# Modeling of non-stationary heat field in a plane plate for asymmetric problem

DAGMAR JANÁČOVÁ, HANA CHARVÁTOVÁ, KAREL KOLOMAZNÍK
VLADIMÍR VAŠEK, *PAVEL MOKREJŠ
Faculty of Applied Informatics
*Faculty of Technology
Tomas Bata University in Zlín
CZECH REPUBLIC
janacova@fai.utb.cz, http://www.fai.utb.cz

*Abstract:* - This In the paper we deal with a problem of non-stationary heat conduction in a plane plate. In the first part of paper we formulated mathematic model for the case of asymmetric heating or cooling of a solid plate. In the second part, we computed a temperature field in the plate during the mentioned process by mathematic software Maple. Finally, we verified course a real technological process - moulding of plastic material by use of the formulated model.

*Key-Words:* - Modeling, Non-stationary Heat Conduction, Plastic Moulding

## 1 Introduction

Heat conduction in a wall of the solid body, caused by heat action of a surrounding body, is a part of many technological operations during which the processed material is heated or cooled. It occurs by processing of metals, plastics, rubbers etc. Course of the process always depends on given conditions.

In the paper we focus on one of the most frequent case of non-stationary heat conduction in a solid material. It is unsteady heat conduction in a solid body which is caused by double-sided heating or cooling of he surrounding environments of different temperatures. Experimental determination of the process course is very difficult, nevertheless this information is necessary for suggestion of its optimal technological procedure. Therefore we formulated mathematic model based on physical patterns of given process and verified its validity. We used analytical solution of the model for assessment of a concrete case of plastic material processing as shown in following text.

## 2 Theoretical part

We solved the asymmetric problem of heat conduction in a solid plane plate made from isotropic material. Length and width of the plate are much longer than its thickness δ. The plate of initial temperature *tp* is suddenly exposed double-sided heat action of surrounding environment, whereas we supposed that temperature of surrounding environment on the left side from plate to1 is different to temperature on the right side from plate $t_{o2}$. Temperatures of both environments don't depend on time and in addition they differ from initial temperature of the plate. Geometry sketch of the mentioned problem is in Fig. 1.

With respect to above mentioned assumptions, the heat transfer across the wall will be asymmetric in accordance with axis of the wall. We used Fourier-Kirchhoff equation of heat conduction (1) with initial and boundary conditions (2) – (4) for modeling of the problem.

$$\frac{\partial t(x,\tau)}{\partial \tau} = a\frac{\partial^2 t(x,\tau)}{\partial x^2} \tag{1}$$

$$t(x,0) = t_p \tag{2}$$

$$t(0,\tau) = t_{o1} \tag{3}$$

$$t(\delta,\tau) = t_{o2} \tag{4}$$

Equation (2) is assumption of the initial uniform temperature distribution in a heated or cooled body. Conditions (3) and (4) assume that temperature of a wall margin is constant and it equals surrounding temperature.

***a*** - temperature diffusivity of the heated (cooled) material $m^2.s^{-1}$

*t* - temperature of the heated (cooled) body, °C;

$t_{o1}$ - temperature of surrounding environment on the left side from heated (cooled) body, °C;

$t_{o2}$ - temperature of surrounding environment on the right side from heated (cooled) body, °C;

$t_p$ - initial temperature of the heated (cooled) body, °C;

$x$ - direction coordinate, m;

$\delta$ - thickness of the heated (cooled) body, m;

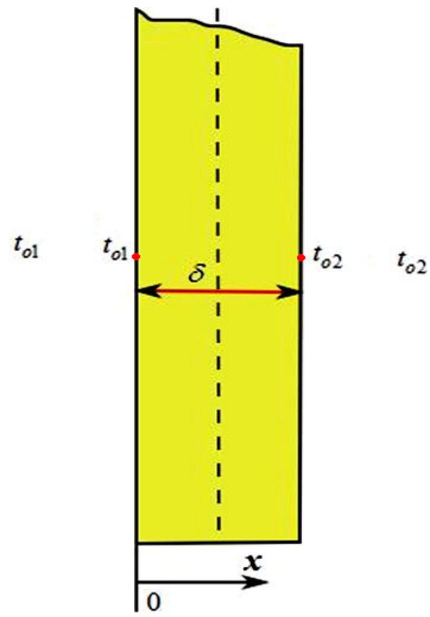



Fig. 1 Geometry sketch of the non-stationary asymmetric heat conduction in a plane plate

By use of Laplace transformation we obtained analytical solution of the formulated model. Temperature yield in a wall during heating (cooling) $t(x, \tau)$ is given by equation (5):

$$t(x,\tau) = t_p + \frac{(x-\delta)(t_p - t_{o1})}{\delta} + $$
$$+2\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\cdot\pi} \sin\left(\frac{(\delta - x)}{\delta} n\cdot\pi\right) \; e^{-\left((n\cdot\pi)^2 \frac{a\cdot\tau}{\delta^2}\right)} (t_p - t_{o1}) +$$
$$+\frac{x(t_{o2} - t_p)}{\delta} +$$
$$+2\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n\cdot\pi} \sin\left(-\;\frac{x}{\delta}\cdot n\cdot\pi\right) \; e^{-\left((n\cdot\pi)^2 \frac{a\cdot\tau}{\delta^2}\right)} (t_{o2} - t_p) \quad (5)$$

$$a = \frac{\lambda}{\rho \cdot c_p} \quad (6)$$

where:

$c_p$ - specific thermal capacity, J.kg-1.K-1;

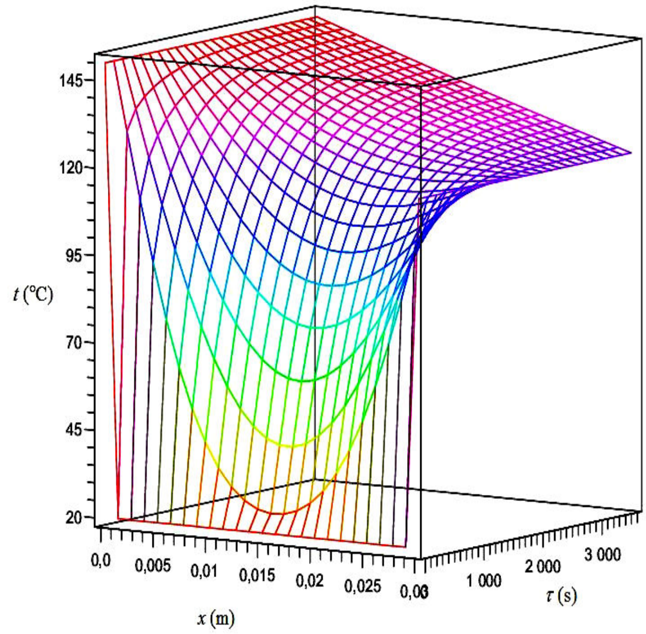
$\lambda$ - thermal conductivity, W.m-1.K-1;

$\rho$ - density, kg.m-3;

$\tau$ - time, s.

## 2.1 Modeling of the asymmetric temperature field

We used mathematic software Maple for description of the asymmetric heat conduction in a solid plate. In Maple user environment we solved and visualised the temperature field in a solid plate in accordance with relation (5). Potential cases of the temperature fields in a plane plate which can ocure by asymmetric temperature action of the surrounding environment we show in Fig. 2, 3 and 4.



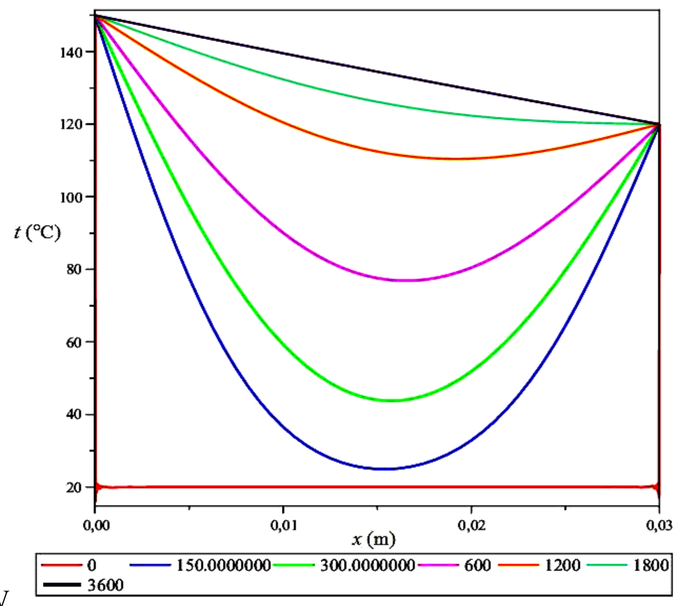




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Fig. 2: 2D Temperature field in a plate during asymmetric heating . *Parameters:* $t_p = 20$ °C, $t_{o1} = 150$ °C, $t_{o2} = 120$ °C

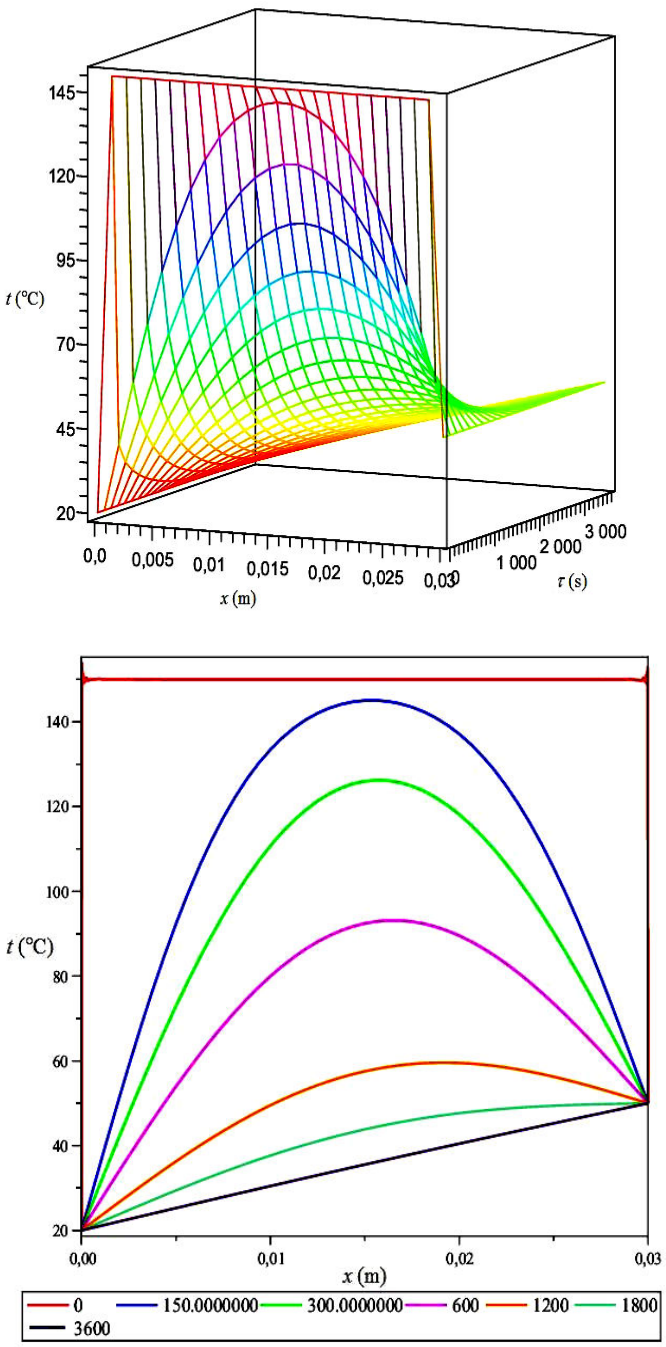


Fig. 3 3D and 2D Temperature field in a plate during asymmetric cooling .
Par*ameters*: $t_p$ = 150 °C, $t_{o1}$ = 20 °C, $t_{o2}$ = 50 °C

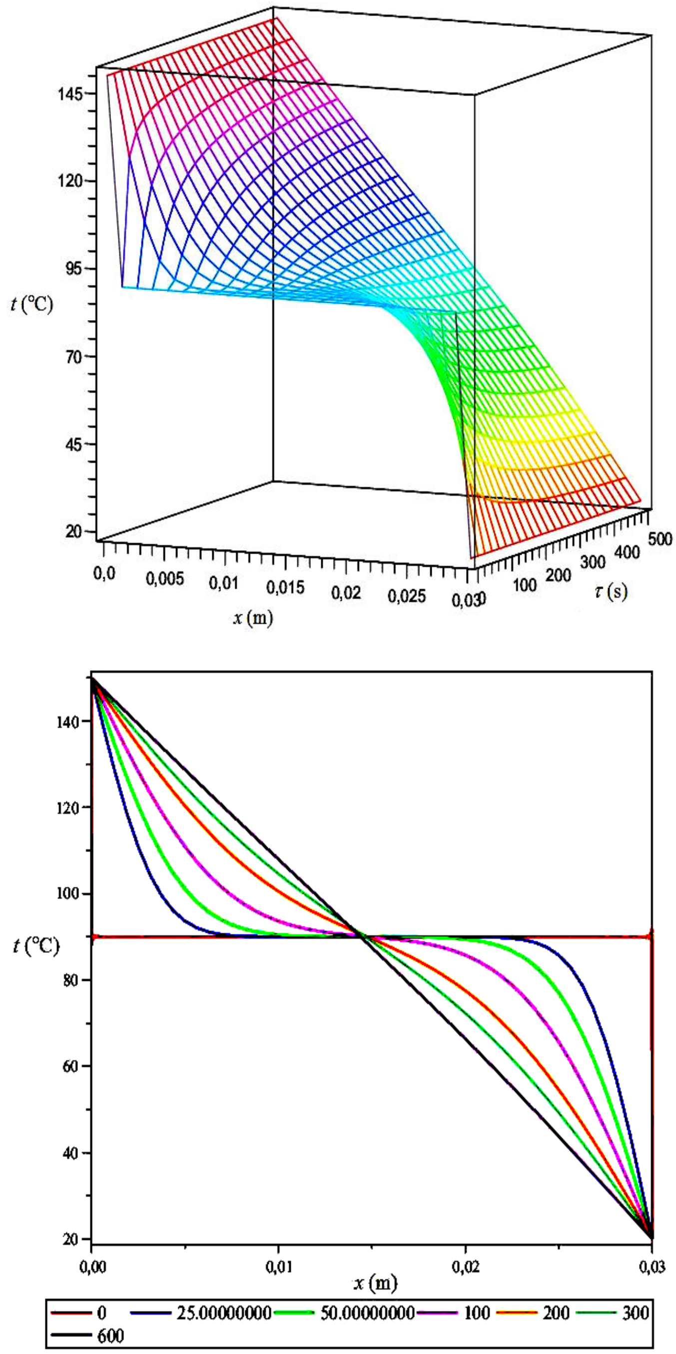


Fig. 4 Temperature field in a plate during its simultaneous heating and cooling.
*Parameters*: $t_p$ = 150 °C, $t_{o1}$ = 20 °C, $t_{o2}$ = 50 °

## 3 Experimental part

We applied the formulated model of asymmetric heat conduction for asessment of the course of real technological operation - injection moulding combined with subsequent blow moulding during moulding of button of plastic small bottles by their manufacturing (Fig. 5).

In practice, first the oven-ready food for blowing will be prepared. After them, the form will be slightly open, the shape will be sticked up by jaws and blow moulding will follow. Time of the submited cycle took 40 second. Expected time in practice manufacturing takes 40 second. For this purpose, we verified if it is possible to dissipate needed heat quantity in 20 second.

We solved temperature under the conditions:

Temperature of force: 130 °C,

Temperature of form: 38 °C

Temperature of melt: 230 °C

Thickness of bottom: 2.8 mm

Properties of polypropylene:

Density: 910 kg.m-3, Specific thermal capacity: 1700 $J.kg^{-1}.K$-1, thermal conductivity: 0.22 $W.m^{-1}.K^{-1}$

Temperature field in the bottom of bottle is shown in Fig. 6. The temperatures field in time 20 second and 40 second are shown in Fig. 7. It is evident that the cycle time 20 second is sufficient. In time 40 seconds, the process will be steady.

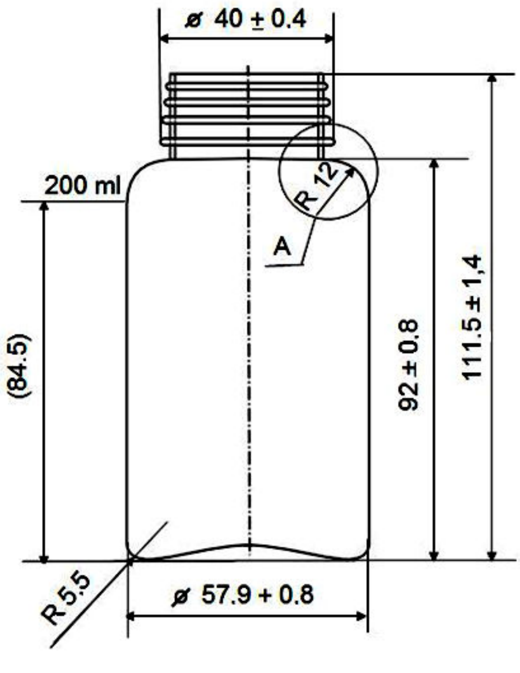


Fig. 5 Scheme of the tested polypropylene bottle

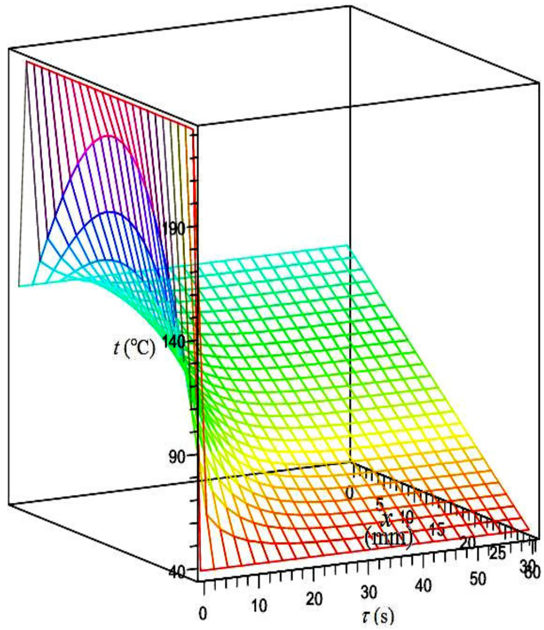


Fig. 6 Temperature field in a plate during asymmetric

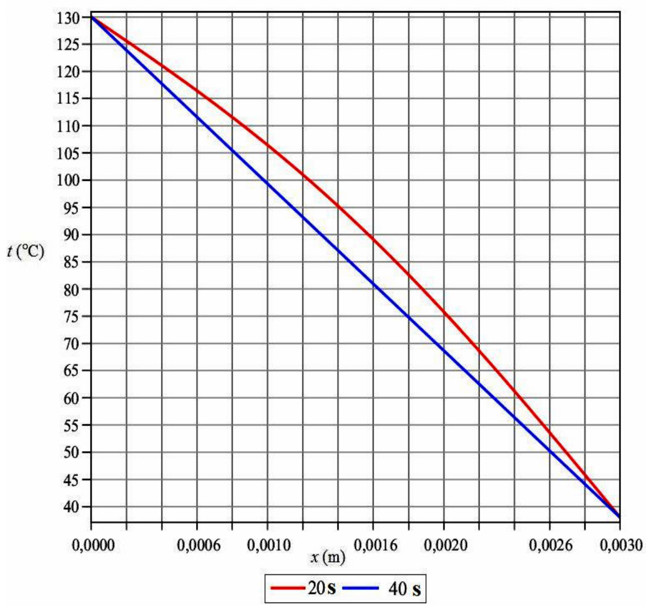


Fig. 7 Temperature field in the bottom of polyethylene bottle during moulding process

## 4 Conclusion

The formulated mathematic model is suitable for description of asymmetric heat conduction in a plane plate. The mentioned model can be used for examination of any process of non-stationary heating or cooling of a solid plane which are based on the same mechanism as we solved in this paper.

We demonstrated our model using by evaluation of moulding process course. Results of our computation were in accordance with experimental data.

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***Acknowledgement:*** This work was supported by MSM, project No. 7088352102: Modeling and controlling of natural and synthetic polymers processing and by FR VŠ, project. No. 17: Software aids for support of teaching of subject Process engineering.