

# Estimating Local Thickness for Finite Element Analysis

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*Abstract:* Within the development of motor vehicles, crash safety is one of the most important attributes. To comply with the ever increasing requirements of shorter cycle times and costs reduction, car manufacturers keep intensifying the use of virtual development tools, such as, for crash simulations, the explicit finite element method (FEM). The accuracy of the simulation process is highly dependent on the accuracy of the model, including the midplane mesh. One of the roughest approximations typically made is the actual part thickness which, although most frequently modelled as a constant value, can, in reality, vary locally. Availability of per element thickness information, which does not exist explicitly in the FEM model, is one key enabler and can significantly contribute to an improved crash simulation quality, especially regarding fracture prediction.

Although not explicitly available, thickness can be inferred from the original CAD geometric model through geometric calculations. This paper proposes and compares two thickness estimation algorithms based on ray tracing and nearest neighbour 3D range searches. A systematic quantitative analysis of the accuracy of both algorithms is presented, as well as a thorough identification of particular geometric arrangements under which their accuracy can be compared. These results enable the identification of each technique's weaknesses and hint towards a new, integrated, approach to the problem that linearly combines the estimates produced by each algorithm.

*KeyWords:* Automotive crash simulations, structural modelling, FEA mesh, thickness estimation, ray tracing

## 1 Introduction

Today, the automotive industry is challenged with a continuous rising number of demands taking a strong influence on the development process. The need for CO<sub>2</sub> and vehicle weight reduction as well as the need to constantly improve crashworthiness makes it necessary to fully utilize the deployed materials as efficiently as possible. For crash simulations the explicit Finite Element Method (FEM) has been applied for a long time. However, this process can only be successful if the numerical methods are capable and have a high confidence level.

The properties of plastic parts are particularly difficult to predict due to the intrinsic complex behaviour of those materials. However, as plastics are pervasive in many different automotive applications, it becomes vital to model and simulate those parts under service

conditions. Within the area of crash simulation of thermoplastic parts, the actual local thicknesses play a significant role for accurate deformation and fracture behaviour prediction. Since most thermoplastic parts are injection moulded, the actual thickness can vary significantly throughout a part. That thickness distribution is not explicitly available within 2D crash meshes (often used in full car crash simulations), but it exists implicitly in the full part geometry in CAD files (like IGES). However, currently there are no automated and precise ways to extract that local thickness distribution from the CAD files, seriously limiting the precision – and thus, the potential benefit – of crash simulations.

Finite Element Analysis (FEA) is a numerical approach for calculating approximate solutions of partial differential equations and integral equations, enabling the numerical solution of many complex problems in

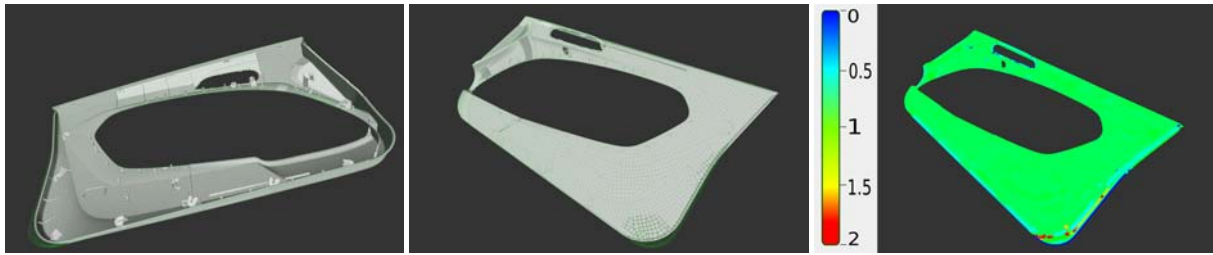


Figure 1: Different views of a car door geometry (grey) and the respective midplane mesh (green). Ideally the midplane mesh runs inside the part's surface, but in reality it might extend outside (dark green regions). The figure at the right presents, for each element of the midplane mesh, the estimated thickness as a pseudo color map.

structural mechanics; FEA is the standard approach for complex systems, particularly in the industry setting [6] and the main method for vehicle crash simulation of thermoplastic parts.

The entire geometric domain of the part/system under study is discretized and modelled by a mesh comprised of a large set of finite elements that intersect at points termed nodes [7]. Elements are then assigned properties, such as thickness, density, tensile strength, etc. This method was initially proposed in the 1950s for airframe and structural analysis [2]. In 1973 a rigorous mathematical foundation was provided, enabling its expansion to many new applications [3]. Besides structural mechanics, FEA has been used in a large variety of fields such as acoustics, fluid dynamics, medicine and many others [4, 5].

This paper proposes and compares two techniques to estimate local thicknesses from geometric models of automotive thermoplastic parts and make these estimates available to vehicle crash FEA simulations. The geometry of the parts is described as a closed surface on a CAD file. FEA simulators use discrete approximate representations of this geometry, which are meshes of polygons that, ideally, run in the middle of the closed surface, being referred to as midplane meshes (figure 1). Midplane meshes do not contain any information about the part's local thickness. However, this information is crucial to allow accurate behaviour prediction in automotive crash simulations. This paper addresses automatic estimation of thickness on a per-element basis, using as inputs the CAD geometry model and the midplane mesh. The proposed thickness estimation techniques, based on ray tracing and nearest neighbour 3D range searches, allow tagging each mesh element with its associated local thickness, thus empowering accurate vehicle crash simulations. The paper contribution is a systematic quantitative analysis of the accuracy of both techniques, as well as a thorough identification of particular geometric arrangements under which the methods' accuracy can be compromised. These re-

sults enable identifying weaknesses and suggesting new approaches to the problem.

The next section presents the two thickness estimation algorithms, the methodology used to assess their respective accuracies and an analysis of the obtained results. Section 3 proposes improvements that significantly increase accuracy and the paper closes with some concluding remarks.

Note that all numerical information about parts' thicknesses, including final estimates, is normalized so that real values are 1; thus listed values do not represent the parts' real thicknesses.

## 2 Thickness estimation algorithms

Ideally, the midplane mesh runs inside the closed surface and parallel to it; intuitively, in such cases the thickness at the centroid of each midplane mesh element is the sum of the distances between this centroid and some surface point on each side of the element. With this definition in mind two thickness estimation algorithms are proposed.

**Ray Tracing (RT)** - a ray is shot for each side of the midplane mesh element, with origin on the element's centroid and direction equal to the element's normal [10]. Ideally each of these rays intersects the part's geometry; the sum of both intersections' distance is taken as an estimate of the part's local thickness.

**Nearest Neighbor (NN)** - this algorithm performs a search on the surface geometry to locate which point is nearer to the mid-plane mesh element centroid [9]. Actually, two such searches are performed to locate two points, each on a different side of the mid plane mesh element. The sum of the distances from the element's centroid and these two points is taken as an estimate of the part's local thickness.

Since the geometry models are themselves represented as a mesh of polygons a kd-tree is used to accelerate each of these algorithms [9].

In the ideal case both algorithms return exactly

the same thickness estimate. Real parts, however, include complex geometric configurations and/or incorrect midplane mesh approximations. Correctly handling such cases requires a systematic quantitative analysis of the proposed algorithms behaviour and accuracy. Using real automotive parts is difficult because the exact local thicknesses are unknown. Instead seven simple synthetic parts, whose exact thicknesses are known (normalized to 1 mm for all parts), were modelled and used throughout the whole validation process. For each part three different midplane meshes were supplied, corresponding to different meshing granularities, termed “coarse”, “medium” and “fine”. The “medium”, respectively “coarse”, mesh edge length is 2.5, respectively 4.0, times the edge length of the “fine” mesh. This allows studying the thickness estimate accuracy for different representations. Parts 1 to 4 (figure 2) have the mid-

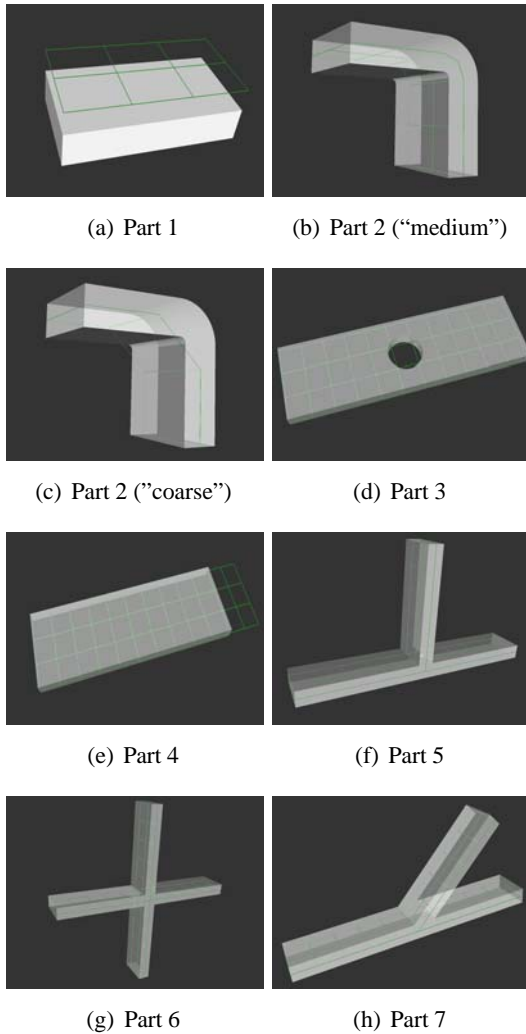


Figure 2: Synthetic parts used for quantitative analysis of the thickness estimate accuracy.

plane mesh either total or partially outside the part’s

surface, whereas parts 5 to 7 include ribs – thickness is not exactly defined at the regions where the rib intersects the main surface.

## 2.1 Metrics for quantitative analysis

Knowledge of the exact thickness of the synthetic parts allows for a quantitative analysis of the thickness estimation process using two metrics: arithmetic mean and the root mean square error (RMSE). The objective function is RMSE, which must be minimized.

**Arithmetic Mean (AM)** - since the actual thickness has been normalized to 1 across the whole surface for all the synthetic parts, the arithmetic mean  $\bar{T}$ , calculated as the average of the estimated thickness,  $\tilde{T}_i$ , across all  $N$  elements of the mid-plane mesh, gives a fast hint of whether estimates are converging towards the correct value. Being a global metric it does not capture whether there are local errors on the estimates that can be smoothed away by the averaging process. Furthermore, it would not convey useful information if the real thickness varied from element to element.

**Root Mean Square Error (RMSE)** - RMSE takes the square of the individual differences, also called residuals, between the estimated and the real thickness at each element and aggregates them onto a single metric that is perceived as a good measure of accuracy [8] (equation 1). RMSE heavily weights outliers (i.e., particularly bad local estimates) due to the squaring of the residuals, whereas small residuals are attributed very small weights; this is a desirable property since for Finite Element Analysis of structural properties outliers can strongly affect the simulation result.

$$RMSE = \sqrt{\frac{\sum_{i=1}^N (\tilde{T}_i - T_i)^2}{N}} \quad (1)$$

## 2.2 Results Analysis

Table 1 presents results for all synthetic parts, midplane mesh resolutions and the two thickness estimation algorithms. In those cases where the midplane mesh runs outside the part’s surface (parts 1 to 4) both algorithms fail to find valid points on both sides of the mesh and, consequently, fail to estimate the thickness - an estimate  $\tilde{T}_i = 0$  is generated for these cases. Figures 3(a) and 3(b) illustrate this for Part 3 over the hole region. For Part 1 estimates can not be generated for any element, since all of them are outside the part, thus resulting on  $\bar{T} = 0.0$ . These situations will be handled explicitly in section 3.

Parts 5 to 7 illustrate situations where ribs are present. RT fails when the midplane mesh element’s

| Part nbr. |        | Ray tracing |       | Nearest Neighbor |       |
|-----------|--------|-------------|-------|------------------|-------|
|           |        | $\bar{T}$   | RMSE  | $\bar{T}$        | RMSE  |
| 1         | fine   | 0.000       | 1.000 | 0.000            | 1.000 |
|           | medium | 0.000       | 1.000 | 0.000            | 1.000 |
|           | coarse | 0.000       | 1.000 | 0.000            | 1.000 |
| 2         | fine   | 1.000       | 0.001 | 0.883            | 0.169 |
|           | medium | 0.999       | 0.001 | 0.999            | 0.001 |
|           | coarse | 0.667       | 0.577 | 1.040            | 0.070 |
| 3         | fine   | 0.942       | 0.241 | 0.911            | 0.204 |
|           | medium | 0.969       | 0.174 | 1.029            | 0.294 |
|           | coarse | 1.000       | 0.000 | 0.874            | 0.333 |
| 4         | fine   | 0.857       | 0.378 | 1.194            | 0.865 |
|           | medium | 0.833       | 0.408 | 1.287            | 0.883 |
|           | coarse | 0.857       | 0.378 | 1.246            | 0.651 |
| 5         | fine   | 1.833       | 3.062 | 0.895            | 0.162 |
|           | medium | 1.000       | 0.000 | 0.999            | 0.000 |
|           | coarse | 1.000       | 0.000 | 0.999            | 0.000 |
| 6         | fine   | 2.250       | 4.330 | 0.912            | 0.141 |
|           | medium | 1.000       | 0.000 | 1.000            | 0.000 |
|           | coarse | 1.000       | 0.000 | 1.000            | 0.000 |
| 7         | fine   | 1.097       | 0.312 | 0.887            | 0.179 |
|           | medium | 1.134       | 0.368 | 1.019            | 0.053 |
|           | coarse | 1.000       | 0.000 | 1.000            | 0.000 |

Table 1: Results for thickness estimation algorithms (normalized).

centroid is aligned with the rib: rays, which are shot along the element's normal, will run inside the part, finding an intersection at distant points of the part's surface and overestimating thickness (figure 4(a)). This is particularly evident for the finer granularity meshes. The NN algorithm will still be able to find nearest points near the rib's junction with the part's surface, thus avoiding large thickness estimation errors (figure 4(b)). Figures 3(c) and 3(d) clearly show that the NN algorithm outperforms RT at these particular regions.

Surprisingly the error of the NN algorithm tends to increase as the mesh granularity becomes thinner. When searching for the nearest point in the part's surface, the elements that are close to the mesh boundaries find the part's lateral surface as its closest neighbor (see figure 5): the thickness estimate is thus smoothed, suggesting a round edge. This divergence, whose result can be visualized in figures 3(b) and 3(d), is addressed in section 3.

Summarizing, both algorithms produce wrong thickness estimates when the mid-plane mesh is a very inaccurate approximation of the part's geometry and runs outside it. Additionally, the ray tracing algorithm overestimates thickness in the presence of ribs, whereas the nearest neighbor algorithm underestimates it near the midplane mesh boundaries.

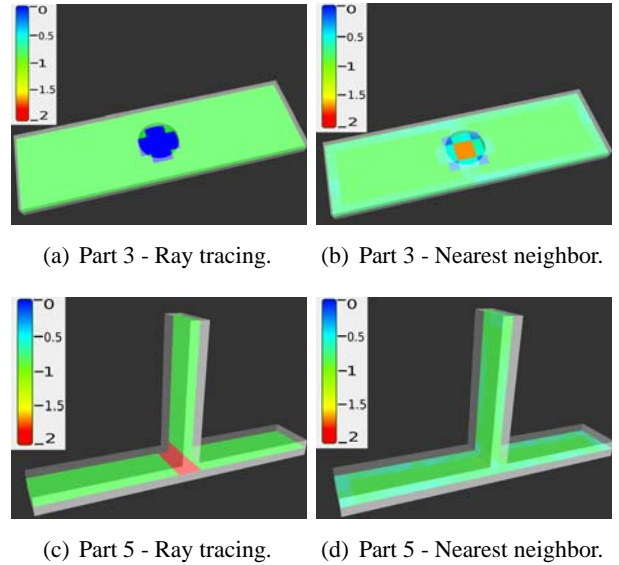


Figure 3: Thickness estimation within holes and near ribs - pseudo color maps (normalized).

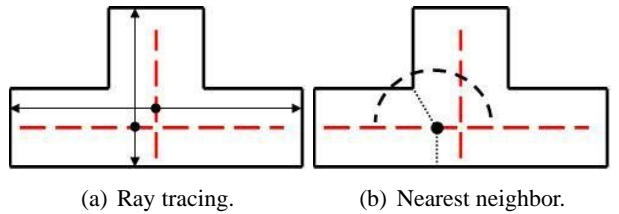


Figure 4: Thickness estimation near ribs. Black lines represent the part's geometry, whereas the red dashed line represents the midplane mesh.

### 3 Estimation improvement

#### 3.1 Inaccurate midplane meshes

Using midplane meshes which are poor approximations of the parts' geometries leads to thickness estimation errors, as shown in the previous section. Two different cases may occur: either the mesh is outside the surface but it still encompasses the part's geometry (parts 1 and 2, figures 2(a) and 2(c)), or the mesh runs outside the surface but this does not correspond to any region of the part's geometry (parts 3 and 4, figures 2(d) and 2(e)).

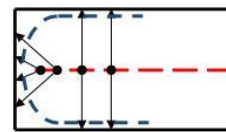


Figure 5: Divergence of the NN algorithm close to the midplane mesh boundaries.

Detecting whether an element's centroid is contained within the part's volume is a generalization of the point in polygon problem and can be solved by resorting to RT: if a ray is shot from the element's centroid along the normal direction and if it intersects the closed surface an odd number of times then the centroid is inside the surface, else it is outside [1]. For each centroid, one ray is shot along the element's normal direction for each side of the element. If both rays intersect the surface an even number of times, then the centroid is outside the surface, but if at least one of the rays intersects the surface more than zero times, the side of the centroid whose ray reported the closest intersection is selected as the one closest to the surface and thickness is estimated as the difference between the two closest intersections of that ray (figure 6).

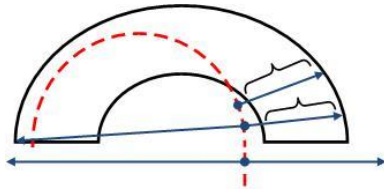
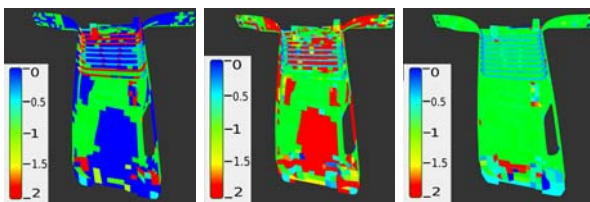


Figure 6: Detection of whether the midplane mesh is outside the part's surface. The brackets represent the estimated thicknesses by subtracting the distances found by the two closest intersections along the same ray.

Parts 1 and 2 illustrate two cases where the midplane mesh is outside the part's surface but still encompasses it. By detecting whether each element's centroid is outside the part the exact thickness is found and a RMSE equal to zero is obtained. The effectiveness of the RT corrected thickness estimation is also shown with a real part representing a B-pillar trim where a significant number of elements of the midplane elements are outside the part's surface. Figure 7 shows thickness estimations obtained with RT and NN (left and center) and with the detection of elements outside the part's surface (right).



(a) Ray Tracing. (b) Nearest Neighbor. (c) RT corrected.

Figure 7: Thickness estimation for the B-pillar - pseudo color (normalized).

For some elements of the mid-plane mesh no intersections are found on either side, e.g., within the hole of part 3 and on the right of part 4 (see also the bottom of figure 6). Such elements are tagged as "Incorrect" and can later be post-processed either manually or automatically.

### 3.2 Nearest neighbor divergence

In order to limit the divergence occurring near the midplane mesh boundaries with the NN algorithm, a limitation has been imposed on the maximum acceptable angle between the element's normal and the direction defined by the element's centroid and the surface nearest point. By limiting this angle it is expected that the part's lateral surface is rejected as a nearest neighbor, thus forcing the algorithm to expand its search onto regions of the surface that are farther away from the mid-plane element (Figure 8).

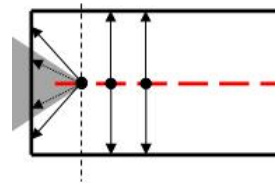


Figure 8: Limiting acceptable angle for NN: the gray triangle represents the unacceptable angle domain.

This technique requires some precaution. Some real part's geometries are modeled with polygons that have an area orders of magnitude larger than the respective midplane elements area. If the limitation of the angle is too strict, then some midplane elements could reject the surface polygon and overestimate local thickness. In the presence of ribs the angle rejection technique might also reject the correct nearest neighbor, resulting in overestimating thickness. The angle rejection threshold must thus be carefully selected. Table 2 presents the RMSE obtained for

|               | Part 2 | Part 5 | Part 6 | Part 7 |
|---------------|--------|--------|--------|--------|
| RT            | 0.0008 | 3.0619 | 4.3301 | 0.3118 |
| NN (no limit) | 0.1688 | 0.1618 | 0.1414 | 0.1792 |
| NN (80°)      | 0.0860 | 0.0741 | 0.0704 | 0.0848 |
| NN (65°)      | 0.0273 | 0.0461 | 0.0447 | 0.0577 |
| NN (45°)      | 0.0065 | 0.0118 | 0.0163 | 0.0493 |

Table 2: RMSE results for NN angle limitation with fine midplane meshes.

4 different parts, fine grained midplane meshes and three angle thresholds: 80°, 65° and 45°. Although

a threshold of  $45^\circ$  produces the smaller RMSE, for complex real parts such a large threshold results in many rejections and, consequently, in many local errors. A threshold of  $80^\circ$  does not induce such errors and still its impact on the RMSE more than halves it.

## 4 Conclusion

This paper presents and analyzes two techniques, based on ray tracing (RT) and 3D nearest neighbor range search (NN), to estimate local thicknesses from geometric models of automotive parts. Experimental results identified three cases that lead to poor thickness estimation: the midplane mesh runs outside the part's surface preventing both algorithms to find valid points on the part's surface, NN diverges on the midplane boundaries and RT fails on ribs.

The first problem was addressed by using RT to detect whether an element's centroid is outside the surface. In such cases the difference between the two closest intersections detected on the same side of the element is used as thickness estimation. There are still some elements where no intersections are found on either side: these are tagged as "Incorrect" for manual post-processing. NN's divergence near boundaries was minimized by limiting the maximum angle allowed between the element's normal and the direction defined by the element's centroid and the nearest point on the part's surface: RMSE was significantly reduced while avoiding other geometric errors. The ribs inaccuracies associated with RT were not addressed explicitly since these are completely avoided by the NN approach.

The above results suggest that: (i) NN fails on the midplane mesh boundaries but RT provides good estimates at these locations; limiting the angle does not make NN as accurate as RT (see part 2 in table 2); (ii) RT fails on ribs, but NN provides good estimates at these locations.

Both algorithms complement each other: if each algorithm's best estimate can be selected for each element of the midplane mesh then RMSE is reduced. Per-element manual selection of either the RT or NN estimate that minimizes the difference to the correct thickness for Part 5 resulted in a very good overall estimate, with an average mean of 1.0091 and RMSE equal to 0.0325.

### 4.1 Future work

The above conclusion suggests that if a criterion can be found that allows automatic selection of either the RT or the NN estimate for each element, then RMSE can be significantly reduced and the whole results

would be much more reliable. Such a criterion is not evident however, due to the complexity of real world parts, having lots of details and particular configurations that make it very difficult to establish which is the best estimate - particularly, the real local thicknesses are not known, since this is exactly the quantity that is being measured. Analysis of the local geometries in order to identify ribs and/or midplane mesh boundaries may also reveal too complex to be performed accurately. A promising approach is to estimate thicknesses using both algorithms and then assign each estimate a confidence weight given their relative variations within some neighborhood. Future work will entail studying such alternative criteria, which will allow integrating the two algorithms presented throughout this paper.

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