Two step, PID and model predictive control applied on fed batch process

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Abstract: The paper presents different control methods applied on a chemical exothermic semi batch reactor. Because of a strongly exothermic chemical reaction the in-reactor temperature is rising very fast depending on the reaction component dosing. Thus, the temperature control is necessary. The system control is generally difficult because of its nonlinear behavior. To simulate the real process a mathematical model including reaction kinetics was used. Three control methods were simulated – two step control with penalization, PID control and model predictive control.

Key-Words: Predictive control, PID control, two step control, chemical semi-batch reactor, process modelling

1 Introduction
Batch reactors provide flexible means of producing high value-added products in specialty chemical, biotechnical, and pharmaceutical industries. To realize the production objectives, these batch reactors have to be operated optimally in a precise fashion. However, due to the following characteristics: 1. intrinsic nonlinearity; 2. lack of steady-state operating conditions; 3. uncertainties in reaction dynamics, or modeling error; 4. unknown disturbances; 5. constraints on process variables; 6. and limited on-line measurement information, the optimization and control of batch reactors present some of the most interesting and challenging problems for both academia and industry in the process control arena [1].

The interest in the control of batch reactors has increased in recent years because of the expansion of small-volume specialty chemicals. In the biotechnology area, batch reactors are used on both small- and largescale fermenters because of the inherent superiority of batch fermentation over continuous fermentation in most systems. Many of these batch reactors are “semibatch” or “fedbatch” reactors in which an initial amount of material is placed in the reactor, the liquid is heated to the desired temperature, and then additional feed of fresh reactant is gradually added to the vessel. The result is a time-varying process with variable volume. If heating and/or cooling is achieved by heat transfer from the vessel liquid into a heating/cooling medium in a surrounding jacket, the time-varying volume means that the heat-transfer area is also changing with time. The optimum operation of many fed-batch reactors is an operating strategy that minimized the batch time. This corresponds to feeding the fresh feed into the reactor as quickly as possible. The feed rate is often limited by heat transfer. If the reaction is exothermic, heat must be removed. The rate of heat transfer depends on three factors [2]: 1. The temperature difference between the reaction liquid and the jacket coolant. The latter depends on the coolant flow rate, the inlet coolant temperature, and the heat-transfer rate. 2. The overall heat-transfer coefficient, which depends on agitator mixing in the vessel and the flow rate of coolant in the jacket. 3. The heat-transfer area. If jacket cooling is used, the effective heat-transfer area in a fed-batch reactor varies during the course of the batch directly with the volume of liquid in the vessel.

Due to the complexity of the reaction mixture and the difficulties to perform on-line composition measurements, control of batch and fed-batch reactors is essentially a problem of temperature control. The temperature profile in batch reactors usually follows three-stages [3]: (i) heating of the reaction mixture until the desired reaction temperature, (ii) maintenance of the system at this temperature and (iii) cooling stage in order to minimize the formation of by-products. Any controller used to control the reactor must be able to take into account these different stages.

2 Process model
In this paper, a fedbatch reactor model is used to study different control approaches. The model input data comes from a real process - the chromium waste recycling process [4].

Let us consider single input – single output (SISO) system of chemical exothermic semi-batch reactor (figure 1). The reactor has a double wall for cooling medium and the paddle stirrer for the reaction mass stirring. As can be seen from the figure, the working area is limited by the height of the cooling double wall, thus the actual maximum working volume of the reaction mass is 2,1166 m³.
The chemical reaction carried in the reactor is given by the following scheme:

\[
2C_7H_NO + 5K_2Cr_2O_7 + Cr_2O_3 + 23H_2SO_4 \rightarrow 6CO_2 + N_2 + 28H_2O + 5K_2SO_4 + 6Cr_2(SO_4)_3
\]  

(1)

where \( C_7H_NO \) is the protein and \( Cr_2O_3 \) is the chromium trioxide that are main compounds of the chromium sludge. The sulphuric acid (\( H_2SO_4(aq) \)) and the potassium dichromate (\( K_2Cr_2O_7(aq) \)) are main compounds of the reactor charge. For the reactor working volume there were computed following amounts of reactants, 641.7 kg of the chromium sludge, 535.2 kg of the 96% aqueous solution of the sulphuric acid, 335.0 kg of the potassium dichromate and 940.8 kg of water. Thus, the total weight of the reactor charge is 1811.0 kg.

Water, which flows in the double wall part, is used for the cooling of the reactor. After applying usual simplifications the mathematical model of this system can be written by equations (2)-(5). The illustrative scheme of the reactor is provided in the figure 2 (where the \( m_B \) stands for weight of reactor charge).

\[
\frac{dm(t)}{dt} = F_I
\]  

(2)

\[
\frac{da(t)}{dt} = \frac{F_I}{m(t)} - A \cdot e^{\frac{E}{RT(t)}} \cdot a(t)
\]  

(3)

\[
\frac{dT(t)}{dt} = \frac{F_I}{m(t)} \cdot c \cdot T(t) + \frac{A \cdot e^{\frac{E}{RT(t)}} \cdot \Delta H_r \cdot a(t)}{m(t) \cdot c} - \frac{K \cdot S \cdot T(t)}{m(t) \cdot c} + \frac{K \cdot S \cdot T_C(t)}{m(t) \cdot c}
\]  

(4)

\[
\frac{dT_C(t)}{dt} = \frac{F_C \cdot T_C(t)}{m_C \cdot c_C} + \frac{K \cdot S \cdot T(t)}{m_C \cdot c_C} - \frac{K \cdot S \cdot T_C(t)}{m_C \cdot c_C} - \frac{F_C \cdot T_C(t)}{m_C}
\]  

(5)

where \( m \) is the total weight of reaction components in the reactor, \( a \) is the mass concentration of the reaction component in the reactor, \( c = 4500 \text{ J.kg.K}^{-1} \) is the specific heat capacity of the reactor content, \( T \) is the temperature of the reactor content, \( F_I \), \( T_I = 293.15 \text{ K} \) and \( c_I = 4400 \text{ J.kg.K}^{-1} \) is the reaction component input mass flow rate, temperature and specific heat capacity. \( F_C = 1 \text{ kg.s}^{-1}, T_C = 288.15 \text{ K}, T_C, c_C = 4118 \text{ J.kg.K}^{-1} \) and \( m_C = 220 \text{ kg} \) is the cooling water mass flow rate, input temperature, output temperature, specific heat capacity and weight of the cooling water in the cooling system of the reactor, respectively. Other constants: \( A = 219.588 \text{ s}^{-1}, E = 29967.5087 \text{ J.mol}^{-1}, R = 8.314 \text{ J.mol}^{-1}.\text{K}^{-1}, \Delta H_r = 1392350 \text{ J.kg}^{-1}, K = 200 \text{ kg.s}^{-1}.\text{K}^{-1}, S = 7.36 \text{ m}^2.\text{K}^{-1}.\text{s}^{-1}\).
reactor takes about two-thirds of the in-reactor volume and the reactor is treated as ideally stirred, so we can do this simplification.

3 Control methods
Three different control methods were simulated to control the fed-batch reactor – two step control with penalization, PID control and model predictive control using artificial neural network. Also a two step control without penalization was applied, but was not satisfactory, so we skip that one. The task was to control the in-reactor temperature $T$ by reaction component dosing $F_I$. The desired value of temperature $T$ was 270K and the maximum value shouldn't exceed 273K. The actuating variable $F_I$ was from the interval $<0,3> \text{ kg.s}^{-1}$.

3.1 Two step control with penalization
The two step control with penalization provided these results: the upper-most in-reactor temperature $T$ reached 372.93 K, the maximum chromium sludge concentration $a$ was 0.0762 and the total batch time made 25727 seconds. The in-reactor temperature oscillated around the desired value in the subrange of 7 Kelvin degrees. The result process control diagrams are displayed in figure 1 and figure 2.

![Fig. 1 The in-reactor temperature development – two step control with penalization](image1)

![Fig. 2 The chromium sludge concentration development – two step control with penalization](image2)

3.2 PID control
The results of PID control were following: the upper-most in-reactor temperature $T$ reached 370.22 K, the maximum chromium sludge concentration $a$ was 0.0439 and the total batch time made 25491 seconds. The maximum and minimum actuating variable values were 1.546 kg.s$^{-1}$ or 0 kg.s$^{-1}$ respectively. The steady state actuating variable value made approximately 0.032 kg.s$^{-1}$. The PID control diagrams are displayed in figure 3 and figure 4.

![Fig. 3 The in-reactor temperature development – PID control](image3)

![Fig. 4 The in-reactor chromium sludge concentration development – PID control](image4)

3.3 Model predictive control
The basic idea of model predictive control (MPC) is to use a model to predict the future output trajectory of a process and compute a series of controller actions to minimize the difference between the predicted trajectory and a user-specified one, subject to constraints [5]. Generally we can say that MPC use artificial neural network (ANN) as the plant model in order to get its output predictions. The controller then calculates the control input that will optimize the performance criterion over a specified future time horizon. Typical form of the performance criterion $J$ is as follows:
\[ J = \lambda \sum_{j=N_1}^{N_2} [y_j(k+j) - \hat{y}(k+j)]^2 + \]
\[ + \rho \sum_{j=1}^{N_u} [u_j(k+j-1) - u_j(k+j-2)]^2 \]
\[ \text{(5)} \]

where \( N_1, N_2 \) and \( N_u \) define horizons over which the tracking error and the control increments are evaluated. The \( u_t \) variable is the tentative control signal, \( y_r \) is the desired response and \( j \) is the ANN predictor response. The \( \lambda \) and \( \rho \) parameters determine the contribution that the particular sum has on the performance index.

Due to the particular plant behaviour, the size of the control signal had to be penalized in the beginning of the batch. Thus, the third part of the criterion was added where the \( \gamma \) parameter determines the contribution that the third sum has on the performance index. However, in order to avoid the permanent control error the \( \gamma \) parameter was during the control gradually decreased up to zero. In other words, the third sum in the beginning of the control has the maximum value, and after initial phase it equals to zero.

\[ J = \lambda \sum_{j=N_1}^{N_2} [y_j(k+j) - \hat{y}(k+j)]^2 + \]
\[ + \rho \sum_{j=1}^{N_u} [u_j(k+j-1) - u_j(k+j-2)]^2 + \gamma \sum_{j=1}^{N_u} u_j(k+j) \]
\[ \text{(6)} \]

As can be seen from the figure 5 and 6, the MPC results were: the upper-most in-reactor temperature \( T \) reached 370.78 K, the maximum chromium sludge concentration \( a \) was 0.0461 and the total batch time made 25499 seconds. The maximum and minimum actuating variable values were 0.9375 kg.s\(^{-1}\) or 0 kg.s\(^{-1}\) respectively. The steady state actuating variable value made approximately 0.031 kg.s\(^{-1}\).

4 Conclusion
It is difficult to distinguish which one of the shown control method was the best. The shortest process time provided the PID control method, but the difference with regard to MPC was only 8 seconds. The total process time took over 7 hours, so the difference 8 seconds can be neglected. The best control performance was obtained by MPC, but simulation of this method is quite hardware demanding today. The simulation using CPU 2500 MHz computer took almost 2 hours. The cheapest solution for an industrial application could be the two step control with penalization, but just in the case we don’t need precise control performance.

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References: