Application of Kronecker Summation Method in Computation of Robustly Stabilizing PI Controllers for Interval Plants

RADEK MATUŠŮ, ROMAN PROKOP
Department of Automation and Control Engineering
Faculty of Applied Informatics
Tomas Bata University in Zlín
Nad Stráněmi 4511, 76005 Zlín
CZECH REPUBLIC
{rmatusu; prokop}@fai.utb.cz

KATARÍNA MATEJIČKOVÁ, MONIKA BAKOŠOVÁ
Department of Information Engineering and Process Control
Faculty of Chemical and Food Technology
Slovak University of Technology in Bratislava
Radlinského 9, 81237 Bratislava
SLOVAKIA
{katarina.matejickova; monika.bakosova}@stuba.sk

Abstract: - The contribution deals with design of continuous-time robustly stabilizing PI controllers for interval systems using the combination of Kronecker summation method, sixteen plant theorem and an algebraic approach to controller tuning. The effectiveness and practical applicability of the proposed method is demonstrated in control of a 3rd order nonlinear electronic plant.

Key-Words: - Robust Stabilization, PI Controllers, Interval Systems, Kronecker Summation Method

1 Introduction

Despite the development of many advanced control technologies, the engineers from practice still clearly prefer the application of controllers with simple PI or PID structure. This kind of controllers is very popular because of their easy implementation and sufficient performance at the same time, even under conditions of uncertainty, and thus the investigation of an effective tuning method remains very topical.

A possible approach to robust control design for systems with interval uncertainty [1], [2] consists of computation of all robustly stabilizing controllers and consequently the selection of the final one on the basis of user demands. The calculation of robustly stabilizing controllers can be done using the stability boundary locus as published in [3], [4] or alternatively with the assistance of Kronecker summation method [5]. The approach from [3], [4] has been analyzed in [6], [7], while this paper studies alternative method [5] and verifies it on the same laboratory apparatus as in [6], [7]. Furthermore, a technique for controller choice itself can be adopted from algebraic approach [8], [9], [10]. This method is based mainly on general solutions of Diophantine equations in the ring of proper and Hurwitz stable rational functions (R_{PS}). An advantage is that the controller can be further tuned through the only positive scalar tuning parameter *m*.

The contribution is focused on computation of continuous-time robustly stabilizing PI controllers for interval plants using Kronecker summation method, sixteen plant theorem and several algebraic tools. Originality of the proposed approach lies in combination of Kronecker summation method for obtaining the stability boundary and the choice of the final controller via an algebraic methodology. However, the work deals not only with theoretical background but also with the practical application in laboratory conditions. A nonlinear electronic plant, considered as the 3rd order interval system, has been controlled in various operational points with the assistance of the designed PI algorithms which have been realized using the Simatic automation system by Siemens Company.

2 Computation of Stabilizing PI Controllers Using Kronecker Summation

Consider the traditional closed-loop control system as depicted in fig. 1.

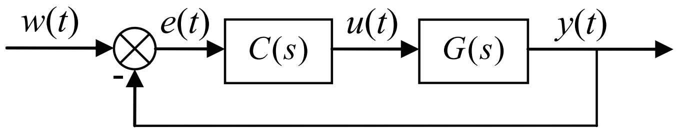

Fig. 1: Feedback control loop

The controlled plant is described by:

$$G(s)=\frac{B(s)}{A(s)} \tag{1}$$

and controller is supposed to be in a PI form:

$$C(s)=k_P+\frac{k_I}{s}=\frac{k_P s+k_I}{s} \tag{2}$$

The initial task is to determine the parameters of PI controllers which guarantee stability of the feedback system.

An approach to computation of stabilizing PI controllers which is based on interesting features of Kronecker summation has been published in [5].

First, remind that Kronecker summation of two square matrices Y (of size k-by-k) and Z (l-by-l) is generally defined as

$$Y\oplus Z=Y\otimes I_l+I_k\otimes Z \tag{3}$$

where I_k, I_l are identity matrices of size k-by-k and l-by-l, respectively, and where $\otimes$ denotes the Kronecker product [11], e.g. concisely:

$$Y\otimes I_l=\begin{bmatrix} y_{11}I_l & \cdots & y_{1k}I_l \\ \vdots & \ddots & \vdots \\ y_{k1}I_l & \cdots & y_{kk}I_l \end{bmatrix} \tag{4}$$

The momentous property of the obtained square matrix $Y\oplus Z$ (kl-by-kl) is that it has kl eigenvalues which are pair-wise combinatoric summations of the k eigenvalues of Y and l eigenvalues of Z. It means the Kronecker summation operation induces the "eigenvalue addition" feature to the matrices. One can exploit this attribute to obtain the equation for which all pairs (k_P,k_I) leading to purely imaginary roots comply.

The characteristic equation of the closed-loop system from fig. 1 is:

$$\begin{aligned} P_{CL}&=A(s)s+B(s)\left(k_P s+k_I\right)= \\ &=f_n(k_P,k_I)s^n+\cdots+f_1(k_P,k_I)s+f_0(k_P,k_I)=0 \end{aligned} \tag{5}$$

Define:

$$\begin{aligned} x_1'&=x_2 \\ x_2'&=x_3 \\ &\vdots \\ x_n'&=-\frac{f_0(k_P,k_I)}{f_n(k_P,k_I)}x_1-\frac{f_1(k_P,k_I)}{f_n(k_P,k_I)}x_2-\cdots-\frac{f_{n-1}(k_P,k_I)}{f_n(k_P,k_I)}x_n \end{aligned} \tag{6}$$

and transform (5) into matrix differential equation:

$$X'=MX \tag{7}$$

where M is n-by-n matrix:

$$M=\begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 \\ \vdots & 0 & 0 & 1 & \cdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \ddots & 0 \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ -\frac{f_0(k_P,k_I)}{f_n(k_P,k_I)} & -\frac{f_1(k_P,k_I)}{f_n(k_P,k_I)} & -\frac{f_2(k_P,k_I)}{f_n(k_P,k_I)} & \cdots & \cdots & -\frac{f_{n-1}(k_P,k_I)}{f_n(k_P,k_I)} \end{bmatrix} \tag{8}$$

and $X'=\left[x_1',x_2',\ldots,x_n'\right]^{\mathrm{T}}$, $X=\left[x_1,x_2,\ldots,x_n\right]^{\mathrm{T}}$. The equations (5) and (7) are linked via:

$$P_{CL}=f_n(k_P,k_I)\det\left(sI-M\right)=0 \tag{9}$$

Obviously, the same complex variable s is both the root of (5) and the eigenvalue of M. Owing to the fact that M is a constant matrix, the complex conjugates of s must also satisfy (9).

$$\det\left(s^*I-M\right)=0 \tag{10}$$

On that account, as it has been presented in [5], if $s=j\omega$ is the root of (5) it must be the eigenvalue of M. Moreover, $s^*=-j\omega$ is also the root of (5) and the eigenvalue of M. As the sum of two eigenvalues $s=j\omega$ and $s^*=-j\omega$ equals to zero, the Kronecker summation of two matrices must be singular when such correspondence of k_P, k_I and ω occurs. Thus:

$$\det\left(M\oplus M\right)=0 \tag{11}$$

defines the stability boundary in (k_P,k_I) plane, because every couple of (k_P,k_I) satisfying (11) means that the same couple inserted into (5) will lead to the pair of conjugate purely imaginary roots or zero roots. Those are the only positions where the system stability can shift. Generally, the stability boundary splits the (k_P,k_I) plane into the stable and unstable regions. The determination of the stabilizing area (or areas) can be done via a test point, leading to a polynomial to verify, within each region.

3 Robust Stabilization of Interval Plants

The previous section has outlined calculation of region of stabilizing compensator parameters only for a system with fixed coefficients. Nevertheless, the works [3], [4], [5] have embellished an arbitrary stabilization technique also for interval plants simply by using its combination with the sixteen plant theorem [1], [12], [13]. According to this rule, a first order controller robustly stabilizes an interval plant:

$$G(s,b,a)=\frac{B(s,b)}{A(s,a)}=\frac{\sum_{i=0}^{m}\left[b_i^-,b_i^+\right]s^i}{\sum_{i=0}^{n}\left[a_i^-,a_i^+\right]s^i};\quad m<n \tag{12}$$

where $b_i^-, b_i^+, a_i^-, a_i^+$ represent respectively lower and upper bounds for parameters of numerator and denominator if and only if it stabilizes its 16 Kharitonov plants, which are defined as:

$$G_{i,j}(s)=\frac{B_i(s)}{A_j(s)} \tag{13}$$

where $i,j\in\{1,2,3,4\}$; and $B_1(s)$ to $B_4(s)$ and $A_1(s)$ to $A_4(s)$ are the Kharitonov polynomials for the numerator and denominator of the interval system (12).

Remind that the construction of Kharitonov polynomials e.g. for the numerator interval polynomial:

$$B(s,b)=\sum_{i=0}^{m}\left[b_i^-;b_i^+\right]s^i \tag{14}$$

is based on use of the lower and upper bounds of interval parameters in compliance with the principle [14]:

$$\begin{aligned}B_1(s)&=b_0^-+b_1^-s+b_2^+s^2+b_3^+s^3+\cdots\\B_2(s)&=b_0^++b_1^+s+b_2^-s^2+b_3^-s^3+\cdots\\B_3(s)&=b_0^++b_1^-s+b_2^-s^2+b_3^+s^3+\cdots\\B_4(s)&=b_0^-+b_1^+s+b_2^+s^2+b_3^-s^3+\cdots\end{aligned} \tag{15}$$

As can be seen, the stabilization of an interval plant directly follows from the simultaneous stabilization of all 16 fixed Kharitonov plants. Thus the final area of stability for original interval plant is given by intersection of all 16 related partial areas obtained individually using the Kronecker summation method from the previous section.

4 Algebraic Design of PI Controller

So far, the methodologies from sections 2 and 3 allow calculating all robustly stabilizing combinations of proportional and integral gains in PI controller. Nonetheless, the final selection of a controller is another problem. An effective solution is represented by algebraic approach to control design [8], [9], [10], which is based on general solutions of Diophantine equations in R_{PS}, Youla-Kučera parameterization and conditions of divisibility in the specific ring. A merit of the technique is that the controllers can be tuned by the only positive scalar parameter *m*.

Due to the limited space the paper can not provide full details on this method [7], [10]. It exploits only one specific result, i.e. the coefficients of feedback PI controller (2) can be computed according to:

$$k_P=\frac{2m-a_0}{b_0};\quad k_I=\frac{m^2}{b_0} \tag{16}$$

where the parameters a_0 and b_0 of the first order nominal controlled plant:

$$G(s)=\frac{b_0}{s+a_0} \tag{17}$$

are supposed to be known and where the tuning parameter *m* can be chosen on the basis of several approaches such as trivial “trial-and-error”, user knowledge and experience, or using recommendation [15]:

$$m=ka_0 \tag{18}$$

Appropriate coefficient *k* depends on the size of first overshoot of the output (controlled) variable. Some of its values can be found in table 1.

Table 1: Relation between *k* and overshoot

Overshoot [%]	*K*
0	1.00
1	1.62
2	1.87
3	2.14
5	2.80
10	7.38

5 Real Control Experiments

The presented theoretical tools have been tested in laboratory conditions during robust control of a nonlinear electronic model while the control loop has been realized using Simatic S7-300 automation system.

The utilized plant, constructed at Slovak University of Technology in Bratislava, has included a 3rd order system with a variable time constant, adjustable from 5s to 20s, and a model of nonlinear valve. The real visual appearance of this model is shown in fig. 2 and the block

scheme of the process is in fig. 3, where signals are denoted as follows:
V – control signal for valve opening (0 – 10V)
F – signal representing the valve opening (0 – 10V)
P – output of the process (0 – 10V)
U – disturbance (0 – 10V)

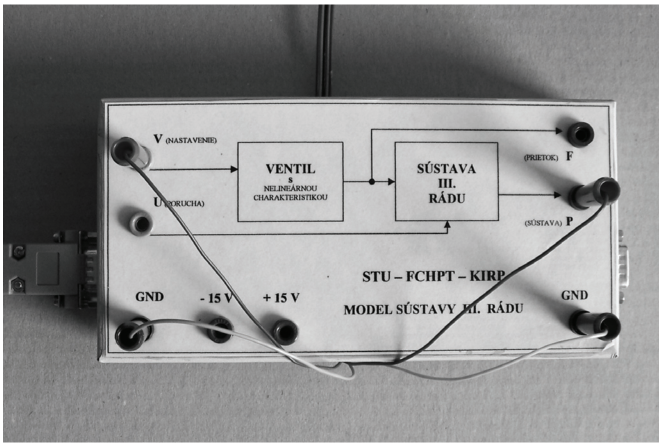

Fig. 2: Electronic laboratory model

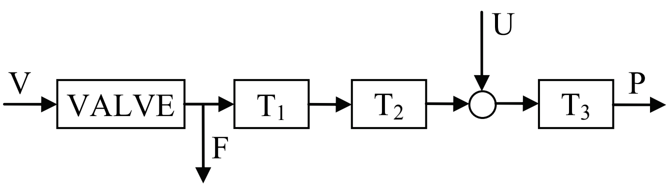

Fig. 3: Block scheme of laboratory model

The plant has been identified as the third order interval system which has led to the approximate mathematical model [6], [7], [16]:

$$G_I(s,b,a) = \frac{[0.35, 5.5]}{[83, 268]s^3 + [104, 171]s^2 + [19, 25]s + 1} \quad (19)$$

The first of its 16 Kharitonov plants (13) can be simply constructed:

$$G_{1,1}(s) = \frac{0.35}{268s^3 + 171s^2 + 19s + 1} \quad (20)$$

The closed-loop characteristic equation (5) is:

$$268s^4 + 171s^3 + 19s^2 + (1 + 0.35k_P)s + 0.35k_I = 0 \quad (21)$$

From here, the matrix (8) follows:

$$M = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -0.001306k_I & -0.003731 - 0.001306k_P & -0.0709 & -0.6381 \end{bmatrix} \quad (22)$$

The stability boundary is given by (11). The position of such pairs (k_P, k_I) which fulfil (11) are shown in fig. 4.

As can be verified, the stabilizing PI controllers for the plant (20) are in the inner space.

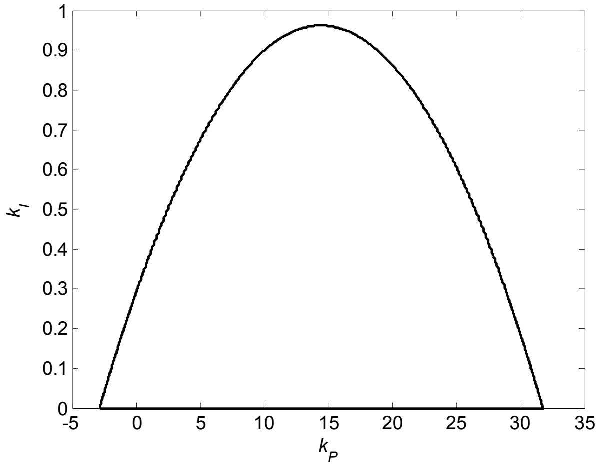

Fig. 4: Stability boundary for the plant (20)

Generally, one must repeat an analogical procedure for all 16 Kharitonov plants (13). Nonetheless, in such specific case, only 8 plants are enough to test. It is thanks to the fact that the numerator of (19) represents just zero order polynomial with two extreme values and thus it is not necessary to deal with all 4 Kharitonov polynomials for this numerator. The regions of stability regions for all 8 Kharitonov plants are plotted in fig. 5.

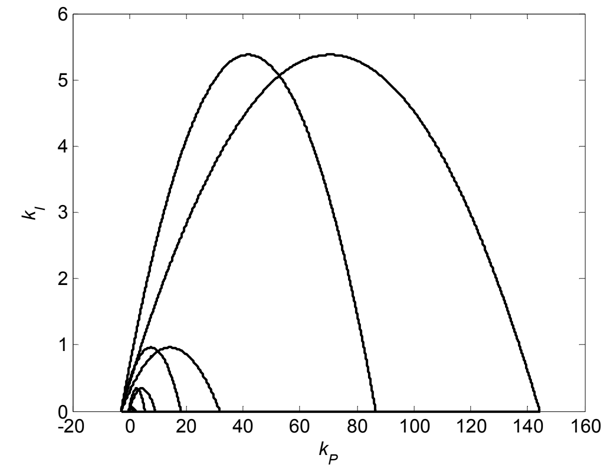

Fig. 5: Stability areas for 8 Kharitonov plants

The intersection of all these stability areas is zoomed and depicted in fig. 6. It determines the final region of robustly stabilizing PI controllers for the original interval model (19).

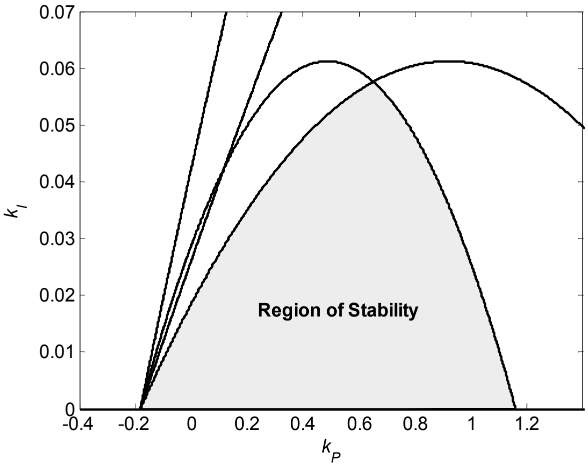

Fig. 6: Stability region for the interval system (19)

Quite naturally, the following step brings the question of how to find the practically convenient PI controller from the obtained robust stability region. Among possible methods, the algebraic approach from the Part 4 has been utilized for this purpose.

However, this algebraic synthesis requires the model of controlled system in the form of first order transfer function in order to obtain the final controller of appropriate (first) order (PI type). So the simplest approximation of (19) has been applied. It results in:

$$G_A(s,b,a) = \frac{[0.35, 5.5]}{[19, 25]s+1} \tag{23}$$

Computing the average values of interval parameters then lead to the nominal plant for control design:

$$G_N(s) = \frac{2.925}{22s+1} \doteq \frac{0.133}{s+0.04545} \tag{24}$$

First, the assumption of 0% first overshoot in output variable for the case of nominal system, application of appropriate parameter k from table 1, and furthermore equations (18) and (16) give the transfer function of the controller:

$$0\% \Rightarrow m = 0.04545 \Rightarrow C_1(s) \doteq \frac{0.3417s+0.01553}{s} \tag{25}$$

Then analogically, 1% first overshoot requirement results in:

$$1\% \Rightarrow m \doteq 0.07363 \Rightarrow C_2(s) \doteq \frac{0.7655s+0.04076}{s} \tag{26}$$

The fig. 7 depicts the positions of the controllers (25) and (26) in the stability area from fig. 6. As can be seen, they lie on the curve hypothetically connecting the other potential controllers tuned by various parameters $m > 0$.

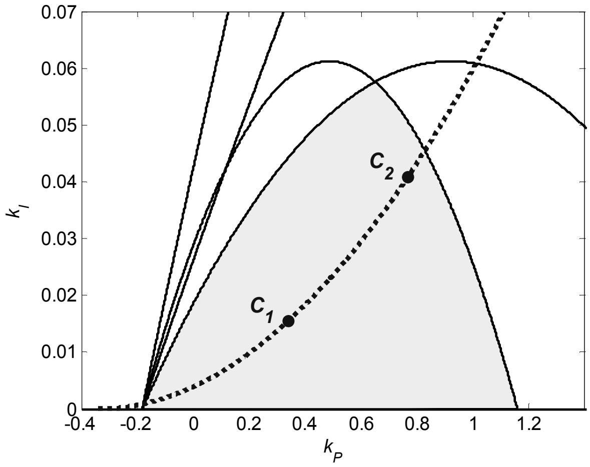

Fig. 7: Positions of controllers (25) and (26) in stability region

Finally, three control experiments have been executed under different working points using the chosen controllers and PLC Simatic S7-300.

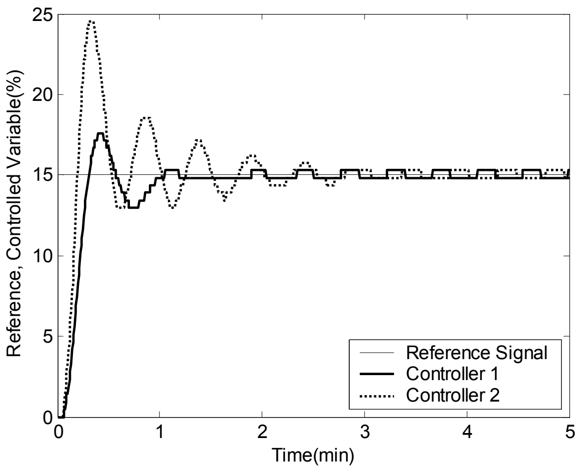

Fig. 8: Real control results (for 15% reference point)

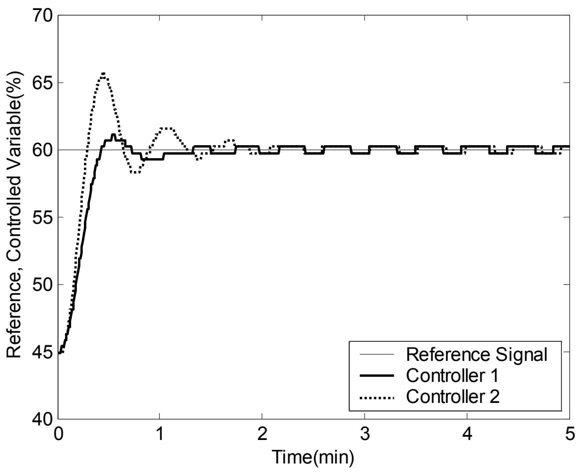

Fig. 9: Real control results (for 60% reference point)

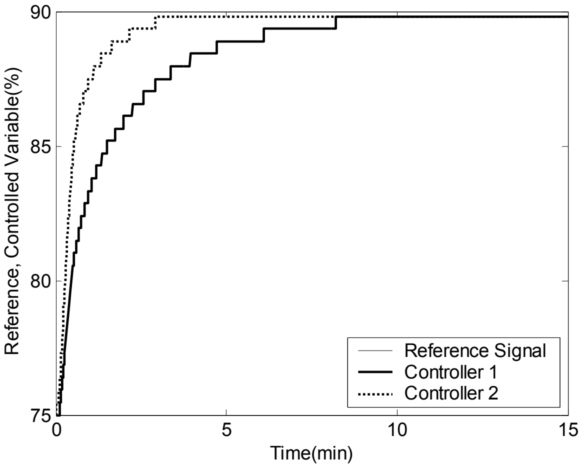

Fig. 10: Real control results (for 90% reference point)

The nominally prescribed overshoots have not been measured in real conditions. Actually it was expected, because the controlled plant has had highly nonlinear behaviour and these recommendations strictly hold true only for the nominal linear system. Figs. 8-10 indicate that the "less aggressive" controller C_1 provides very good results mainly in the mean set points, but it has comparatively long settling time in higher operational areas. On the other hand, the controller C_2 is much "faster" here, however it is more oscillating in the lower levels. Altogether, both compensators have been able to control the nonlinear process robustly stable and with acceptable performance. The definitive selection of the controller would depend on the main operational area.

6 Conclusion

The paper has dealt with an approach to computation of robustly stabilizing PI controllers. The proposed method has been based on combination of calculating the stability boundary via Kronecker summation, its extension for interval systems using 16 plant theorem, and the choice of the final regulator through the single-parameter tuning algebraic approach. The developed synthesis represents easy but effective way of designing the controllers for interval systems. On the other hand, coincident nominal performance and robust stability can not be assured in advance. They have to be verified during the design process which can be considered as a method demerit. However, the applicability has been shown on laboratory experiments in which a nonlinear 3rd order electronic model has been successfully controlled in various operational points.

Acknowledgements:
The authors would like to gratefully acknowledge the support from the Ministry of Education, Youth and Sports of the Czech Republic under Research Plan No. MSM 7088352102 and the Scientific Grant Agency of the Slovak Republic under Grants No. 1/0537/10 and 1/0071/09.

References:

[1] B. R. Barmish, *New Tools for Robustness of Linear Systems*, Macmillan, New York, USA, 1994.

[2] S. P. Bhattacharyya, H. Chapellat, L. H. Keel, *Robust control: The parametric approach*, Prentice Hall, Englewood Cliffs, NJ, USA, 1995.

[3] N. Tan, I. Kaya, Computation of stabilizing PI controllers for interval systems, In: *Proceedings of the 11th Mediterranean Conference on Control and Automation*, Rhodes, Greece, 2003.

[4] N. Tan, I. Kaya, C. Yeroglu, D. P. Atherton, Computation of stabilizing PI and PID controllers using the stability boundary locus, *Energy Conversion and Management*, Vol. 47, No. 18-19, 2006, pp. 3045-3058.

[5] J. Fang, D. Zheng, Z. Ren, Computation of stabilizing PI and PID controllers by using Kronecker summation method, *Energy Conversion and Management*, Vol. 50, No. 7, 2009, pp. 1821-1827.

[6] R. Matušů, K. Vaneková, R. Prokop, M. Bakošová, Robust proportional-integral control of a laboratory model using programmable logic controller SIMATIC S7-300, In: *Proceedings of the 6th IFAC Symposium on Robust Control Design*, Haifa, Israel, 2009.

[7] R. Matušů, K. Vaneková, R. Prokop, M. Bakošová, Design of robust PI controllers and their application to a nonlinear electronic system, *Journal of Electrical Engineering*, Vol. 61, No. 1, 2010, pp. 44-51.

[8] M. Vidyasagar, *Control system synthesis: a factorization approach*, MIT Press, Cambridge, M.A., USA, 1985.

[9] V. Kučera, Diophantine equations in control - A survey, *Automatica*, Vol. 29, No. 6, 1993, pp. 1361-1375.

[10] R. Prokop, J. P. Corriou, Design and analysis of simple robust controllers, *International Journal of Control*, Vol. 66, No. 6, 1997, pp. 905-921.

[11] D. S. Bernstein, *Matrix Mathematics: Theory, Facts, and Formulas with Application to Linear Systems Theory*, Princeton University Press, NJ, USA, 2005.

[12] B. R. Barmish, C. V. Hollot, F. J. Kraus, R. Tempo, Extreme point results for robust stabilization of interval plants with first order compensators, *IEEE Transactions on Automatic Control*, Vol. 37, No. 6, 1992, pp. 707-714.

[13] M. T. Ho, A. Datta, S. P. Bhattacharyya, Design of P, PI and PID controllers for interval plants, In: *Proceedings of American Control Conference*, Philadelphia, USA, 1998.

[14] V. L. Kharitonov, Asymptotic stability of an equilibrium position of a family of systems of linear differential equations, *Differentsial'nye Uravneniya*, Vol. 14, 1978, pp. 2086-2088.

[15] R. Matušů, R. Prokop, Single-parameter tuning of PI controllers: from theory to practice. In: *Proceedings of the 17th IFAC World Congress*, Seoul, Korea, 2008.

[16] K. Vaneková, M. Bakošová, J. Závacká, Robust PI control of a laboratory process using control system Simatic, In: *Proceedings of 16th International Conference on Process Control*, Štrbské Pleso, Slovakia, 2007.