Measurement of the stereoscopic rangefinder beam angular velocity using the digital image processing method

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Abstract: - This paper deals with the measurement of the mechanical construction kinematical characteristics using the digital image processing method. This method has been utilized to determine the angular velocity and the angular position of the stereoscopic rangefinder beam during its operation.

Key-Words: - vibration, measurement, digital, image, processing, cross-correlation

1 Introduction

In last years a project aimed on the research of the passive tracking systems has been solved at the University of Defense in Brno, Czech Republic [1]. This project is focused on the research and development in the area of the weapons and fire control systems design (e.g. [2], [3], [4] and [5]). During the solution of this project the functional model of the passive optoelectronic rangefinder has been developed, which has been using for determining the position of the target and for tracking of this target as well.
The design of the rangefinder mentioned above is obvious from Fig. 1.

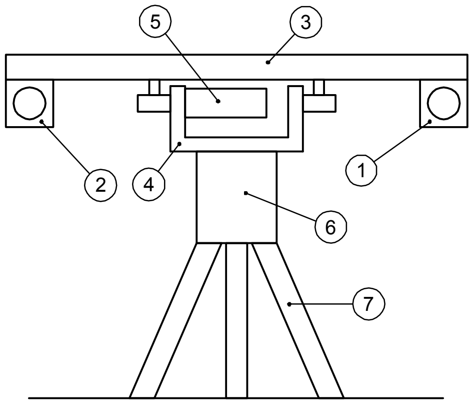

Fig. 1 Optoelectronic rangefinder

Two digital cameras 1 and 2 are placed on the beam 3 of the rangefinder. The beam is connected by means of the rotational part 4 to the head of the tripod 6. The spatial position of this beam is established by means of two synchronous brushless servomotors. One servomotor 5 is used for rotating the beam around the horizontal axis (so called elevation motion); the other servomotor (placed inside of the tripod head 6) is used for rotating the beam around the vertical axis (so called the traversal motion). The whole rangefinder structure is mounted on the stationary tripod 7.
The position of the target with respect to the rangefinder is determined from the pair of images, acquired synchronously from the digital cameras, using the methods of the digital stereophotogrammetry [6], [7].
The tracking of the target is achieved by setting the appropriate angular velocity to the servomotors so as the angular motion of the rangefinder was in accord with the angular motion of the target with respect to the rangefinder position, whereas the angular velocity of the servomotors was determined from control deviation α in both vertical and horizontal direction. For the control deviation in the horizontal direction the following formula is valid (lens distortion is not considered)

$$a_x = \arctan\left(\frac{x_i - x_0}{c_x}\right), \qquad (1)$$

where α_x is the horizontal control deviation, x_i is the horizontal image coordinate of the target, x_0 is the horizontal coordinate of the principal point and c_x is the principal distance of the camera in the horizontal direction. For vertical control deviation α_y the relation is analogical.
To control the angular velocity of the both servomotors the proportional-integral (PI) controller is used in the basic form [8]

$$w_d(t) = Ka(t) + \frac{1}{t}\int_0^t a(t)\,dt, \qquad (2)$$

where $\omega_d(t)$ is desired angular velocity of the beam in the particular direction, $\alpha(t)$ is the control deviation determined from the image information, K is the proportional constant of the controller and τ is the time constant of the integral part of the controller.

2 Problem Formulation

The constants K and τ of the PI controller were determined using the software simulation of the controller for extreme values of the probable target velocity and the target distance to achieve the time optimal tracking of the selected target.

Furthermore, the tracked target has to be kept in the field of view of the used camera, whereas this field of view can be further reduced to decrease the amount of computational operations needed for target position determination, therefore to minimize the time consumption and maximize the measurement rate respectively.

The example of the simulation result for the target moving with velocity 20 $m.s^{-1}$ (72 $km.h^{-1}$) at distance 200 m is shown in Fig. 2.

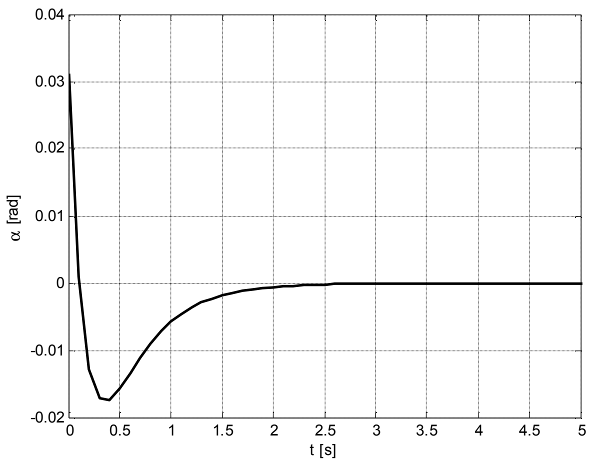

Fig. 2 Control deviation course

Unfortunately, after applying the determined values of the K and τ into the functional model of the rangefinder it has been fond out that the target ran out from the selected area of interest, therefore the tracking algorithm crashed immediately after marking the target. From visual inspection the reason seems to be the higher real angular velocity of the rangefinder beam than it was set to servomotor according to the estimated control deviation. Further effort was aimed on finding the sources of this discrepancy.

3 Problem Solution

Preliminary analysis of the target image coordinates record showed that despite of the set angular velocity the real angular velocity could be significantly higher than the desired one and it changes periodically in addition. The following sources of this periodical motion were assumed:

- vibrations of the rangefinder beam as of the elastic body,
- vibrations of the rangefinder beam as of the solid body elastically mounted to the servo (output shaft of the transmission),
- elastically mounted rangefinder as the solid body on the basis (tripod) and
- incorrectly controlled servo.

To confirm the actual reason of the parasitic motion the series of the angular motion measurements was prepared.

3.1 The angular velocity of the rangefinder beam measurement analysis considering the usage of image information

The angular displacement and angular velocity of the rangefinder beam was determined by means of two methods:

a) evaluating the signal from the servomotor's resolver and
b) computing the rotational motion from the image acquired from digital camera, mounted on the appropriate place on the rangefinder construction.

Ad a) The servomotor is equipped with so called resolver, which is the rotary electrical transformer used for measuring degrees of rotation. The instantaneous position of the servomotor shaft can be read throughout the controller bus as the integer number, whereas this number represents the number of increments with respect to the set absolute zero. The angular displacement of the rangefinder beam α can be computed using the formula

$$a = \frac{2ip}{n_c r}, \qquad (3)$$

where i is the number of increments read from the resolver, n_c is the number if increments for one turn of the servomotor shaft, r is the ratio number of the gear placed between the servomotor and the beam and π is the circular constant.

The resolver is relatively sensitive and precise sensor of angular displacement. For n_c = 2^{16} and r = 135 the minimum theoretically distinguishable value of the beam angular displacement is approximately 0.71 μrad. On the other hand, the resolver signal is affected by motion of the rangefinder structure in the feedback and it cannot be used for estimating the angular motion of the structure especially in the case of the elastic beam vibrations. Therefore, the resolver signal was used only as the supplemental information and the actual angular motion of the rangefinder structure was determined by the optical method presented hereafter.

Ad b) If the digital camera is placed on the moving structure, the relative motion of the camera with respect to the observed object takes effect in the change of the image coordinates of the object. In case of the general spatial motion the reconstruction of the structure motion is relatively complicated process, which usually requires the usage of greater number of the cameras. Also the calibration of this measurement is quite complicated with emphasis on the precise determination their relative position.

If the structure motion is simple (e.g. rotation around one axis only), the situation significantly simplifies and we can reconstruct the structure motion using only one camera, placed on the appropriate place. Hereafter we will suppose only rotating motion of the rangefinder around the vertical axis. The camera is mounted on the rangefinder so that the columns of the camera detector are parallel to the rotation axis.

If the rotation axis goes through the projection centre of the camera, the angle of the beam rotation α can be determined using the formula

$$\alpha = \arctan\left(\frac{c\left(p_{i+1}-p_i\right)}{c^2+p_{i+1}p_i}\right), \tag{4}$$

where c is the principal distance of the camera, p_i is the horizontal image coordinate of the observed object in the beginning of the rotation and p_{i+1} is the horizontal image coordinate of the observed object in the end of the rotation.

If the projection centre of the camera lies off the rotation axis, the angle of the beam rotation α can be determined solving the formula in its implicit form

$$\begin{aligned} &p_{i+1}\left(-\left(X_i-b\right)\sin\alpha+\left(Z_i-a\right)\cos\alpha+a\right)=\\ &=c\left(\left(X_i-b\right)\cos\alpha+\left(Z_i-a\right)\sin\alpha+b\right), \end{aligned} \tag{5}$$

where b is the distance between the projection centre and the rotation axis in direction parallel to the detector plane, a is the distance between the projection centre and the rotation axis in direction perpendicular to the detector plane, X_i is the observed object coordinate in the direction parallel to the detector plane in the beginning of the motion and Z_i is the observed object coordinate in the direction perpendicular to the detector plane in the beginning of the motion. The other variables are identical with formula (4).

Comparing the formula (5) to formula (4) it is obvious that the value α depends not only on the position of the projection centre with respect to the rotation axis, but also on the position of the observed object with respect to the camera. Therefore, even if we know the coordinates of the projection centre a and b (for example from the design documentation) and if we use the relation between X_i and Z_i according to the formula

$$X_i=\frac{p_i}{c}Z_i, \tag{6}$$

we still have only one conditional equation (5) for two unknown variables α and Z_i. It means that at least on more conditional equation has to be obtained, e.g. using some supplemental measurement.

Substituting (6) into (5) we get the modified form of conditional equation

$$\begin{aligned} &p_{i+1}\left(-\left(\frac{p_i}{c}Z_i-b\right)\sin\alpha+\left(Z_i-a\right)\cos\alpha+a\right)=\\ &=c\left(\left(\frac{p_i}{c}Z_i-b\right)\cos\alpha+\left(Z_i-a\right)\sin\alpha+b\right), \end{aligned} \tag{7}$$

and if we extract the Z_i at the both sides of this equation, we get the final formula

$$\begin{aligned} &p_{i+1}\left(-\left(\frac{p_i}{c}-\frac{b}{Z_i}\right)\sin\alpha+\left(1-\frac{a}{Z_i}\right)\cos\alpha+\frac{a}{Z_i}\right)=\\ &=c\left(\left(\frac{p_i}{c}-\frac{b}{Z_i}\right)\cos\alpha+\left(1-\frac{a}{Z_i}\right)\sin\alpha+\frac{b}{Z_i}\right). \end{aligned} \tag{8}$$

It can be clearly seen that for $Z_i >> a$ and $Z_i >> b$ it is valid

$$\frac{a}{Z_i}\to 0 \quad \text{and} \quad \frac{b}{Z_i}\to 0, \tag{9}$$

therefore the formula (8) can be written in form

$$\begin{aligned} &p_{i+1}\left(-\frac{p_i}{c}\sin\alpha+\cos\alpha\right)=\\ &=c\left(\frac{p_i}{c}\cos\alpha+\sin\alpha\right), \end{aligned} \tag{10}$$

which leads directly to the formula (4). In the other words, if the observed reference object is placed far enough comparing to the design dimensions of the camera with respect to the rotation axis, the angle of structure rotation α can be computed directly by means of the formula (4) using the evaluated image coordinates of the observed object from one camera, which significantly simplifies the measurement.

The simulation of the measurement has been made, when for the known rangefinder design dimensions and given range of the reference object coordinates the image coordinates of the reference object were computed as the function of the angle α. Then the angle α was inversely computed from these image coordinates using the both formulas (4) and (5). The difference of these computations is shown in Fig. 3 as the Z_i coordinate dependence. From this diagram it can be seen that if the object distance is longer than 15 m, the relative error in angle α evaluation is lower than 0.1%.

If we know the course of the angle α with respect to time as the vector of discrete values $\boldsymbol{\alpha}$ with respect to the vector of discrete values $\mathbf{t}$, the angular velocity can be computed as the numerical derivation of the vector $\boldsymbol{\alpha}$ with respect to time. Because the angular displacement of the rangefinder beam between two consecutive frames is important, the simplest method of numerical

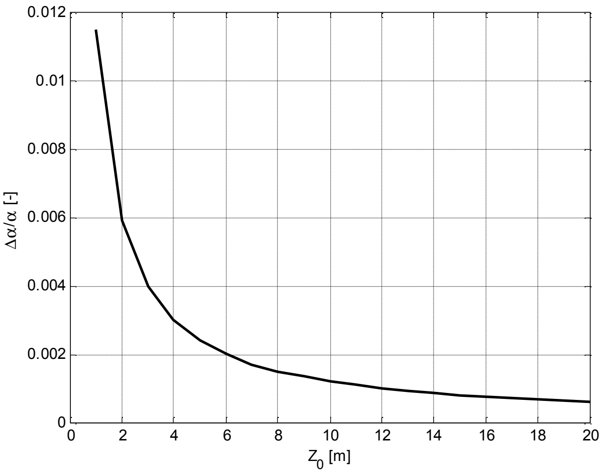

Fig. 3 Relative error in angle α evaluation

derivation was used to compute the angular velocity ω in form

$$w_i = \frac{a_{i+1} - a_i}{t_{i+1} - t_i}, \quad \text{for } i = 0..n-2, \qquad (11)$$

where α_i and α_{i+1} are the elements of the vector **α**, t_i is the element of the vector **t** and n is the size of these vectors.

3.2 The angular velocity of the rangefinder beam measurement implementation

To verify the hypotheses mentioned in the beginning of the Chapter 3 of this article the following measurements has been arranged.

A) Hypothesis: The rangefinder beam vibrates as the elastic body. Two cameras are mounted at the ends of the rangefinder beam. If the beam behaves as the elastic body (the motion in basic natural shape of vibration is supposed mainly), the record of the angle α from the first camera will show the phase shift with respect to the record of the other camera. Otherwise, the beam can be considered solid.

B) Hypothesis: The rangefinder beam vibrates as the solid body elastically mounted at the output shaft of the transmission. One camera is mounted on the end of the beam; the other is mounted on the output shat of the transmission. Again, if the record of the angle α is phase-shifted with respect to the record of the angle α from the other camera, the connection between the output shaft of the servo is elastic. Otherwise, the connection is solid and the structure behaves as the solid body.

C) Hypothesis: Elastically mounted rangefinder as the solid body on the basis (tripod). One camera is mounted on the end of the beam; the other is mounted on the head of the tripod. Theoretically, the head of the tripod should stay still during the rotation motion of the rangefinder. If the head of the tripod moves synchronously with the rangefinder, the reason of the parasite vibrations could be the elasticity of the tripod.

D) Hypothesis: Incorrectly controlled servomotor. The rangefinder beam is dismounted from the output shaft of the transmission, so as it cannot retroactively effect the servo behavior by its inertia. Camera is mounted on the output shaft of the transmission. If the servo response, measured by means of the camera, is phase or amplitude shifted with respect to the desired value of the angular velocity, the servomotor is controlled incorrectly. Otherwise, the servo motor is controlled correctly, or the non-linear response of the servo is caused only by inertia of the rangefinder beam.

The series of measurements has been made for configurations mentioned above, whereas for each configuration the desired angular velocity of the beam has been set to values -0.005 rad.s^{-1}, -0.01 rad.s^{-1}, -0.05 rad.s^{-1} and -0.1 rad.s^{-1}, which represents the range of the angular velocities used for tracking of the ground targets. As the example, in the following pictures the results of the evaluated angular velocity of the rangefinder beam analysis for desired angular velocity ω = 0.01 rad.s^{-1} are shown for variant when the camera 0 is mounted at one end of the beam and the camera 1 is mounted at the other end of beam (variant A).

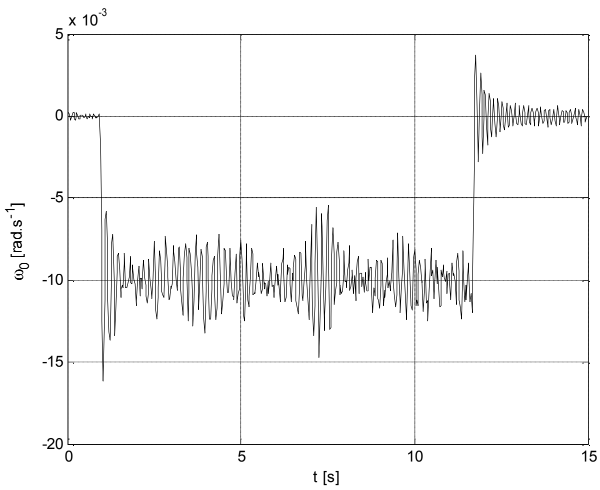

Fig. 4 Angular velocity of camera 0 - A

In Fig. 5 the angular velocity of the beam evaluated using the image information from camera 1 is shown.

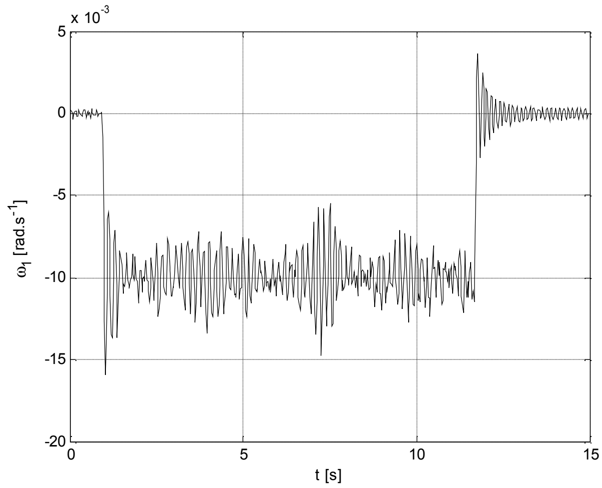

Fig. 5 Angular velocity of camera 1 - A

The similarity of these courses was compared using the normalized cross-correlation method according to the formula [9]

$$R_{w_0 w_1}\left(T_j\right)=\frac{\sum_{i=0}^{n-2}\left(w_0\left(t_i\right)-\bar{w}_0\right)\left(w_1\left(T_j+t_i\right)-\bar{w}_1\right)}{\sqrt{\sum_{i=0}^{n-2}\left(w_0\left(t_i\right)-\bar{w}_0\right)^2 \sum_{i=0}^{n-2}\left(w_1\left(T_j+t_i\right)-\bar{w}_1\right)^2}} \tag{12}$$

where $\bar{w}_0$ and $\bar{w}_1$ are mean values of the vectors $\boldsymbol{\omega}_0$ and $\boldsymbol{\omega}_1$ respectively. For comparison only the motion parts of the records were extracted. The result of the cross-correlation is shown in Fig. 6.

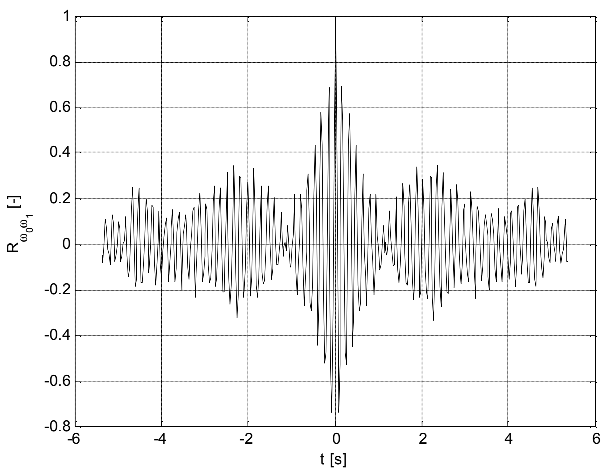

Fig. 6 Cross-correlation of ω_0 and ω_1 - A

In can be seen from Fig. 6 that the cross-correlation response displays almost perfect match in both amplitude and phase part of the ω_0 and ω_1 records. It means that the rangefinder beam behaves as the solid body and the parasitic vibrations, displayed in the Fig. 4 and Fig. 5, are caused by elasticity of another part of the rangefinder design. The B) measurement configuration was analyzed with the same results, i.e. the connection between the beam and the output shaft of the transmission behaves as absolutely stiff in the operational range of the tracking velocities.

The record of the angular velocity ω_0 of the camera mounted at the end of the beam and the angular velocity ω_1 of the camera mounted at the tripod head is shown in Fig. 7 and Fig. 8.

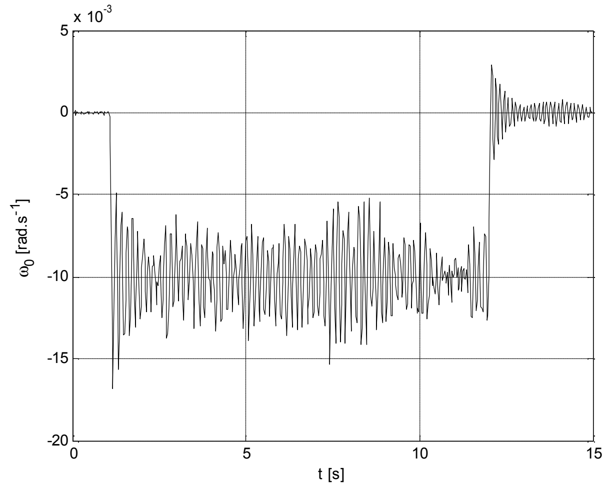

Fig. 7 Angular velocity of camera 0 - C

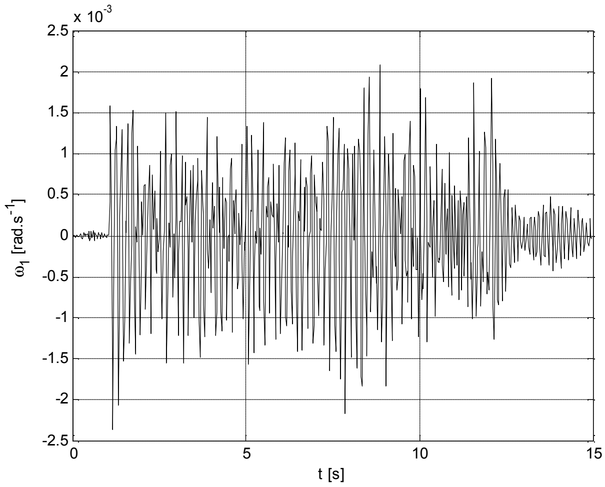

Fig. 8 Angular velocity of camera 1 - C

Also in this case only the motion part of these records was used for analysis. Already from these time records it is obvious that the tripod, which should be absolutely stiff theoretically, shows the vibrations of relatively significant level.

The correlation response for the measurement configuration C) is displayed in Fig. 9. It can be seen from this diagram that even if the maximum of the correlation response is in the zero point, the linear dependency between ω_0 and ω_1 is weaker than in case of the variants A) and B).

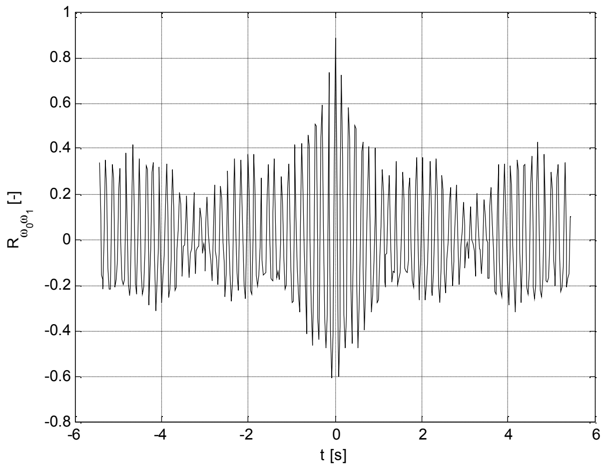

Fig. 9 Cross-correlation of ω_0 and ω_1 - C

3.3 The angular velocity of the rangefinder beam measurement evaluation

Taking into account the results of the measurements mentioned above the following partial conclusions can be made.

Variant A) Because the records of the rangefinder beam angular velocities at the ends of the beam do not show the difference in the both amplitude and phase part, it can be said that the beam behaves as the solid body in the range of the operating tracking velocities.

Variant B) Because the records of the angular velocities at the end of the beam and at the output shaft of the transmission do not show the significant difference in the both amplitude and phase part, it can be said that the connection of the rangefinder beam to the output shaft of the transmission can be considered solid in the range of the operating tracking velocities.

Variant C) Because the record of the angular velocity of the tripod head is not zero, it is obvious that the tripod is not stiff enough. Furthermore, the record of the angular velocity at the end of the rangefinder beam shows significantly higher amplitude swing than the record of the angular velocity at the head of the tripod and these records show the phase shift in the transition part. Therefore, it can be said that the low stiffness of the tripod can be considered as the main reason of the parasitic vibrations of the rangefinder beam. In addition, the amplitude and phase shift between the motion of the beam and tripod head can mean that another flexible connection exists between the output shaft of the transmission and the tripod head (it could be likely the backlash of the transmission).

4 Conclusion

The simple method of the vibration characteristics measurement using the digitized image information is presented in this article. This method can be used for the fast evaluation of the angular displacement and velocities of the civil and mechanical structures, where the direction of the motion vector can be estimated in advance and when the added camera mass does not effect the dynamic characteristics of the structure significantly.

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