Essay on Teletraffic Models (I)

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Abstract: - Teletraffic (traffic for short) modeling plays a role in telecommunications. There are two categories of the models of traffic. One is stochastic modeling and the other deterministic modeling (Li and Borgnat [1]). Two have their specific applications to practical issues in computer networks. This essay gives a short tutorial of traffic modeling in this regard.

Key-Words: - Traffic modeling; Fractal time series; Bounded modeling of traffic.

1 Introduction
Traffic modeling is a fundamental work in telecommunication systems (Akimaru and Kawashima [2]). A modern telecommunication system is the Internet that is an infrastructure in modern societies. A server serves arrival traffic either in the aggregated case from the point of view of network management or at individual connection level (or the specific class of connections) so that the quality of service (QoS) of the specific connection (or the specific class of connections) can be guaranteed. Therefore, in principle, techniques of traffic modeling are application dependent. We shall brief two categories of the models of traffic, namely, the stochastic modeling and the deterministic one in this paper.

Let \( x(t_i) \) be an arrival traffic function, implying the number of bytes in the \( i \)th packet arriving at \( t_i \) (\( i = 0, 1, 2, \ldots \)), where \( t_i \) is the timestamp of the \( i \)th packet (Li et al. [3]). The function \( x(t_i) \) may represent either aggregated traffic, that is, it consists of arrival packets of all connections at the input of a server, or arrival packets of a specific connection or a specific class of connections. The former is called aggregated traffic while the later traffic at connection level. Network management concerns about stochastic modeling in the aggregated case while QoS relates to bounded modeling at connection level. Without confusions, we use \( x(t) \) and \( x(i) \) to represent a traffic trace in the continuous case and the discrete case, respectively.

The pioneering work of bounded modeling refers to Cruz [4] and that of TAMU (Raha et al. [5,6]). This type of models is developing towards to stochastically bounded modeling, see e.g., Yang [7], Yang and Liu [8], Li et al. [9,10], Wang et al. [11,12], Starobinski and M. Sidi [13], Yaron and Sidi [14], Parekh and Gallager [15], Li and Zhao [16].

As far as the stochastic modeling of traffic was concerned, Partridge used to announce that the self-similar process may be the end of the stochastic modeling of traffic [17]. Unfortunately, that may be over-optimistically. While the early work of self-similar process modeling of traffic (Leland et al. [18], Crovella and Bestavros [19], Beran et al. [20]) attracted attention of researchers, the limitation of this model was noted, see e.g., Paxson and Floyd [21], Tsybakov and Georganas [22]. Rather than the self-similar process modeling, locally self-similar processes may be more agreement with real-traffic data, such as the generalized Cauchy process (Li [23], Li and Lim [24-26]), alpha stable processes (Karasaridis and Hatzinakos [27]), Levy flights (Terdik and Gyires [28], Kogon and Manolakis [29]).

As a 6-page paper, covering too many results in the field of traffic modeling may be unrealistic. Therefore, we aim at providing results adequately for a short tutorial. In the remaining of this easy, Section 2 will give the fractal models of traffic based on fractional Brownian motion (fBm). Bounded modeling is interpreted in Section 3. Discussions and conclusions are given in Section 4.

2 Fractal Models based on FBM
Fractal models of traffic are mainly for aggregated traffic. Traffic on old telephony networks obeys the Poisson model. It has been successfully used in the design of infrastructure of old telephony networks for years as can be seen from [2] and Gibson [30]. It is such a success on old telephony networks that it has
almost been taken as an axiom for modeling traffic in communication systems. Due to unsatisfactory performances of the Internet (e.g., unpleasantly traffic congestions), people began doubting about the Poisson model. To re-evaluate traffic models in the Internet, people began measuring the Internet at different sites during different periods of times (Paxson [31,32] and Internet Traffic Archive at http://www.sigcomm.org/ITA/). Experimental processing real –traffic traces reveals that traffic has fractal properties.

2.1 Fractional Brownian Motion
The early work of fractal modeling of traffic is based on fractional Brownian motion (fBm) and its increment process called fractional Gaussian noise (fGn), which were introduced by Mandelbrot and van Ness [33].

Denote the standard Brownian motion by \( B(t) \) for \( t \geq 0 \) and \( B(0) = 0 \). Let \( B_\alpha(t) \) be the fractional Brownian motion (fBm) defined by using the Weyl integral. Then \([33, Li [34])\), for the Hurst parameter \( 0 < H < 1 \),

\[
B_H(t) - B_H(0) = \frac{1}{\Gamma(H + 1/2)} \left\{ \int_{-\infty}^{0} [(t - u)^{H-0.5} - (-u)^{H-0.5}] dB(u) + \int_{0}^{t} (u)^{H-0.5} dB(u) \right\}. \tag{1}
\]

Denote the autocorrelation function (ACF) of \( B_\alpha(t) \) by \( r_{B_\alpha,W}(t,s) \). Then,

\[
r_{B_\alpha,W}(t,s) = \frac{V_H}{(H + 1/2)\Gamma(H + 1/2)}\left[ |t|^{2H} + |s|^{2H} - |t-s|^{2H}\right], \tag{2}
\]

where

\[
V_H = \text{Var}[B_\alpha(1)] = \Gamma(1-2H)\cos(\pi H)\pi H. \tag{3}
\]

Denote by \( S_{B_{\alpha,W}}(t,\omega) \) the power spectrum density (PSD) function of \( B_{\alpha}(t) \). Then (Flandrin [35]),

\[
S_{B_{\alpha,W}}(t,\omega) = \frac{1}{|\omega|^{2H+1}}(1-2^{-2H}\cos \omega). \tag{4}
\]

From (2) or (3), we see that either the ACF or the PDF of \( B_{\alpha}(t) \) is time varying. Therefore, \( B_{\alpha}(t) \) is nonstationary.

Note that \( B_{\alpha}(t) \) is self-similar because it satisfies the definition of self-similarity [34]. As a matter of fact, from (1), one has

\[
B_{\alpha}(at) = a^\alpha B_{\alpha}(t), \quad a > 0, \tag{5}
\]

denotes equality in the sense of probability distribution. We cite Norros [36], Song and Ng [37] for the traffic using fBm model.

From (4), one sees that the PSD of fBm is divergent at \( \omega = 0 \), exhibiting a case of \( 1/f^a \) noise, see Csabai [38] for the early work of \( 1/f \) noise of traffic. The relationship between the fractal dimension of fBm, denoted by \( D_{\text{fBm}} \), and its Hurst parameter denoted by \( H_{\text{fBm}} \) is given by

\[
D_{\text{fBm}} = 2 - H_{\text{fBm}}. \tag{6}
\]

2.2 Fractional Gaussian Noise
Denote by \( C_\alpha(t; \varepsilon) \) the ACF of fGn. Then,

\[
C_\alpha(t; \varepsilon) = \frac{V_H \varepsilon^{2H-2}}{2} \left[ \left( \frac{|t|}{\varepsilon} + 1 \right)^{2H} + \left( \frac{|t|}{\varepsilon} - 1 \right)^{2H} - 2 \frac{|t|^{2H}}{\varepsilon^{2H}} \right], \tag{6}
\]

where \( \varepsilon > 0 \) is used by smoothing fBm so that the smoothed fBm is differentiable. The PSD of fGn is given by (Li and Lim [39])

\[
S_{\text{fGn}}(\omega) = V_H^2 \sin(H \pi) \Gamma(2H + 1)|\omega|^{-2H}. \tag{7}
\]

The literatur of fGn modeling of traffic is rich, see e.g., [18-22], Willinger and Paxson [40], Li et al. [41,42], Li [43,44], Adas [45], Michiel and Laevens [46].

The ACF of the discrete fGn (dfGn for short) is given by

\[
r_{\text{dfGn}}(k) = \frac{V_H^2}{2} \left[ (|k| + 1)^{2H} + (|k| - 1)^{2H} - 2 |k|^{2H} \right]. \tag{8}
\]

Its PSD is given by

\[
S_{\text{dfGn}}(\omega) = 2C_f(1-\cos \omega) \sum_{r\in\mathbb{Z}} 2\pi r + \omega |\omega|^{-2H-1}. \tag{9}
\]

where \( C_f = V_H^2 (2\pi)^{-1} \sin(\pi H) \Gamma(2H+1) \) and \( \omega \in [-\pi, \pi] \) (Sina [47]).

2.3 Generalized FGN
The standard fGn has its limitation in modeling small lags of a traffic series. To release that limitation, Li [48] introduced the generalized fGn. Its ACF in the discrete case is given by

\[
r_{\text{gGn}}(k;H,a) = \frac{V_H^2}{2} \left[ \left( |k| + a \right)^{2H} - 2 |k|^{2H} + \left( |k| - a \right)^{2H} \right], \tag{10}
\]

where \( 0 < a \leq 1 \). It can be easily seen that the above \( r_{\text{gGn}}(k;H,a) \) becomes the ACF of the standard fGn if \( a = 1 \).

2.4 Local Hurst Function
Traffic has multifractal properties, see e.g., Abrey and Veitch [49], Taqqu et al. [50], Feldmann et al.
Due to this, the above fBm or fGn, which are in the sense of monofractal because the self-similarity holds for all time scales, we need considering processes that are locally self-similar. One of possible processes is to generalize fBm by replacing the Hurst parameter $H$ by a continuously deterministic function $H(t)$ (Lim and Muniandy [52]). The function $H(t)$ satisfies $H: [0, \infty) \rightarrow (0, 1)$. Denote the generalized fBm by $X(t)$, instead of $B_H(t)$, so as to distinguish it from the standard one. Then,

$$X(t) = \frac{1}{\Gamma(H(t) + 1/2)} \left\{ \int_0^t [(t-u)^{H(t)-0.5} - (-u)^{H(t)-0.5}] dB(u) \right\},$$

(11)

The following ACF holds for $\tau \rightarrow 0$

$$\mathbb{E}[X(t)X(t+\tau)] = \frac{V_{H(t)}}{(H(t)+1/2)\Gamma(H(t)+1/2)}[\Gamma^{2H(t)}(\tau) + \tau^{2H(t)} - \Gamma^{2H(t)}(\tau)].$$

(12)

In fact, $H(t)$ can be regarded as a tool to characterize local properties of fBm. This can be seen when the self-similarity is expressed by $X(at) = a^{H(t)}X(t)$, $a > 0$.

(13)

Based on the local growth of the increment process, one may write a sequence expressed by

$$S_k(j) = \frac{m}{N-1} \sum_{i=1}^N |X(i+1) - X(i)|, \quad 1 < k < N,$$

(14)

where $m$ is the largest integer not exceeding $N/k$. Then, $H(t)$ at point $t = j/(N-1)$ is given by

$$H(t) = -\frac{\log(\sqrt{\pi/2}S_k(j))}{\log(N-1)},$$

(15)

see Peltier and Levy-Vehel [53]. Fig. 1 is a plot of a real-traffic trace. Fig. 2 shows its $H(t)$, which clearly gives the evidence of the multifractal property of traffic (Li et al. [54]).

Fig. 1. Traffic time series $X(i)$ of BC-pAug89.

The properties of traffic are listed as follows.
1) Non-summable autocorrelation functions.
2) $1/f$ law of power spectra.
3) Heavy-tailed distributions.
4) Infinite variances.
5) Large time-scale phenomena (robustness of traffic dynamics over wide range).
6) Highly local irregularity.

Note that the property 4) should be understood in the engineering sense (Li [55]). The multifractal property of traffic has been applied to anomaly detection of distributed denial of service attacking, see e.g., Li [56,57], Xia et al. [58].

3 Bounded Modeling of Traffic

3.1 Bounded Model of TAMU

Denote by $x(t)$ the arrival traffic. If $x(t_0 + t) - x(t_0) \leq F(t)$ for $t > 0$ and $t_0 > 0$, $F(t)$ is called the maximum traffic constraint function of $x(t)$ (Wang et al. [59], Tang et al. [60]). We term that as the model of TAMU since it was first introduced by the scholars in TAMU [5]. It has wide applications to various issues in networking, such as end-to-end delay [5], intrusion detection (Bettati et al. [61], Li et al. [62]), admission control (Raha et al. [63]), integrated routing (Jia et al. [64]), and so on.

3.2 Bounded Model of Cruz

Note that the amount of traffic generated in the interval $[0, t]$ is upper-bounded by

$$\int_0^t x(u)du = X(t) \leq \sigma + \rho t,$$

(16)

where $\sigma$ and $\rho$ are constants and $t > 0$. The parameter $\sigma$ characterizes the burstness of traffic while $\rho$ the long-term average rate. This kind of model has applications in many aspects of networking, see e.g., Boudec and Patrick [65] and references therein.

Two types of models have their advantages. On the one hand, the maximum traffic constraint...
function $F(t)$, i.e., the model of TAMU, does not have a concrete form. Hence, it may contain other possible forms of functions, including the one on the right side of (16), i.e., the one of Cruz. On the other hand, the model of Cruz has a concrete function form that contains two parameters to separately characterize the local property and the global one of traffic. One advantage in common is that both need not have prior information of the statistics of traffic. Hence, they have particular applications to the traffic at connection level.

4 Discussions and Conclusions

Two parts of the basic elements of traffic modeling have been essayed. We discussed the stochastic modeling based on fBm. For the bounded modeling, we described the model of TAMU and the one of Cruz. The stochastically bounded modeling is active in the field, see e.g., [8] and [10]. For the stochastic modeling, spatial modeling (Uhlig [66]) and the prediction of traffic (Li and Li [67], Elbiaze et al. [68]) are challenge. Besides, measurement accuracy of traffic is worth noting (Janowski and Owezarski [69], Li and Zhao [70], Li [71,72]).

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