Locally Optimal Fuzzy Control of a Heat Exchanger

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Abstract: The paper presents a fuzzy control based on parallel distributed fuzzy controllers for a heat exchanger. First, a Takagi-Sugeno fuzzy model is employed to represent a system. Each subcontroller is LQR designed and provides local optimal solutions. The stability of the system with the proposed fuzzy controllers is discussed. Finally, simulation results illustrate the validity and applicability of the presented approach.

Key-Words: Takagi-Sugeno fuzzy model, Lyapunov function, LMI problem, shell heat exchanger, Takagi-Sugeno parallel distributed fuzzy LQR, PID control.

1 Introduction
Fuzzy controllers have found popularity in many practical situations. Many complex plants have been controlled very well using fuzzy controllers without any difficult analysis common in classical control design. Fuzzy controllers are general nonlinear ones and their benefits are well-known [9]. In spite of these advantageous properties of fuzzy controllers, the main crisis of them was the absence of a formal method for proving the system's stability. However, after introducing the fuzzy plant modeling in [11], some methods for stable controller design have arisen.

The described approach is based on fuzzy modelling of a nonlinear plant as a sum of nonlinear-weighted linear subsystems. Following this approach, one can design a linear controller for each subsystem and satisfying some constraints expressible as Linear Matrix Inequalities (LMIs), stability of the whole system can be proved [14, 6]. This is what is called Takagi-Sugeno (TS) fuzzy controller. The idea is similar to traditional gain scheduling method in which controlling gains change according to the state of the controlled system [8].

A heat exchanger is a device in which energy is transferred from one fluid to another across a solid surface. Exchanger analysis and design therefore involve both, convection and conduction. The heat exchangers are widely used in many industrial power generation units, chemical, petrochemical, and petroleum industries. These types of heat exchangers are robust units that work for wide ranges of pressures, flows and temperatures [10].

2 Problem Formulation
Fuzzy modelling is a framework in which different modelling and identification methods are combined, providing a transparent interface with the designer or operator and. It is a flexible tool for nonlinear system modelling and control too. The rule-based character of fuzzy models allows for a model interpretation in a way that is similar to the one humans use to describe reality.

Using fuzzy systems it is possible to define very general nonlinearities. In order to be able to derive any analytical useful results it is necessary to constrain the classes of nonlinearities that one consider. The class of systems that has achieved most attention is linear and affine Takagi-Sugeno systems on state-space form. For these systems both stability and synthesis results are available based on Lyapunov theory.

Quadratic Lyapunov functions are very powerful if they can be found. In many cases it is very difficult to find a common global Lyapunov function. The feasible solution is to use a piecewise quadratic Lyapunov function that is tailored to fit the cell partition of the system [4]. The search for piecewise quadratic Lyapunov function can also be formulated as an LMI-problem.

2.1 Fuzzy Plant Model
Consider a nonlinear controller. We can assume that the plant can be represented by a fuzzy plant model. Our goal is designing a nonlinear state feedback controller. The continuous fuzzy dynamic model, proposed by [11] is described by fuzzy if-then rules. It can be seen as a combination of linguistic modelling and mathematical regression, in the sense that the
antecedents describe fuzzy regions in the input space in which consequent functions are valid. The $i^{th}$ rule is of the following form [15].

Plant Rule $i$:

\[
\begin{align*}
\text{if } & z_1(t) \text{ is } M_1^i \text{ and ... and } z_k(t) \text{ is } M_k^i \text{ then } \\
\dot{x}(t) &= A_i x(t) + B_i u(t) \\
y(t) &= C_i x(t) \\
\end{align*}
\]

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in \mathbb{R}^n$ is the state vector, $u(t) = [u_1(t), u_2(t), \ldots, u_m(t)]^T \in \mathbb{R}^m$ is the control input, $y(t) = [y_1(t), y_2(t), \ldots, y_p(t)]^T \in \mathbb{R}^p$ is the controlled output, $M_i^j$ are fuzzy sets, $z(t) = [z_1(t), z_2(t), \ldots, z_k(t)]$ are the premise parameters, $A_i \in \mathbb{R}^{nxn}$ is the state transition matrix, $B_i \in \mathbb{R}^{nxm}$ is input matrix, $C_i \in \mathbb{R}^{pxn}$ is output matrix. Let us use product as $t$-norm operator of the antecedent part of rules and the center of mass method for defuzzification. The final output of the fuzzy system is inferred as follows [12, 15]:

\[
\begin{align*}
\dot{x}(t) &= \frac{\sum_{i=1}^{N} \mu_i |z(t)| A_i x(t) + B_i u(t)}{\sum_{i=1}^{N} \mu_i |z(t)|} \\
y(t) &= \frac{\sum_{i=1}^{N} \mu_i |z(t)| C_i x(t)}{\sum_{i=1}^{N} \mu_i |z(t)|} \\
\end{align*}
\]

where

\[
\begin{align*}
h_i |z(t)| &= \frac{\mu_i |z(t)|}{\sum_{i=1}^{N} \mu_i |z(t)|} \\
\mu_i |z(t)| &= \prod_{j=1}^{k} M_i^j |z_j(t)| \\
\sum_{i=1}^{N} h_i |z(t)| &= 1
\end{align*}
\]

2.2 Quadratic Stability

After defining the model, the conditions are found under which the system is stable.

Theorem 1. The continuous uncontrolled $(u=0)$ fuzzy system of (1) - (3) is globally quadratically stable if there exists a common positive definite matrix $P=PT$ such that

\[
A_i^T P + P A_i < 0, \quad i = 1, \ldots, N
\]

This is equivalent to saying that one must find a single function $V(x) = x^T P x$ as a candidate for Lyapunov function.

Finding a common $P$ can be considered as linear matrix inequality (LMI) problem. Matlab LMI toolbox presents simple appliance for solving this problem [3].

2.3 Parallel distributed compensation

Having TS plant model, it can be used parallel distributed compensation control defined as follows: Control Rule $j$:

\[
\begin{align*}
\text{if } & z_1(t) \text{ is } M_1^j \text{ and ... and } z_k(t) \text{ is } M_k^j \text{ then } \\
u(t) &= -K_j x(t) \\
\end{align*}
\]

Hence, the fuzzy controller is given

\[
\begin{align*}
u(t) &= -\sum_{j=1}^{N} h_j |z(t)| K_j x(t) \\
\end{align*}
\]

in which $K_j$ are state feedback gains. We can see it as local gains of gain scheduling design which overall control signal is made from combining each local control signal with different weights according to the closeness to the each rule's region.

The closed loop system can be expressed by combining (2) and (7) as following system

\[
\begin{align*}
\dot{x}(t) &= \sum_{i=1}^{N} \sum_{j=1}^{N} h_{ij} |z(t)| h_j |z(t)| \left[ A_i - B_i K_j \right] x(t) = \\
&= \sum_{j=1}^{N} h_j^2 |z(t)| G_{ij} x(t) + \\
&\quad + \sum_{i=1}^{N} \sum_{j=1}^{N} h_{ij} |z(t)| h_j |z(t)| \left[ \frac{G_{ij} + G_{ji}}{2} \right] x(t)
\end{align*}
\]

where $G_{ij} = A_i - B_i K_j$.

It is easy to obtain the following result using Theorem 1: The fuzzy system (2), (3) with fuzzy control of (9) is globally stable if there exists $P=PT$ such that
\[
\left( \frac{G_j + G_{ji}}{2} \right)^T P + P \left( \frac{G_j + G_{ji}}{2} \right) < 0, \quad i=1, \ldots, N \quad j=1, \ldots, N \quad (11)
\]

2.4 Locally optimal design

Steady state linear quadratic regulator (LQR) problem can be formulated as \cite{1, 2, 5}:

\[
J_{x(t), u(t)} = \int_0^\infty (x^T Q x + u^T R u) \, dt \quad (12)
\]

The optimal gain is \( K = R^{-1} B^T P \) in which \( P \) is solution of the algebraic Ricatti equation

\[
PA + A^T P + Q - PBR^{-1} B^T P = 0 \quad (13)
\]

3 Simulations and Results

3.1 Shell heat exchangers

Consider two shell heat exchangers in series where a liquid is steam heated. The measured and controlled output is temperature from second exchanger. The control objective is to keep the temperature of the output stream close to a desired value 353 K. The control signal is input volumetric flow rate of the heated liquid. Assume ideal liquid mixing and zero heat losses. We neglect accumulation ability of exchangers walls. Hold-ups of exchangers as well as flow rates and liquid specific heat capacity are constant.

Under these assumptions the mathematical model of the exchangers is given as

\[
\frac{dT_1}{dt} = q \frac{T_0 - T_1}{V_1} + \frac{A_1 k}{V_1 \rho C_p} (T_p - T_1) \quad (14)
\]

\[
\frac{dT_2}{dt} = q \frac{T_1 - T_2}{V_2} + \frac{A_2 k}{V_2 \rho C_p} (T_p - T_2) \quad (15)
\]

where \( T_1 \) is temperature in the first exchanger, \( T_2 \) is temperature in the second exchanger, \( T_p \) is liquid temperature in the inlet stream of the first tank, \( q \) is volumetric flow rate of liquid, \( \rho \) is liquid density, \( V_1, V_2 \) are liquid volumes, \( A_1, A_2 \) are heat transfer areas, \( k \) is heat transfer coefficient, \( C_p \) is specific heat capacity. The superscript \( s \) denotes the steady-state values in the main operating point.

Parameters and inputs of the exchangers are enumerated in Table 1.

Table 1: Parameters and inputs of heat exchangers

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_1 )</td>
<td>5 m³</td>
</tr>
<tr>
<td>( V_2 )</td>
<td>5 m³</td>
</tr>
<tr>
<td>( \rho )</td>
<td>900 kg ( \text{m}^3 )</td>
</tr>
<tr>
<td>( T_0^s )</td>
<td>293 K</td>
</tr>
<tr>
<td>( T_p^s )</td>
<td>373 K</td>
</tr>
<tr>
<td>( T_1^s )</td>
<td>313 K</td>
</tr>
<tr>
<td>( T_2^s )</td>
<td>328 K</td>
</tr>
</tbody>
</table>

3.2 Takagi-Sugeno fuzzy model

The system was approximated by nine fuzzy models

\[
\text{if } x \text{ is } M_1^i \text{ and } u \text{ is } M_2^i \text{ then }
\]

\[
\dot{x}(t) = A_i x(t) + B_i u(t) \quad i=1, \ldots, 9 \quad (16)
\]

The bell curve membership functions for the premise variables \( x \) and \( u \) in each rule are adopted:

\[
f(x; a, b, c) = \left( 1 + \frac{|x-c|}{a} \right)^{-b} \quad (17)
\]

The parameters \( a, b \) and \( c \) for bell shaped membership functions are listed in the Table 2 and membership functions are shown in Figures 1, 2.

The consequent parameters are given in Table 3 and the resulting plot of the output surface of a described fuzzy inference system is presented in Figure 3.
Table 2: Bell curve membership functions parameters

<table>
<thead>
<tr>
<th>x</th>
<th>u</th>
</tr>
</thead>
<tbody>
<tr>
<td>ai</td>
<td>bi</td>
</tr>
<tr>
<td>ai</td>
<td>bi</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>0.13</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>0.14</td>
<td>2</td>
</tr>
<tr>
<td>6</td>
<td>2</td>
</tr>
<tr>
<td>0.14</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 3: Consequent parameters

<table>
<thead>
<tr>
<th>Ai</th>
<th>Bi</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.73</td>
<td>-13.41</td>
</tr>
<tr>
<td>0.27</td>
<td>-4.94</td>
</tr>
<tr>
<td>-0.33</td>
<td>9.44</td>
</tr>
<tr>
<td>0.01</td>
<td>0.22</td>
</tr>
<tr>
<td>0.13</td>
<td>-3.04</td>
</tr>
<tr>
<td>-0.37</td>
<td>11.11</td>
</tr>
<tr>
<td>-0.25</td>
<td>0.72</td>
</tr>
<tr>
<td>2.33</td>
<td>-74.17</td>
</tr>
<tr>
<td>2.05</td>
<td>-59.04</td>
</tr>
</tbody>
</table>

Fig.3: Output surface of a fuzzy inference system

\[ x' = f(x,u) \]

After obtaining \( A_i, B_i \), gains \( K_j \) were calculated for each subsystem using LQR design and then tested for stability of the whole system.

The problem was solved using LMI optimization toolbox in Matlab software package. Consider the linearized model of the heat exchangers in the form

\[ x(t) = Ax(t) + Bu(t), \quad x(t_0) = x_0 \]
\[ y(t) = Cx(t) + Du(t) \]

with matrices \( A, B, C, D \):

\[
A = \begin{bmatrix}
-0.2667 & 0 \\
0.2000 & -0.2667
\end{bmatrix} \quad B = \begin{bmatrix}
-4 \\
-3
\end{bmatrix} \\
C = \begin{bmatrix}
0 & 1
\end{bmatrix} \quad D = 0
\]

The comparison of LQR controller with TS controller was made using \( iae \) and \( ise \) criteria described as follows:

\[
iae = \int_0^T |e|dt
\]
\[
ise = \int_0^T e^2 dt
\]

The results for different performance measures are compared in Table 4.

Table 4: Performance comparison between linear and fuzzy LQR controllers

<table>
<thead>
<tr>
<th>performance measure</th>
<th>LQR</th>
<th>TS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q=I*1(2,2)</td>
<td>K= -0.6995</td>
<td>K= -0.981</td>
</tr>
<tr>
<td>R=1</td>
<td>iae = 2.26e3</td>
<td>iae = 1.694</td>
</tr>
<tr>
<td></td>
<td>ise = 2.00e3</td>
<td>ise = 1.674</td>
</tr>
<tr>
<td>Q=100*I(2,2)</td>
<td>K= -9.98</td>
<td>K= -9.98</td>
</tr>
<tr>
<td>R=1</td>
<td>iae = 1.4e3</td>
<td>iae = 1.4e3</td>
</tr>
<tr>
<td></td>
<td>ise = 1.65e4</td>
<td>ise = 1.654</td>
</tr>
<tr>
<td>Q=40*I(2,2)</td>
<td>K= -2.292</td>
<td>K= -9.98</td>
</tr>
<tr>
<td>R=40</td>
<td>iae = 1.48e3</td>
<td>iae = 1.42e3</td>
</tr>
<tr>
<td></td>
<td>ise = 1.65e4</td>
<td>ise = 1.654</td>
</tr>
<tr>
<td>Q=40*I(2,2)</td>
<td>K= -4.6224</td>
<td>K= -6.3054</td>
</tr>
<tr>
<td>R=1</td>
<td>iae = 1.73e3</td>
<td>iae = 1.68e3</td>
</tr>
<tr>
<td></td>
<td>ise = 1.96e4</td>
<td>ise = 1.96e4</td>
</tr>
</tbody>
</table>

3.3 PID control

For feedback controller tuning, the approximate model of a system with complex dynamics can have the form of a first-order-plus-time-delay transfer function (17). The process is characterised by a steady-state gain \( K \), an effective time constant \( T \) and an effective time delay \( D \).

\[
G_p(s) = \frac{K}{T s + 1} e^{-Ds}
\]

The transfer function describing the controlled heat exchangers was identified from step response data in the form (21) with parameters: \( K = -38.57, T = 11.3 \) min, \( D = 2 \) min. These parameters were used for feedback controller tuning. The feedback PID controllers were tuned by various methods [7]. Two controllers were used for comparison: PID controller (22) tuned using Rivera-Morari method.
with parameters $K_C = -0.1063$, $T_I = 12.3$, $T_D = 0.91$ and PID controller tuned using Ziegler-Nichols method with parameters $K_C = -0.17$, $T_I = 4$, $T_D = 1$. The transfer function of the used PID controller is following

$$G_C(s)=K_C\left(1+\frac{1}{T_Is}+T_Ds\right)$$

(22)

The step changes of the reference $y_r$ were generated and the LQR, fuzzy and PID controllers were compared. Figure 4 presents the comparison of the simulation results obtained by LQR controller, TS controller and PID controllers tuned using Rivera-Morari and Ziegler-Nichols methods. Figure 5 presents the comparison control inputs generated by above mentioned controllers. The control results obtained by LQR controller and fuzzy LQR controller are practically identical.

Figure 6 presents the simulation results of the LQR, fuzzy LQR and PID control of the heat exchanger in the case when disturbances affect the controlled process. Disturbances were represented by temperature changes from 373 K to 353 K at $t=25$ min, from 353 K to 383 K at $t=75$ min and from 383 K to 368 K at $t=125$ min. The comparison of the controllers output is shown in Figure 7.

The described controllers were compared using $iae$ and $ise$ criteria. The $iae$ and $ise$ values are given in Table 5.


Table 5: Comparison of the simulation results by integrated absolute error $iae$ and integrated square error $ise$

<table>
<thead>
<tr>
<th></th>
<th>$iae$</th>
<th>$ise$</th>
</tr>
</thead>
<tbody>
<tr>
<td>control method</td>
<td></td>
<td></td>
</tr>
<tr>
<td>LQR: $K = -4.6224$</td>
<td>1.73e3</td>
<td>1.96e4</td>
</tr>
<tr>
<td>fuzzy LQR: $K = -6.3054$</td>
<td>1.68e3</td>
<td>1.96e4</td>
</tr>
<tr>
<td>PID (Rivera-Morari)</td>
<td>3.66e3</td>
<td>4.09e4</td>
</tr>
<tr>
<td>PID (Ziegler-Nichols)</td>
<td>5.17e3</td>
<td>5.78e4</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>$iae$</th>
<th>$ise$</th>
</tr>
</thead>
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<tr>
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<tr>
<td>LQR: $K = -4.6224$</td>
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<td>1.65e4</td>
</tr>
<tr>
<td>fuzzy LQR: $K = -6.3054$</td>
<td>1.42e3</td>
<td>1.65e4</td>
</tr>
<tr>
<td>PID (Rivera-Morari)</td>
<td>3.94e3</td>
<td>3.67e4</td>
</tr>
<tr>
<td>PID (Ziegler-Nichols)</td>
<td>5.04e3</td>
<td>5.51e4</td>
</tr>
</tbody>
</table>

Used fuzzy controller (and linear LQR too, presented results and criteria values are very similar) is simple, and it offers the smallest values $iae$ and $ise$ or equal to linear LQR. The disadvantage of the LQR controllers is, that using these controllers can lead to nonzero steady-state errors, but without overshoots practically. In the case of the heat exchanger control in the presence of disturbances, the control responses with LQR controllers do not show any overshoots and undershoots.

4 Conclusion

In this paper, a stable nonlinear fuzzy controller based on parallel distributed fuzzy controllers is proposed. Each subcontroller is LQR designed and provides local optimal solution. The Takagi-Sugeno fuzzy model is employed to approximate the nonlinear model of the controlled plant. Based on the fuzzy model, a fuzzy controller is developed to guarantee not only the stability of fuzzy model and fuzzy control system for the heat exchanger but also control the transient behaviour of the system. The design procedure is conceptually simple and natural. Moreover, the stability analysis and control design problems are reduced to LMI problems. Therefore, they can be solved very efficiently in practice by convex programming techniques for LMIs. Simulation results shows that the proposed control approach is robust and exhibits a superior performance to that of established traditional control methods. Comparison of the LQR simulation results with classical PID control demonstrates the effectiveness and superiority of the proposed approach.

References:


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