optimize the road network flow and the critical lane volume based performance function to evaluate the intersections.

Key words: - Intersection performance, road network, user equilibrium, optimization, critical lane volume, traffic flow, origin & destination.

1. Introduction

A road network consists of links and nodes. Nodes are the road intersections where drivers change their course of travel from one road link to another. Driver chooses their path between a pair of origin and destination is such a way that their travel time is minimized. Generally, the link travel time is considered as the time required traversing the link and summation of all the traversed link travel time provides the total travel time for the driver from their origin to the destination. The path which yields shortest travel time is known as shortest path. The link travel time changes with the volume of traffic. When there is more than one origin and destination pair, there exist multiple shortest paths. Some of the links of the shortest paths might be common and therefore the volume in that link will be more than what it is supposed to be for one set of shortest path between a pair of origin and destination. This leads to increased travel time in that link and thus increases the probability of the availability of some other alternate path with lesser travel time. In this situation drivers will start exploring other paths which have the potential of yielding lesser travel time. The process will continue until no driver can improve their travel time by unilaterally shifting to some other path [1 and 2]. At that moment the road network will attain equilibrium with respect to travel time. This is known as user equilibrium (UE) [1]. There are different available algorithms for the UE. Most of these algorithms considered link travel time in the estimating the travel impedance [1, 2, 3, 4 and 5].

Nodes or intersections are another important component of the road network. Drivers have to negotiate the intersections to proceed to the next link of the shortest path. One of the ways to measure the performance of an intersection is by critical lane volume (CLV) [6]. The CLV governs the level of service (LOS) of the intersection. It changes with change in traffic volume traversing through the intersection. The available UE algorithms either do not consider the performance of the intersection or consider it as constant. None so far found considers the variation of intersection efficiency with the traffic volume. This study aims to incorporate the dynamics of intersection performance in the search of shortest paths between multiple set of origin and destination pair.

2. Link Performance Function

The impedances associated with the links which represents a road network can be travel time, safety, cost of travel, stability of flow etc. Some of these components can be efficiently modeled by mathematical objective functions, while others are subjective and difficult to model. One of the components widely used in representing the link performance is the travel time. The reasons for using travel time are threefold. First, empirical studies seem to indicate that it is a primary deterrent for flow of traffic. Second, almost all other possible factors are highly correlated with travel time and thus exhibit the same trend. Third, it is easier to measure. Furthermore, impedances other than travel time can be used explicitly in some parts of a road network. For example, the appropriate impedance measure for links in the transit network is transit (in vehicle) time, waiting time, fare and so on [7]. Because of congestion, travel time is an increasing function of flow. Therefore, a performance function is associated with each of the links representing the road network. A typical performance function for a typical route is shown in Fig. 1. This function captures the time spent in traveling along the road segment under consideration.

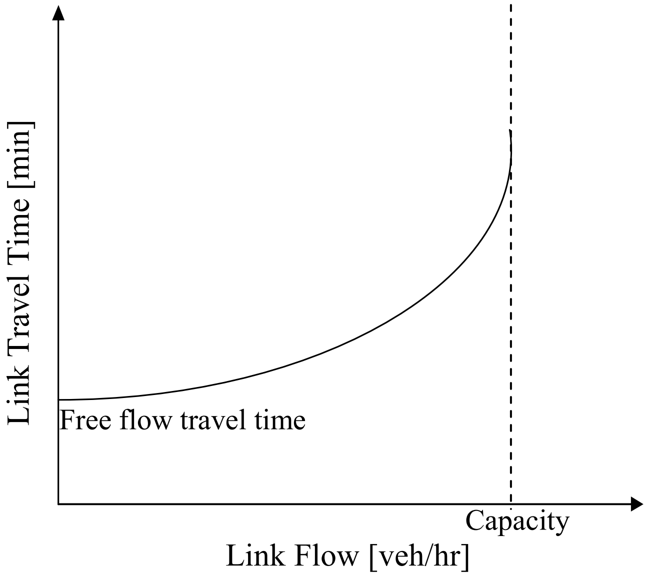

Fig. 1: Link travel time vs. link flow [2]

The travel time at zero flow is known as the free flow travel time. At this condition, a driver won't experience any delay related to interaction with other vehicles on the road. The only source of delay is the time associated with traversing the road segment. As the flow increases, the interaction among the vehicles increases and thus travel time increases. Characteristically, the performance function is asymptotic for a certain level of flow known as the capacity of the road segment under consideration. The mathematical relationship between travel time and the flow in the road segment can be represented as follows [1 and 2]:

$$t_a(x_a) = k_a\left[1 + d\left(\frac{x_a}{b_a}\right)\right] \qquad \forall a \qquad (1)$$

where,

$t_a(.)$ = Travel time function for road segment a

x_a = Flow on road segment a

k_a = Free flow travel time on road segment a

b_a = Capacity of road segment a

d = Constant based on lane characteristic

3. Intersection Performance Function

The performance of an intersection depends on the traffic volume using the intersection to traverse from one road link to another. Driver can drive left, right or straight through the intersection. Some of the maneuvers are conflicting to each other while others concurrent in nature. The conflicting maneuvers dictate the performance of the intersection [6]. Summation of all conflicting maneuvers in the peak hour period for an intersection is known as critical lane volume (CLV). The capacity of an intersection is defined by the maximum CLV it can handle at any point of time. It depends on factors like the geometry, traffic characteristics, driver behavior and so forth. The performance or level of service (LOS) of an intersection is subjective and dependent on the CLV and intersection capacity. The ratio of CLV and intersection capacity (v/c ratio) will govern the LOS [6]. A v/c ratio based LOS will be used in this study to evaluate the performance of an intersection.

The CLV of an intersection is dependent on the peak hour volume and the turning movement at that intersection. If $TM^i_{j,k}$ represents the j-type peak hour turning movement volume of i^{th} intersection from k^{th} approach in a road network, then the objective function for intersection performance evaluation can be written as follows:

$$CLV_{TM}^{i} = f\begin{pmatrix} TM_{1,1}^{i}, TM_{2,1}^{i}, ..., TM_{J,1}^{i}, TM_{1,2}^{i}, \\ TM_{2,2}^{i}, ..., TM_{J,2}^{i}, ..., TM_{J,K}^{i} \end{pmatrix} \qquad (2)$$

where,

CLV_{TM}^{i} = Peak hour CLV of i^{th} intersection in the road network
J = Type of turning movement from K^{th} approach
K = Number of approach
TM = Peak hour turning movement

The performance function of an intersection depends on CLV and intersection capacity. If the capacity of the i^{th} intersection is given by c^{i}, then the performance function of the intersection will be:

$$f\left(CLV_{TM}^{i}, c^{i}\right) = \frac{CLV_{TM}^{i}}{c^{i}} \qquad (3)$$

The total v/c ratio of all the intersections in a road network is given by:

$$TVC_{\text{int}} = \sum_{i=1}^{m} f\left(CLV_{TM}^{i}, c^{i}\right) \qquad (4)$$

where,

TVC_{int} = Total volume to capacity ratio of all intersections
m = Total number of intersections in a road network

4. Problem Formulation

The problem is formulated to find the flows in the road links that satisfy the user equilibrium criteria when all the origin destination (O-D) pair traffic volume are appropriately assigned. The link flow volume can be obtained by solving the following UE equivalent mathematical program:-

Minimize

$$Z(x) = \sum_{a} \int_{0}^{x_a} t_a(\omega)\, d\omega + \sum_{i=1}^{m} TF \times f\left(CLV_{TM}^{i}, c^{i}\right) \qquad (5)$$

Subject to:

$$\sum_{k} f_k^{rs} = q_{rs} \qquad \forall r, s \qquad (6)$$

$$f_k^{rs} \geq 0 \qquad \forall r, k, s \qquad (7)$$

The definitional constraints $x_a = \sum_{r}\sum_{s}\sum_{k} f_k^{rs} \delta_{a,k}^{rs}$ $\forall a$; is also part of this formulation.

where,

x_a = Flow in link a
f_k^{rs} = Flow in route k connecting origin-destination pair r and s
q_{rs} = Peak hour trip volume from origin r and destination s
ω = Obtained link flow
TF = Equivalent factor to present v/c ratio to the unit of travel time

$$\delta_{a,k}^{rs} = \begin{cases} 1 & \text{if road segment a belong to route k between r and s} \\ 0 & \text{otherwise} \end{cases}$$

The objective function in the above formulation does not have any intuitive economical behavioral interpretation. It should be viewed strictly as a mathematical construct that will be utilized to solve equilibrium problems.

Equation (6) represents a set of flow conservation constraints. These constraints state that the flow on all paths connecting each O-D pair has to be equal to the O-D trip rate. In other words all O-D trip rates have to be assigned to the network. The non-negativity conditions in equation (7) are required to ensure that the solution of the program will be physically meaningful [8].

5. Algorithm [1]

Step 0: *Initialization*

Perform "all-or-nothing" assignment based on travel-time $t_a = t_a(0)$, $\forall a$.

This yields flow vector $\{x_a^1\}$.

Set counter n=1

Step 1: *Update*

Find the travel time of a road link and the v/c ratio of the nodes associated to the link.

Set $t_a^n = t_a\left(x_a^n\right) + TF \times \left(\frac{CLV_{TM}^{i}}{c^{i}} + \frac{CLV_{TM}^{i+1}}{c^{i+1}}\right) \qquad \forall a$

Step2: *Direction finding*

Perform "all-or-nothing" assignment based on travel-time$\{t_a^n\}$.

This yields a set of (auxiliary) flows$\{y_a^n\}$

Step3: *Line search.*

Find α_n that solves $\min\limits_{0\le\alpha\le1}\sum\limits_a\int_0^{x_a^n+\alpha\left(y_a^n-x_a^n\right)} t_a\left(\omega\right)\mathrm{d}\,\omega$

Step4: *Move*

Set $x_a^{n+1}=x_a^n+\alpha_n\left(y_a^n-x_a^n\right), \qquad \forall a$

Step5: *Convergence test*

If convergence criterion is met, STOP (the current solution, $\left\{x_a^{n+1}\right\}$, is the set of equilibrium link flows).

Otherwise, set n = n+1 and GO TO step 1.

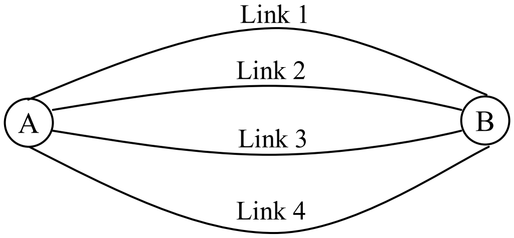

Fig. 2

6. Sample Test Problem

The methodology proposed in this study was applied to a simple road network. Suitable factors for road link characteristics, capacity of an intersection and factor to convert v/c ratio to travel time are used in the solution. For the network shown in figure 2, and 1000 peak hour volume from node A to node B the link flows, obtain by using the computer program based on the solution algorithm, is as follows:

```
The number of trips produced:-1000
The number of links:- 4
The free flow travel time in link 1:- 35
The flow capacity in link 1:-            500
The free flow travel time in link 2:- 10
The flow capacity in link 2:-            200
The free flow travel time in link 3:- 20
The flow capacity in link 3:-            400
The free flow travel time in link 4:- 25
The flow capacity in link 4:-            300

SOLUTION OF USER EQUILIBRIUM IN
TABULAR FORM:-
Travel Time  35.010.020.025.0
Flow            0        1000    0       0
Travel Time  35.0947.5     20.025.0
Flow            0        0       1000    0
Flow            0        403     597     0
Travel Time  35.034.834.825.0
Flow            0        0       0       1000
Flow            0        338     500     161
Travel Time  35.022.327.325.3
Flow            0        1000    0       0
Flow            0        362     483     155
Travel Time  35.026.126.425.3
Flow            0        0       0       1000
Flow            0        355     473     173
Travel Time  35.024.825.925.4
Flow            0        1000    0       0
Flow            0        359     469     171
FINAL FLOW

Travel Time  35.025.625.725.4
FINAL TRAVEL TIME
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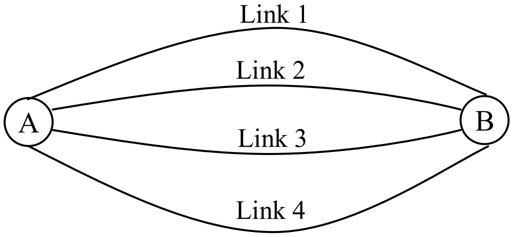

Figure 3

7. Conclusion

The developed methodology has been applied to a very simple road network. The outcome of the test problem is promising and proves efficient and effective optimization of road network flow using the developed methodology. More computational effort is needed to apply the developed methodology to a real road network. However, the developed methodology has a potential to be considered as one of the revised traffic assignment models. This methodology also provides opportunity to explore intersection performance and utilize in assessing road network.

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