Friction Induced Vibrations of a Two Degrees of Freedom System

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Abstract: The influence of the dry friction on the dynamics of a system with two degrees of freedom is proposed. The model system consists of a body of mass $m_1$, constrained by means of a spring and a damper to a driving support, moving relatively to its counterpart of mass $m_2$. In the conditions stability of the position of equilibrium vibrations due the static friction and the support’s velocity have been pointed out.

Key–Words: Self Excited Vibrations, Dry Friction, Friction Instability, Limit Cycle, Stick-Slip.

1 Introduction

In many mechanical systems the dry friction can induce self-excited vibrations which often are unwanted. In [1, 6] it has been analyzed a system with one degree of freedom in presence of a friction force characteristic. This force field has been assumed to be piecewise linear function of the relative speed. The method used for this analysis has mostly been a geometrical type method. Suitable conditions have been fixed upon the phase trajectory in the discontinuity points of friction characteristic. In this paper, the static friction influence upon the two degrees system dynamical behavior is analyzed. The system represented in Figure 1 is composed of two masses $m_1$ and $m_2$; the first one is undergo to an elastic and viscous force field. The second one is attached to a fixed wall by a spring and a viscous damper with constants, respectively, $k_2$ and $\sigma_2$. The friction force to the interface is represented in Figure 2. The most important result obtained in this work is that also under conditions of stability of the equilibrium position, self excited vibrations of the slides can reveal themselves. It has been shown that the system exhibits self excited vibrations increasing the ratio static friction/ support speed. So, it can be asserted that such ratio value is the basic parameter to analyze the bifurcation conditions of the system.

2 Mathematical Model

Let $X_1$ and $X_2$, respectively, the displacement of the slides of mass $m_1$ and $m_2$ in the reference frame system indicated in Figure 1. The motion equations can

![Figure 1: System Model](image)

![Figure 2: Friction Force Characteristic](image)
be written so as indicated in the following relations:

\[
\begin{cases}
    m_1 \ddot{X}_1 + \sigma_1 (\dot{X}_1 - v) + k_1 (X_1 - vt) + F (\dot{X}_1 - \dot{X}_2) = 0 \\
    m_2 \ddot{X}_2 + \sigma_2 \ddot{X}_2 + k_2 X_2 - F (\dot{X}_1 - \dot{X}_2) = 0
\end{cases}
\]

The friction characteristic is assumed to be piecewise linear function as shown in Figure 2. This function is analytically expressed by the followings relationships:

\[
F (\dot{X}_1 - \dot{X}_2) = \begin{cases}
    F_c & X_1 - \dot{X}_2 > 0 \\
    F_s & |F| < F_s, X_1 - \dot{X}_2 \equiv 0 \\
    -F_s & X_1 - \dot{X}_2 = 0^- \\
    -F_c & X_1 - \dot{X}_2 < 0
\end{cases}
\]

Putting:

\[
\begin{align*}
x_1 &= X_1 - vt \\
x_2 &= X_2 \\
\frac{\sigma_1}{m_1 \omega_1} &= 2 s_1 \\
\frac{\sigma_2}{m_2 \omega_2} &= 2 s_2 \\
\frac{F_c}{m_1 \omega_1 v} &= f c_1 \\
\frac{F_s}{m_1 \omega_1 v} &= f s_1 \\
\omega_1^2 &= \frac{k_1}{m_1} \\
\omega_2^2 &= \frac{k_2}{m_2} \\
\frac{m_1}{m_2} &= r \\
\frac{\omega_1}{\omega_2} &= \zeta \\
\frac{\beta}{m_1 \omega_1} &= 2 \mu_1 \\
\left( \frac{x_1}{\omega_1} \right) &= \eta_1 (\tau) \\
\left( \frac{x_2}{\omega_1} \right) &= \eta_2 (\tau)
\end{align*}
\]

the equations (3) can be rewritten as follows:

\[
\begin{cases}
    \ddot{\eta}_1 + 2 s_1 \dot{\eta}_1 + \eta_1 + \frac{1}{m_1 \omega_1 v} F \left\{ \left( \dot{\eta}_1 - \dot{\eta}_2 \right) + 1 \right\} = 0 \\
    \ddot{\eta}_2 + 2 s_2 \dot{\eta}_2 + \frac{1}{\xi^2} \eta_2 + \frac{r}{m_1 \omega_1 v} F \left\{ -v \left( \dot{\eta}_1 - \dot{\eta}_2 \right) + 1 \right\} = 0
\end{cases}
\]

By the integration of the (4) it is possible to determine the dynamic behavior of the system for assigned initial conditions. The system (4) is a dynamical system with piecewise linear structure. Such systems, because of the friction force discontinuity, are difficult to be analyzed analytically and numerically. In this work we have debugged a numerical procedure that take advantage of the uncoupling of the motion equations in all the phases space points in which the following relationship isnt verified:

\[
\dot{\eta}_1 - \dot{\eta}_2 = -1
\]
As it is deduced by the Figure 3, the fixed (for the integration) initial conditions are such that the phase trajectory run through the points of the space in which (5) is verified. In such way, the system dynamics will be influenced by the static friction and any limit cycles will be evident. The critical parameters sets, as it results from the Figure 3 [1,2], are those to which a phase trajectory that is tangent to

$$\dot{\eta}_1 - \dot{\eta}_2 = -1.$$  

In Figure 4 the dynamic state evolution is brought as we have increased only the parameter $f_{s1}$ value, point $B$ of Figure 3b. The system, in this case, exhibits a limit cycle which slides vibrations correspond.

3 Conclusions

The dry friction influence upon the dynamic behavior of a two degrees of freedom mechanical system has been analyzed. From the system proposed results that:

1. The system can exhibit limit cycles for rigidity and damping values greater than critical one;

2. Vibrations occur if at least one of the parameters $s_1$ or $s_2$ is lower than one. Such vibrations can extinguish for finite perturbations of the dynamic state.

Using the proposed method we will go on, in a next work, debugging the whole stability map in order to be
able to foresee the vibrations onset, when the system parameters are known.

4 Symbols list

- $m_1, m_2$: masses of slides
- $k_1, k_2$: rigidities
- $\sigma_1, \sigma_2$: viscous damping
- $t$: time
- $F_s$: static friction force
- $F_c$: kinetic friction force
- $v$: support speed
- $X, X_1, X_2$: dimensionless coordinates
- $\omega_1, \omega_2$: natural frequencies
- $\tau$: dimensionless time
- $x, x_1, x_2$: dimensionless coordinates
- $f_{s_1}$: dimensionless static friction parameter
- $f_{c_1}$: dimensionless kinetic friction parameter
- $s_1, s_2$: dimensionless viscous damping
- $\varsigma$: natural frequencies ratio
- $r$: mass ratio

References: