Robust High-Gain Observer Based Output Feedback Control

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Abstract: This paper proposes a robust high-gain observer based output feedback controller for nonlinear systems. It is assumed that their states are unmeasurable. The proposed observer has the integrator of the estimation error in dynamics. It improves the performance of conventional high-gain observers and makes the proposed observer robust to noisy measurements, uncertainties and peaking phenomenon as well. Its convergence analysis is performed using Lyapunov theory and Lipschitz condition. In order to verify the effectiveness of the proposed scheme, it is applied to output feedback controllers and some comparative simulation result with the conventional observer based output feedback controllers and state feedback controllers is given.

Key-words: Robust, High-Gain, Observer, Output Feedback, Integrator

1 Introduction

Control engineers often conflict the problems that states are partially or fully unavailable in many practical control problems because the state variables are not accessible for direct connection and, sensing devices or transducers are not available or very expensive. In such cases, the observer based control schemes should be designed to generate estimates of the states. Therefore, the observer design has been a very active field during the last decade and has turned out to be much more difficult than the control problem [1,2]. Estimation theories such as the Luenberger observer and the Kalman filter have been widely applied to various areas; the aerospace industry, the army, the process industry, etc. Since these are based on the linearized structure of nonlinear systems where robustness and convergence properties are difficult to prove [3,4]. The development of more robust and stable methodologies has been required, which is associated with nonlinearities and noise [5-8].

The technique, known as high-gain observer (HGO) is to design the observer gain that makes the observer robust against model uncertainties in nonlinear functions. Hence, it works for a wide class of nonlinear systems. Furthermore, the HGO scheme guarantees that the output feedback controller (OFC) recovers the performance of the state feedback controller (SFC) when the observer gain is sufficiently high [9,10]. However, high gains may excite hidden dynamics and amplify measurement noise: large oscillation in the transient response and sensitivity to measurement noise. Thus, they could not be applicable to practice.

In order to overcome the robust problem, several authors have successfully designed sliding-mode approach to construct observers that are highly robust with respect to noise in the input of the system. However, it turned out that the corresponding stability analysis could not be directly applied when output noise is presented. Therefore, it is still a challenge for the control system community to suggest a manageable technique to analyze the stability of identification error generated by sliding-mode type observers whose structure is obtained by differential-algebra techniques [11,12].

In this paper, a robust HGO design scheme for nonlinear systems is proposed and it is used for output feedback control. The proposed observer adopts the integrator of the error dynamics for improving the transient performance of the conventional HGO in transient response and the robustness against noisy measurements, uncertainties and peaking phenomenon. In addition, the proposed observer presents the Lyapunov theory based convergence analysis that is applicable under perturbations. It is assumed that the states of nonlinear systems are unmeasurable. The effectiveness of the proposed observer is guaranteed by the application to OFC and some comparative simulation results with the conventional OFC and SFC.

The rest of this paper is organized as follows. In the section 2, a problem is stated and the design scheme of the proposed observer is expressed in section 3.
stability analysis of section 4 presents Lyapunov based stability analysis. In section 5, Some comparative simulation results with SFC and the proposed HGO based OFC are given to demonstrate the effectiveness and applicability of the proposed scheme. Finally, we make some conclusion in section 6.

2 Problem Formulation

We consider single-input single-output (SISO) nonlinear systems that have the state space representation as follows:

\[ \dot{x} = Ax + F(x,u) \]
\[ y = Cx \]

where,

\[ A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, \quad F(x,u) = \begin{bmatrix} \vdots \\ \vdots \\ \vdots \\ \vdots \\ \vdots \end{bmatrix}, \]

\[ C = \begin{bmatrix} 1 & 0 & \cdots & 0 \end{bmatrix} \]

\[ x = \begin{bmatrix} x_1 & x_2 & \cdots & x_n \end{bmatrix}^T \in \mathbb{R}^n. \]

\( F(x,u) \) is unknown but bounded continuous nonlinear system. \( u \in \mathbb{R} \) is a control input and \( y \in \mathbb{R} \) is an output of the system respectively. It is assumed that only \( y \) is measurable and the system (1) is observable.

The goal is to design the robust HGO that improves not only robustness against measurement noise but also the convergence rate, and to show its performance through some comparative simulation result. In order to accomplish them, integral-type structure is adopted to the dynamics of the proposed observer and its convergence analysis is derived based on Lyapunov theory.

3 Observer Design

In this section, the robust HGO is developed. it has an integral-type structure in dynamics. Such structure based observer has robustness to the observer against noisy measurements, uncertainties and peaking phenomenon. Moreover, it guarantees to estimate states of the original system fast enough when the observer gain is sufficiently high as a HGO.

The proposed observer system is:

\[ \dot{\hat{x}} = A\hat{x} + F(\hat{x},u) + L(y - \hat{y}) + M\sigma \]
\[ \dot{\sigma} = y - \hat{y} \]
\[ \hat{y} = C\hat{x} \]

where, \( L = \begin{bmatrix} L_1 & L_2 & \cdots & L_n \end{bmatrix}^T \) is an observer gain vector. \( M \in \mathbb{R}^{n\times n} \) is an integral gain vector. \( \sigma \) is a new state describing the integral regulation error between the system output and the observer output, and

\[ E = \begin{bmatrix} \frac{1}{\varepsilon} & 0 & \cdots & 0 \\ 0 & \frac{1}{\varepsilon^2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \frac{1}{\varepsilon^n} \end{bmatrix} \in \mathbb{R}^{n\times n} \]

Theorem 1. The system expressed by (2), (3) and (4) is an asymptotical and robust observer for the system (1). It ensures asymptotical convergence under uncertainties and guarantees to estimate states of the original system fast enough when the observer gain is sufficiently high as a HGO.

Note that the observer gain can be any value, which satisfies that all eigenvalues of \( \det(\varepsilon I - (A + LC)) \neq 0 \) are placed in negative real part. However, when the observer gain is sufficiently high, it guarantees that the OFC recovers the performance of the SFC.

4 Convergence Analysis

Convergence analysis is performed using Lyapunov stability theory and Lipschitz condition. Before proceeding the analysis, the following two remarks are needed.

Remark 1. \( \lambda_{\min}(N) \) and \( \lambda_{\max}(N) \) are the smallest and the largest eigenvalue of \( N \), then it follows from \( N = U\Lambda U^T \) that

\[ \lambda_{\min}(N)\|x\|^2 \leq x^T N x \leq \lambda_{\max}(N)\|x\|^2 \]

where, \( N \) is a positive definite matrix, \( U^T U = I \) and \( \Lambda \) is a diagonal matrix containing the eigenvalues of the matrix \( N \).

Remark 2. According to Lyapunov equation [9], there exist \( P \) and \( Q \), which satisfy that

\[ A^T P + PA = -Q \quad B^T P = C \]

where, \( P \) and \( Q \) are symmetric positive definite matrices.
The observer error is defined as \( e = x - \hat{x} \). Then we get the error dynamics (5) using (1), (2), (3) and (4). An error equation can be obtained as follows.

\[
\begin{align*}
\hat{e} &= (A - LC)e - M\sigma + F(x,u) - F(\hat{x},u) \\
\dot{\sigma} &= y - y_1 \\
\dot{y} &= C\hat{x}
\end{align*}
\]

Let \( z = \begin{bmatrix} \sigma \\ e \end{bmatrix} \) as a new state vector, then the dynamic equation (5), (6) and (7) can take the form (8), which is the proposed observation error dynamics.

\[
\begin{align*}
\dot{z} &= \begin{bmatrix} \hat{e} \\ e \end{bmatrix} = \begin{bmatrix} 0 & C \\ -M & A - LC \end{bmatrix} \begin{bmatrix} \sigma \\ e \end{bmatrix} + B[F(x,u) - F(\hat{x},u)] \\
\dot{z} &= \Gamma z + Bg(z,u)
\end{align*}
\]

Where, \( \Gamma = \begin{bmatrix} 0 & C \\ -M & A - LC \end{bmatrix} \), \( B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \) and \( g(z,u) = F(x,u) - F(\hat{x},u) \)

**Theorem 2.** For (6), consider

\[
V = z^T P z
\]

where, \( V \) : a positive definite and radially unbounded function

\( P \) : a symmetric positive definite matrix

If there is \( k_r \), \( Q \) and \( P \) such that \( k_r \|B\| < \frac{\lambda_{\text{min}}(Q)}{2\lambda_{\text{max}}(P)} \) then,

\[
\dot{V} < 0 \quad \text{which means that } \dot{V} \text{ is negative definite. It guarantees the asymptotic stability of } z \text{ for the equilibrium point } z = 0, \text{ which means } \hat{\sigma} \text{ and } e \text{ go to 0, and } \hat{x} = x, \text{ where } x \text{ is an original state vector. } V \text{ is called a Lyapunov function. Where, } k_r \text{ is a Lipschitz constant.}
\]

**Proof.**

The differentiating \( V \) yields

\[
\dot{V} = z^T P \dot{z} + \dot{z}^T P z
\]

By substituting (6) into (7) and under Remark 1. and Remark 2, \( \dot{V} \) is expressed as

\[
\dot{V} = [z^T \Gamma^T + g(z,u) B^T] P z + z^T [\Gamma z + Bg(z,u)]
\]

\( P \) and \( Q \) can be readily chosen, which satisfy

\[
k_r \|B\| < \frac{\lambda_{\text{min}}(Q)}{2\lambda_{\text{max}}(P)}
\]

Hence,

\[
\begin{align*}
\dot{V} &\leq -\lambda_{\text{min}}(Q)\|z\|^2 + 2k_r \|B\|\lambda_{\text{min}}(P)\|z\| < 0 \\
\therefore \quad \dot{V} &< 0
\end{align*}
\]

According to Lyapunov theory, \( V \) guarantees the asymptotic stability of \( z \) for the equilibrium point \( z = 0 \), which means \( \hat{\sigma} \) and \( e \) go to 0, and \( \hat{x} = x \), where \( x \) is an original state vector. \( V \) is called a Lyapunov function.

**5 Numerical Example**

This section presents the effectiveness of the proposed observer. The simulation example borrowed from [1] is provided. The conventional observer and the proposed observer are applied to output feedback controllers and some comparative simulation result with stated feedback controllers is given.

The illustrative system

\[
\begin{align*}
\dot{x}_1 &= x_2 \\
\dot{x}_2 &= x_2 - x_1 + u \\
y &= x_1
\end{align*}
\]

which can be globally stabilized by the state feedback controller,

\[
u = -x_1^2 - x_1 - x_2
\]

The conventional high-gain observer (HGO) based output feedback controller (OFC) and the proposed HGO based output feedback controller are taken as

The conventional HGO based OFC system

\[
u = -x_1^2 - x_1 - x_2
\]

\[
\dot{x}_i = \frac{2}{\varepsilon}(y - \hat{x}_i) \\
\dot{\hat{x}}_i = \frac{1}{\varepsilon}(y - \hat{x}_i)
\]
The proposed HGO based OFC system

\[
\begin{align*}
    u &= -\hat{x}_2 - \hat{x}_1 - \hat{x}_2 \\
    \dot{x}_1 &= \dot{\hat{x}}_1 + \frac{2}{\varepsilon}(y - \hat{x}_1) - \frac{5}{\varepsilon}\sigma \\
    \dot{\sigma} &= -(y - \hat{x}_1)
\end{align*}
\]

Here, the initial value for \( x_1 \) and \( x_2 \) is 0.1 and that for the observer states \( \hat{x}_1 \) and \( \hat{x}_2 \) is 0. The observer gain assigns the eigenvalues of \( \Gamma - LC \) at \( -\frac{1}{\varepsilon} \) and \( -\frac{1}{\varepsilon} \). The integral gain vector, \( M \) is \( \left[ \frac{-5}{\varepsilon^2} - \frac{5}{\varepsilon^2} \right]^T \) in the proposed HGO based OFC system.

Fig. 1 shows that the proposed HGO based OFC has less overshoot and faster settling time than the conventional HGO based OFC. This is more obvious as \( \varepsilon \) decreases. Comparing Fig. 1(b) with Fig. 1(a), the overshoot of the conventional HGO based OFC increases enormously.

Fig. 2 expresses the speed performance of the proposed scheme depending on \( \varepsilon \) comparing with SFC. The response of the proposed HGO based OFC approaches the response of SFC systems in Fig. 2(a) and Fig. 2(b) presents that as \( \varepsilon \) decreases the rising gets even faster than SFC.

6 Conclusion

In this paper, we proposed the robust high-gain observer based output feedback controller. Lyapunov
theory based convergence analysis of the proposed observer was performed. It was assumed that states of nonlinear systems are unmeasurable. The proposed observer has an integral-type structure to be acceptable perturbations. It improved the robustness and overcomes the lack of the conventional HGO, peaking phenomenon. To verify the effectiveness of the proposed observer, it was applied to OFC, and compared with the conventional HGO and SFC. At last, comparative simulation results successfully confirmed the performance of the proposed scheme.

References: