New Method of the Numerical of Gauss-Lobatto Quadrature Rules With Precision Degree (2n+5)

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Abstract: In this paper we present a new method for numerical solution integration Gauss-Lobatto. A new method increase the precision degree from (2n+1) to (2n+5). The method is based upon to suppose boundaries integration are unknown and add another two unknown to system, with find this unknowns the precision degree get (2n+5).

Key Words: Gauss-Lobatto, precision degree, boundary integration

1. INTRODUCTION

We know the Gauss-Lobatto quadrature rules is

\[
\int_{a}^{b} x^j \, dx = \sum_{i=1}^{n} w_i x_i^j + p a^j + q b^j + \frac{z^{n+1}}{2},
\]

where \( x_i \) is the node point, \( w_i \) is coefficient, \( p \) and \( q \) are two another unknowns with find this unknowns by using undetermined coefficient method by substituting basis function equation (2)

\[
f(x) = x^j, \quad j = 0, 1, \ldots, (2n+1).
\]

The expression the below the Gauss-Lobatto Quadrature rules theorem.

Theorem 1.1. if \( f \in C^{(2n-2)}[-1,1] \) then

\[
\frac{1}{2} \int_{-1}^{1} f(x) dx = \sum_{i=1}^{n} w_i f(x_i) + \frac{p}{2(n-1)} f(-1) + \frac{q}{2(n-1)} f(1) + R(f),
\]

(4)

Where \( p_n(x) \) is Legendre polynomial with degree \( n \) and \( x_j \) is (j-1)th \( \left( p_{(n-1)}(x) \right) \) root (Babolian and dehghan 2004). In addition \( R(f) \) and \( w_j \) to explain by the equations (5).

\[
w_j = \frac{2}{n(n-1)[p_{n-1}(x_j)]^2}, \quad x_j \neq \pm 1
\]

(5)

\[
R(f) = \frac{-n(n-1)^3 2^{2n-1}[(n-2)!]^4}{(2n-1)[(2n-2)!]^3} \int_{-1}^{1} f(2n-2)(\psi),
\]

\[-1 < \psi < 1\]
The special case for the weight function

\[ w(x) = \left(1 - x^2\right) \frac{1}{2} \] is Lobatto-Chebichev Quadratic first kind in equation (6).

\[
\int_{-1}^{1} \frac{f(x)}{\sqrt{1-x^2}} \, dx = \pi \sum_{n=1}^{\infty} \frac{f(x_i)}{(2n-3)(2n-2)!} \psi(\nu),
\]

where \( \psi(\nu) = -1 < \psi < 1 \).

(6)

Nowadays using this method for solution many of applied mathematics for example using for the solution of numerical integration equation. In the present paper, we introduce a new numerical method to increase precision degree Gauss-Lobatto from \((2n+1)\) to \((2n+5)\). The method consists of additional four unknown to equation (3) by unknown boundary integration and add two variable to equation (3) in section 2, we describe the new method

2. THE NEW METHOD WITH PRECISION DEGREE \((2n+5)\)

The basis new method upon the first increase precision degree Gauss-Lobatto Quadrature rules from \((2n+1)\) to \((2n+3)\) by extended a basis function \( f(x) = x^j \) from \( j=0,1,\ldots,2n+1 \) to \( j=0,1,\ldots,2n+3 \), with to suppose boundary integration are unknowns in equation (3), it means substituting equation (7) by means of equation (2).

(babolian 2004).

\[
\frac{b}{a}^{j} = \frac{b^{j+1} - a^{j+1}}{j+1} = \sum_{i=1}^{n} w_i x_i^j + pa^j + qb^j,
\]

(7)

\( j = 0,1,\ldots,2n + 3 \).

where \( a, b, p, q, x_1, x_2, \ldots, x_n \) and \( w_1, w_2, \ldots, w_n \) are unknowns and we find by numerical solution, the solution system (7) given by (babolian 2004) with find unknowns this system since the basis function \( f(x) = x^j \) extended from \( j=0,1,\ldots,2n+1 \) to \( j=0,1,\ldots,2n+3 \), the precision degree two increase It means of the precision degree the Gauss-Lobatto Quadrature rule increasing to \((2n+3)\). Now we added one unknown \( z \) to equation (7) for \( j=0,1,\ldots,2n+4 \) to give equation (8).

\[
\frac{b}{a}^{j} = \frac{b^{j+1} - a^{j+1}}{j+1} = \sum_{i=1}^{n} w_i x_i^j + pa^j + qb^j + z(a + b)^j,
\]

(8)

Where \((a \text{ and } b)\) known from solution system (7).

By solution system (8) with using from solution system (7) and substituting \((a \text{ and } b)\) in equation (8) the precision degree Gauss-Lobatto increasing to \((2n+4)\). We finding the numerical solution nonlinear system (8) by using software maple (9.5) and to design simple algorithm. Now we by using gives from solution system (8) the precision degree Gauss-Lobatto increasing to \((2n+5)\). Therefore we added a another unknown to system (8) then equation (9) substituting equation (8).

\[
\frac{b}{a}^{j} = \frac{b^{j+1} - a^{j+1}}{j+1} = \sum_{i=1}^{n} w_i x_i^j + pa^j + qb^j + z(c)^j,
\]

(9)

Where in addition to unknown system (8) variable \((c)\) is unknown. By solution system (9) with using from solution system (8) the precision degree Gauss-Lobatto increasing to \((2n+5)\). We finding the numerical solution nonlinear system (9) by using software maple (9.5) and to design simple algorithm.
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