Analysis of Extensive Power Factor by FPGA Under Power Quality Disturbance

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Abstract: - This paper used six formulas for power factor measurement, and they will be different according to the characteristics of load. The power quality is more and more serious, because a lot of information technology equipments and power electric converters were used in recent years. Harmonic and load unbalance can affect the measurement result of power factor of utility. The Fast Fourier Transform (FFT) is used to analyze the six formula of power factor. The simulation system was modeled in Hardware Description Language (VHDL) and some novel IP (intellectual property) cores, such as CORDIC core and FFT core by the way of Bottom-Up. The technology of semiconductor was more and more maturely. Design of SOC (System On a Chip) is a trend to achieve the strong and small volume in the future. The measurement data were employed to calculate active, reactive and apparent power. The average error rate of our design is about 0.05%.

Key-Words: - Power Factor, FPGA, Power Quality, Harmonic, Unbalance, FFT.

1 Introduction

Both large-size AC and DC electric arc furnaces (EAFs) have been widely employed in the steel industry to produce steels by melting scraps and pellets. The capacity of an AC or DC EAF in Taiwan could be up to 100 MVA. Load fluctuation, load imbalance, and harmonics are three major disturbances from an EAF [1-4]. Their effects on the electric power quality should be noted because the Taiwan power system is and isolated one. It has been reported that, under a same active power loading condition, harmonic currents and load imbalance will cause more power line loss. Thus, it is doubtful whether traditional metering methods could really reflect the load characteristics of an EAF. It is required that a suitable power factor value can be used to reveal effects of power quality disturbances and system line losses.

The power metering method has also been traditional for large-size customers in Taiwan. If inductive electro-mechanical (rotating disc type) three-phase kWh and kVarh meters are used, the power quantities essentially only contain the fundamental components and neglect effects of imbalance. By the way, the power factor value uses the average of a month. And the largest MW demand is the largest one among all average values of each 15-minute period in a month. The results of power metering cannot really reveal the problems of load fluctuation, load imbalance, and harmonics. It has been reported that if traditional electro-mechanical meters are used in circumstances of non-sinusoidal and unsymmetrical voltages or currents, the errors can reach 20%~30%. Thereafter in recent years, there are many discussions regarding the power factor definitions and calculations. Several definitions have been given in the IEEE Standard 1459-2000, such as effective apparent power, arithmetic apparent power, vector apparent power, and corresponding power factors [5].

To reflect the EAF load characteristics truly, a suitable power factor definition should be required [6, 7]. In this paper, six average power factor definitions, three by the IEEE Standard 1459-2000 [5] and three only considering fundamental components are compared. The effects of harmonic, imbalance, and load fluctuation are investigated. From the calculation results of the AC EAF plant, the highest average power factor value is 0.971 and the lowest is 0.896. The effect of harmonic active power values is not significant. Similarly, for the DC EAF plant, the highest average power factor value is 0.999

and the lowest is 0.954. Whether an AC or a DC EAF, it is found that harmonic distortion is more severe than load imbalance. The effective power factor and the 2^{nd} modified fundamental power factor could be better choices when load fluctuation, load imbalance, and harmonics are fairly considered.

2 Power Factor Definitions and Revenue

2.1 **Power Factor**

The power factor is

$$PF = \frac{P}{S} \neq \cos(\tan^{-1}\frac{Q_B}{P})$$
(1)

The Budeanu's distortion power can be given as

$$D_B = \sqrt{S^2 - P^2 - Q_B^2}$$
 (2)

In the three-phase conditions, there are many definitions under different considerations. In Taiwan, the Taipower does not provide neutral lines to customers in 161-kV, 69-kV, and 11.4/22.8-kV voltage levels. These customers can be seen as three-phase three-wire $(3\phi 3W)$ loads. The arithmetic apparent power of a three-phase three-wire load under non-sinusoidal and unbalanced conditions is [5]

$$S_{A} = V_{R}I_{R} + V_{S}I_{S} + V_{T}I_{T}$$

= $S_{R} + S_{S} + S_{T} = \sqrt{S_{R1}^{2} + S_{RN}^{2}} + \sqrt{S_{S1}^{2} + S_{SN}^{2}} + \sqrt{S_{T1}^{2} + S_{TN}^{2}}$ (3)

It is noted that S_A is the direct sum of each phase apparent power, so that it cannot reveal the load imbalance. But harmonic components are fully considered. The arithmetic power factor is

$$PF_A = \frac{P}{S_A}, \quad P = P_R + P_S + P_T \tag{4}$$

If the three-phase active power, reactive power, and distortion power are considered individually, the vector apparent power can be defined as

$$S_{V} = \sqrt{(P_{R} + P_{S} + P_{T})^{2} + (Q_{BR} + Q_{BS} + Q_{BT})^{2} + (D_{BR} + D_{BS} + D_{BT})^{2}}$$

= $\sqrt{P^{2} + Q_{B}^{2} + D_{B}^{2}} = \sqrt{(P_{1} + P_{H})^{2} + (Q_{1} + Q_{H})^{2} + D_{B}^{2}}$ (5)
And the corresponding vector power factor is

$$PF_{V} = \frac{P}{S_{V}} \tag{6}$$

It is always $S_V \leq S_A$, because signs of active power, reactive power, and distortion power of each phase may be different.

Another definition is based on the effective consideration of voltages and currents [5]. The RMS values are

$$V_{e} = \sqrt{\frac{V_{R}^{2} + V_{S}^{2} + V_{T}^{2}}{3}}$$
(7)

 $I_e = \sqrt{\frac{I_R^2 + I_S^2 + I_T^2}{3}}$ (8)

The effective apparent power is

$$S_{e} = 3V_{e}I_{e} = \sqrt{ (V_{R}I_{R})^{2} + (V_{R}I_{S})^{2} + (V_{R}I_{T})^{2} + (V_{S}I_{R})^{2} + (V_{S}I_{R})^{2} + (V_{S}I_{S})^{2} + (V_{S}I_{T})^{2} + (V_{T}I_{R})^{2} + (V_{T}I_{S})^{2} + (V_{T}I_{T})^{2} }$$
(9)

Since RMS values are used, harmonics components are included. In a relative unbalanced condition, for example, if $I_T = 0$ but V_T is given, it can be found that $S_e \ge S_A$, by comparing Equ. (3) and Equ. (9). It has been reported that the effective apparent power is more suitable to reveal the power line losses caused by unbalanced loads. The effective power factor is

$$PF_e = \frac{P}{S_e} \tag{10}$$

If a three-phase four-wire $(3\phi 4W)$ loads, the effective current and voltage are, respectively,

$$I_{e} = \sqrt{\frac{I_{R}^{2} + I_{S}^{2} + I_{T}^{2} + I_{N}^{2}}{3}}$$
(11)

$$V_{e} = \sqrt{\frac{1}{18} \left[3 \left(V_{R}^{2} + V_{S}^{2} + V_{T}^{2} \right) + \left(V_{RS}^{2} + V_{ST}^{2} + V_{TR}^{2} \right) \right]}$$
(12)

In some circumstances, only fundamental components are considered and power factors are calculated indirectly using fundamental active powers and reactive powers. For examples, if inductive electro-mechanical (rotating disc type) kW and kVar meters are used, only fundamental components could be obtained, considering the frequency responses of meters. Then the fundamental (displacement) power factor is

$$PF_1 = \cos\left(\tan^{-1}\frac{Q_1}{P_1}\right) \tag{13}$$

Where

$$P_1 = P_{R1} + P_{S1} + P_{T1}$$
, $Q_1 = Q_{R1} + Q_{S1} + Q_{T1}$ (14)

Then the corresponding fundamental apparent power is

$$S_1 = \sqrt{P_1^2 + Q_1^2}$$
(15)

However, if unidirectional (anti-reverse) kVar meters for fundamental components are used, the first modified fundamental power factor would be

$$PF_{1m1} = \cos\left(\tan^{-1}\frac{Q_{1m1}}{P_1}\right) \tag{16}$$

Where

$$Q_{1m1}(t) = \begin{cases} Q_1(t), & Q_1(t) \ge 0\\ 0, & Q_1(t) < 0 \end{cases}$$
(17)

And

$$S_{\rm lml} = \sqrt{(P_{\rm l})^2 + (Q_{\rm lml})^2}$$
 (18)

Equation (17) means that leading reactive powers could be accepted by the utility, and only lagging reactive powers should be included in revenue.

If both lagging and leading fundamental reactive

$$PF_{1m2} = \cos\left(\tan^{-1}\frac{Q_{1m2}}{P_1}\right)$$
 (19)

Where

$$Q_{1m2}(t) = |Q_1(t)|$$
 (20)

$$S_{1m2} = \sqrt{\left(P_1\right)^2 + \left(Q_{1m2}\right)^2}$$
(21)

2.2 Harmonic and Unbalanced Powers

Some power quantities are useful to represent the conditions of harmonics and imbalance. The effective representation of three-phase four-wire $(3\phi 4W)$ and three-phase three-wire $(3\phi 3W)$ fundamental voltages and currents can be given as (22)-(23) and (24)-(25), respectively,

$$V_{e1} = \sqrt{\frac{1}{18} \left[3 \left(V_{R1}^2 + V_{S1}^2 + V_{T1}^2 \right) + \left(V_{RS1}^2 + V_{ST1}^2 + V_{TR1}^2 \right) \right]}$$
(22)

$$I_{el} = \sqrt{\frac{I_{Rl}^2 + I_{Sl}^2 + I_{Tl}^2 + I_{Nl}^2}{3}}$$
(23)

$$V_{e1} = \sqrt{\frac{V_{R1}^2 + V_{S1}^2 + V_{T1}^2}{3}}$$
(24)

$$I_{e1} = \sqrt{\frac{I_{R1}^2 + I_{S1}^2 + I_{T1}^2}{3}}$$
(25)

Then the fundamental effective apparent power is

$$S_{e1} = 3V_{e1}I_{e1}$$
(26)

It is noted that S_{e1} is different from S_1 . The non-fundamental effective apparent power to reveal harmonic components is

$$S_{eN} = \sqrt{S_e^2 - S_{e1}^2}$$
(27)

The normalized non-fundamental effective apparent power, S_{eN} / S_{e1} , can be used to reveal the harmonic distortion degree of load powers.

When there is an unbalanced circuit, the fundamental positive-sequence apparent power is

$$S_{1}^{+} = 3V_{e1}^{+}I_{e1}^{+}$$
(28)
$$V_{e1}^{+} = 1/3(V_{R1} + aV_{S1} + a^{2}V_{T1})$$
and

Where

 $I_{e1}^+ = 1/3 (I_{R1} + aI_{S1} + a^2I_{T1})$, $a = 1 \ge 120^{\circ}$ are the fundamental positive-sequence components.

Therefore the unbalanced components can by represented by the fundamental unbalanced apparent power as

$$S_{1U} = \sqrt{S_{e1}^2 - S_1^{+^2}}$$
(29)

By the way, the normalized fundamental unbalanced apparent power, S_{1U} / S_1^+ , can be used to reveal the degree of load imbalance.

3 Application of System and Module

The FFT algorithm is used to calculate the fundamental and harmonic components of each phase voltage and current per power cycle as Figure 1. Each FFT uses 64 samples.

(1) FFT module

The Fast Fourier Transform (FFT) is a computationally efficient algorithm for deriving the Discrete Fourier Transform (DFT). The FFT core developed by Xilinx can compute an *N*-point forward DFT or inverse DFT (IDFT) where $N = 2^m, m = 4 \sim 14$. The FFT core applies the Cooley-Tukey decimation-in-time (DIT) algorithm to determine the DFT.

(2) Quantification module

The FFT module outputs 16-bit frequency domain data samples for both the real and imaginary components are fed into this module that picks out the complex pair corresponding to a target frequency. The square root operation is also implemented using the simplified CORDIC algorithm. The flicker component magnitudes are then computed.

4 Simulation Result

The most important issue in designing the calculation IC is the choice of numerical data processing scheme. Floating-point arithmetic has the advantage of a wide dynamic range, but its hardware realization is very complicated. Fixed-point arithmetic is a more practical solution to most industrial applications than floating-point arithmetic owing to its simple circuit realization. The proper numerical scaling plays a very significant role in synthesizing an integer controller. In this study, numerical variables and parameters must be transformed into approximate integers with finite word lengths.

FPGA has become the main stream in complex logic circuit design owing to its flexibility, ease of use and short time to market. The programmable hard-wired feature of FPGA provides a solution to the conflict between the demanding computation requirements and the cost. Therefore, FPGA can be beneficially applied as part of a digital controller to relieve the microprocessor from time-consuming computations. In the application of an arc furnace power system, the IC should serve as a coprocessor with a general-propose microprocessor to provide interface function. This investigation presents a novel digital circuit design methodology, in which all modules were described by using VHDL, and a synthesis tool, ISE 6.2, was adopted to map these designed codes directly onto FPGA. A design implementation software application, Modelsim, was utilized to obtain results. The logic and timing simulation software is (especially OR particularly) important for the design of complicated digital circuits, because can resolve circuit problems during the early design stage. Xilinx's (XC3S1500) was applied to implement this design.

To verify the effect of harmonic distortion and imbalance load was performed using MATLAB and FPGA simulation methods.

(1) The system is supplied by a $3\phi 3W$ symmetrical voltage source as Figure 2. The base values are 24 kVA and 220 V.

(2) The three-phase load is composed of parallel RLC load black and harmonic current source black.

(3) The measurement block of voltages and currents were performed using the three-phase instantaneous voltages and currents.

The situations of a three-phase non-harmonic system, the fundamental active and reactive power are 5kw and 1.65kVar, respectively. The means power factor is 0.9496.

(4) To reveal the power and power factor by using FPGA measures the instantaneous voltages and currents. The measurement data were employed to calculate the six formula of power factor. Compare the simulation results of FPGA and Matlab, the error rate can be defined as

$$\varepsilon(\%) = \frac{PF_{(\text{FPGA})} - PF_{(\text{Matlab})}}{PF_{(\text{Matlab})}} \times 100\%$$
(30)

Table 1 and Table 2 list the six formula of power factor, which were performed using Matlab and FPGA simulation methods. The calculation results using FPGA were approximately equal to these using Matlab. So the error rate is small. The designed system can prevent frequency leakage.



Figure 1. System Module



Table 1. Power factor using Matlab and FPGA of the first group

$\frac{S_{1U}}{(%)}$	PFe			PFA			PFv		
S_1^+ (70)	Matlab	FPGA	ε(%)	Matlab	FPGA	ε(%)	Matlab	FPGA	ε(%)
0	0.6702	0.6703	0.0189	0.6702	0.6698	-0.0576	0.6702	0.6698	-0.0578
5	0.6698	0.6699	0.0170	0.6700	0.6696	-0.0595	0.6702	0.6698	-0.0593
10	0.6685	0.6686	0.0202	0.6693	0.6689	-0.0571	0.6702	0.6698	0.0582
15	0.6663	0.6665	0.0273	0.6682	0.6679	-0.0489	0.6702	0.6699	0.0489
20	0.6633	0.6634	0.0142	0.6666	0.6663	-0.0523	0.6702	0.6699	-0.0541
25	0.6595	0.6597	0.0270	0.6646	0.6643	-0.0489	0.6703	0.6699	-0.0525
30	0.6550	0.6551	0.0198	0.6621	0.6618	-0.0513	0.6703	0.6699	-0.0538
35	0.6497	0.6498	0.0207	0.6591	0.6588	-0.0470	0.6703	0.6699	-0.0534
40	0.6438	0.6439	0.0158	0.6556	0.6553	-0.0527	0.6703	0.6699	-0.0548
45	0.6373	0.6375	0.0343	0.6517	0.6514	-0.0389	0.6703	0.6700	-0.0446
50	0.6302	0.6303	0.0205	0.6472	0.6469	-0.0420	0.6703	0.6700	-0.0495
55	0.6227	0.6228	0.0145	0.6423	0.6420	-0.0446	0.6704	0.6700	-0.0518
Average Error			0.021			-0.05			-0.053

Table 2. Power factor using Matlab and FPGA of the second group

$\frac{S_{1U}}{S_1^+}$ (%)	PF_1			PF _{1M1}			PF _{1M2}		
	Matlab	FPGA	ε(%)	Matlab	FPGA	ε(%)	Matlab	FPGA	ε(%)
0	0.9496	0.9496	0.0017	0.9496	0.9496	0.0017	0.9496	0.9496	0.0017
5	0.9496	0.9496	0.0006	0.9496	0.9496	0.0006	0.9496	0.9496	0.0006
10	0.9496	0.9496	0.0006	0.9496	0.9496	0.0006	0.9496	0.9496	0.0006
15	0.9497	0.9496	-0.0004	0.9497	0.9496	-0.0004	0.9497	0.9496	-0.0004
20	0.9497	0.9496	-0.0004	0.9497	0.9496	-0.0004	0.9497	0.9496	-0.0004
25	0.9497	0.9496	-0.0004	0.9497	0.9496	-0.0004	0.9497	0.9496	-0.0004
30	0.9497	0.9496	-0.0015	0.9497	0.9496	-0.0015	0.9497	0.9496	-0.0015
35	0.9497	0.9496	-0.0015	0.9497	0.9496	-0.0015	0.9497	0.9496	-0.0015
40	0.9497	0.9496	-0.0015	0.9497	0.9496	-0.0015	0.9497	0.9496	-0.0015
45	0.9497	0.9496	-0.0025	0.9497	0.9496	-0.0025	0.9497	0.9496	-0.0025
50	0.9497	0.9496	-0.0025	0.9497	0.9496	-0.0025	0.9497	0.9496	-0.0025
55	0.9497	0.9496	-0.0036	0.9497	0.9496	-0.0036	0.9497	0.9496	-0.0036
Average Error			-0.001			-0.001			-0.001

5 Conclusion

This study has built a FPGA-based calculation IC for obtaining power factor and average error rate. Adopting VHDL provides sufficient flexibility and speed to construct the designed circuit by altering some IP cores. Because all the modules were newly designed, they can easily be integrated with each other, resulting in a successful design. The major benefit of the proposed approach is that it executes all logic continuously and simultaneously. The simulated and experimental results confirm that the instantaneous voltage vector used is effective.

The designed FPGA-based system can obtain has advantages including concurrent operation, small

hardware requirement, easy and fast circuit modification, more IP cores, a low cost for a complex circuitry and rapid prototyping, making it a favorable choice for power quality calculation. It can be found that $PF_e < PF_A < PF_V < PF_1 = PF_{1m1} = PF_{1m2}$. Since Q_1 is always positive, PF_1 , PF_{1m1} , and PF_{1m2} are the same. It has been checked in details that the difference between PF_e , PF_A , and PF_V is less than 0.001%. So each of PF_e , PF_A , and PF_V can reveal effects of harmonics.

found It can be that $PF_e < PF_A < PF_V = PF_1 = PF_{1m1} = PF_{1m2}$. The smallest one is also PF_e . The more the unbalanced degree is, the more PF_e is less than others. Since P_1 and Q_1 are kept constant and the later is always positive, PF_1 , PF_{1m1} , and PF_{1m2} are also the same. It is noted that PF_V cannot reveal the unbalanced condition. By the way, PF_A is the second choice to reveal load imbalance. The S_e has the largest integral value because it completely contains the harmonic distortion degree of load powers and the degree of load imbalance.

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