The Use of the Hierarchical Structured Dynamic Inversion to the Aircrafts Lateral Movement

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Abstract: - The paper presents a methodology for the flight control law's design for the trajectory pursuit using hierarchical dynamic inversion; this is based on separation of multi-time-scale and multi-loop closing method. It greatly simplifies the flight control design compared with PID conventional approaches. The used dynamic equations are classified into 4 groups according to the stairs of time measuring from the physical point of view [1]. The authors made the analysis of the lateral movement of aircrafts and obtained graphic characteristics which demonstrate the effectiveness of the proposed method.

Key-Words: - aircraft, control law, hierarchical dynamic inversion, lateral movement.

1 Introduction

One knows that it is difficult to stabilize and control an aircraft using constant gain controllers because the aircraft's dynamics vary with the considerable modification of the dynamic pressure and Mach number. That's why a very good method for solve this problem is the determination of the gains of the control system. This is a simple and direct methodology for the design of flight control systems. The technique of the gains' determination is the most important thing today in the area of flight control's design [2], [3].

The technique of gains' determination depends on the designer's experience and on his engineering art. Variables' separation on two time scales combined with the theory of singular perturbation have been subject of research, the attitude being taken as slow variable while angular velocities as fast variables. The slow variables are controlled by the fast ones, which, in turn, are controlled by aerodynamic command surfaces.

2 Formulation of the hierarchical dynamic inversion

One considers the following nonlinear system [4], [5], [6]

$$\begin{aligned} \dot{x} &= f(x,u), \\ y &= h(x), \end{aligned} \tag{1}$$

where $x \in R^n$ is the state variable, $u \in R^m$ is the control input and $y \in R^m -$ the output which will be controlled by the control input u. From equations (1), one gets

$$\dot{y} = \frac{\partial h}{\partial x} f(x,u) = F(x,u) \qquad (2)$$

or

$$u = F^{-1}(x,v), \qquad (3)$$

where v is the auxiliary input of the system. From equations (2) and (3) one yields

$$\dot{y} = F\left(x, F^{-1}(x,v)\right) = v\,. \qquad (4)$$

The auxiliary input may have the classical form

$$v = K(y_c - y), \qquad (5)$$

where K is a gain matrix and y_c the imposed value of y.

The term that compensates the nonlinear dynamics also provides the linearization of the dynamic system and the exterior loop, expressed by equation (5); the system becomes linear and achieves the desired value of the output y_c (fig. 1).

Unfortunately, some input-output equations do not describe the aircraft dynamics with minimum phase because of the aerodynamic forces' derivatives in rapport with control surfaces' deflections. This fact has prevented the direct application of dynamic inversion to the automatic flight control systems. This problem can be avoided by system's separating on two time scales; thus, there are slow variables and fast variables. Fast state variables are used to control the slow state variables while the fast variables are controlled by the command variable. One considers the following two time scales nonlinear system

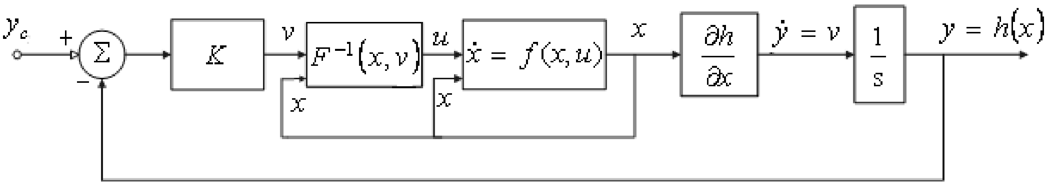

Fig.1 The linear system with dynamic inversion

$$\dot{x}_1 = f_1(x_1, x_2, u),\, \dot{x}_2 = f_2(x_1, x_2, u),\, y = h(x_1), \quad (6)$$

where $x_1 \in R^n$ is the slow state, $x_2 \in R^n$ is the fast state, $u \in R^n -$ the control input and $y \in R^n -$ the controlled output. The input-output equations on the slow scale may be derived as follows

$$\dot{y} = \frac{\partial h}{\partial x_1} f_1(x_1, x_2, u) \equiv F(x_1, x_2, u), \qquad (7)$$

where $F(x_1, x_2, u)$ is invertible in rapport with x_2.

One obtains x_{2c} from the previous equation using the dynamic inversion

$$x_{2c} = F^{-1}(x_1, v_1, u),\, v_1 = K_1(y_c - y), \qquad (8)$$

where v_1 is the auxiliary input for the slow scale controller and $K_1 -$ the feedback gain matrix. If $x_2 = x_{2c}$, the following equation is maintained

$$\dot{y} = F\left(x_1, F^{-1}(x_1, v_1, u)\right) = K_1(y_c - y) = v_1. \qquad (9)$$

Finally, one obtains u_c in the fast scale so that $x_2 \to x_{2c}$

$$\begin{aligned} u_c &= f_2^{-1}(x_1, x_2, v_2), \\ v_2 &= K_2(x_{2c} - x_2), \end{aligned} \qquad (10)$$

where v_2 is the auxiliary input for the fast scale controller $K_2 -$ the feedback gain matrix.

3 The use of hierarchical dynamic inversion to the aircrafts' dynamics

For the conventional aircrafts with fixed wing the command surfaces' deflections has the slowest time scale [4], [5]. These deflections generate aerodynamic moments around aircrafts' axes. The aerodynamic moments generate angular velocities and the angular velocities are integrated in order to obtain the aircraft's attitude. The forces have the same time scale with the accelerations. The attitude is integrated to obtain the velocities and the velocities give the position of the flying object. The variables may be grouped in four layers (time scales): very slow scale (the position of the aircraft X, Y, Z), slow scale (none of the variables), fast scale (velocities U, V, W and angles φ, θ and ψ) and very fast scale (the angular velocities P, Q, R) [3]. The aircraft position is defined by the longitudinal error e_h, the lateral error e_y and the trajectory arc length s. The velocities are defined by the real velocity of the air currents V_{TAS}, the direction angle in rapport with the air currents ψ_a and the trajectory's angle in rapport with the air currents γ_a. V_{TAS} is directly controlled by the thrust force or by the aerodynamic braking. The attitude is defined by three angles: roll angle φ, pitch angle θ and sideslip angle β; all these angles are controlled by angu-

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lar velocities [1]. For the coordinated flight $\beta_c = 0$, the incidence angle α is not considered a state variable as it appears in [3]. This will improve the precision of control because the inertial attitude can be measured with less error than the aerodynamic angles like α. The three angular velocities P, Q, R are controlled by the three command surfaces: rudder, aileron and direction [7].

It is not enough to choose the state variables. This choice may be not optimal for some applications; that's why state transformations will be made.

The state variables are transformed from the initial ones $x \in R^{12}$ in the new state variables $\xi \in R^{12}$. One notes with $T(x)$ the nonlinear transformation which verifies equation $\xi = T(x)$, where ξ is selected so that $T -$ invertible $x = T^{-1}(\xi)$.

$$\begin{aligned} x &= \begin{bmatrix} X & Y & Z & U & V & W & \varphi & \theta & \psi & P & Q & R \end{bmatrix}^T, \\ \xi &= \begin{bmatrix} s & e_y & e_h & V_{TAS} & \psi_a & \gamma_a & \varphi & \theta & \beta & P & Q & R \end{bmatrix}^T. \end{aligned} \tag{11}$$

The control vector u contains four variables representing the deflections of control surfaces

$$u = \begin{bmatrix} \delta_p & \delta_e & \delta_d & \delta_T \end{bmatrix}, \tag{12}$$

where $\delta_p, \delta_e, \delta_d$ and δ_T are the deflections of the rudder, aileron, direction, respectively the gas lever's displacement. Taking into account the multi time scale separation from the previous section, ξ is separated as follows

$$\begin{aligned} \xi_1 &= \begin{bmatrix} e_y & e_h \end{bmatrix}, \xi_2 = \begin{bmatrix} V_{TAS} & \psi_a & \gamma_a \end{bmatrix}^T, \\ \xi_3 &= \begin{bmatrix} \varphi & \theta & \beta \end{bmatrix}^T, \xi_4 = \begin{bmatrix} P & Q & R \end{bmatrix}^T. \end{aligned} \tag{13}$$

In layer $i\ (i = 1,2,3)$ the equations of dynamic models of the subsystems can be defined as

$$\dot{\xi}_i = F_i\left(\xi_i, \xi_{i+1}, u, \tilde{\xi}_i\right), i = \overline{1,3}, \tag{14}$$

where $\tilde{\xi}_i$ is a set of state variables other than ξ_i and ξ_{i+1}. On the other hand the dynamic equations of the inner layer $(i = 4)$ and those for V_{TAS} are given as

$$\dot{\xi}_4 = F_4\left(\xi, \delta_T, \tilde{u}\right), \dot{V}_{TAS} = F_{21}\left(\xi, \delta_T, \tilde{u}\right), \tag{15}$$

where $\tilde{u}$ is the set of control variables. This case $\xi_{(i+1)c}$, $\tilde{u}_c$ and δ_{Tc} are determined from equations [1]

$$\begin{aligned} x_{(i+1)c} &= F^{-1}\left(x_i, v_i, u, \tilde{x}_i\right), v_i = K_i\left(x_{ic} - x_i\right), \\ u_c &= f_n^{-1}\left(x, v_n\right), v_n = K_n\left(x_{nc} - x_n\right). \end{aligned} \tag{16}$$

One obtains

$$\begin{aligned} \xi_{(i+1)c} &= F^{-1}\left(\xi_i, u, \tilde{\xi}_i, v_i\right), v_i = K_i\left(\xi_{ic} - \xi_i\right), \\ u_c &= F_4^{-1}\left(\xi, \delta_T, v_4\right), v_4 = K_4\left(\xi_{4c} - \xi_4\right), \\ \delta_{Tc} &= F_{21}^{-1}\left(\xi, v_{21}, \tilde{u}\right), v_{21} = K_{21}(V_{TASc} - V_{TAS}). \end{aligned} \tag{17}$$

Using Taylor series expansions of F_i

$$F_i\left(\xi_i + \Delta\xi, u + \Delta u\right) \cong F_i(\xi, u) + \frac{\partial F_i}{\partial \xi} + \frac{\partial F_i}{\partial u} \tag{18}$$

and the first equation (17), one gets

$$\begin{aligned} & F_i\left(\xi_i, \xi_{i+1}, u, \tilde{\xi}_i\right) + \frac{\partial F_i}{\partial \xi_{i+1}}\left(\xi_{(i+1)c} - \xi_{i+1}\right) = K_i\left(\xi_{ic} - \xi_i\right), \\ & F_4\left(\xi, \delta_T, \tilde{u}\right) + \frac{\partial F_4}{\partial \tilde{u}}\left(\tilde{u}_c - u\right) = K_4\left(\xi_{4c} - \xi_4\right), \\ & F_{21}\left(\xi, \delta_T, \tilde{u}\right) + \frac{\partial F_{21}}{\partial \delta_T}\left(\delta_{Tc} - \delta_T\right) = K_{21}(V_{TASc} - V_{TAS}). \end{aligned} \tag{19}$$

In the above equation the superior order terms have been neglected and that is why the inversion is inexact.

Solving equation (19) in rapport with $\xi_{(i+1)c}$, $\tilde{u}_c$ and δ_{Tc}, one gets

$$\begin{aligned} \xi_{(i+1)c} &= \xi_{i+1} - \left(\frac{\partial F_i}{\partial \xi_{i+1}}\right)^{-1}\left\{F_i\left(\xi_i, \xi_{i+1}, u, \tilde{\xi}_i\right) - K_i\left(\xi_{ic} - \xi_i\right)\right\}, \\ \tilde{u}_c &= \tilde{u} - \left(\frac{\partial F_4}{\partial \tilde{u}}\right)^{-1}\left\{F_4\left(\xi, \delta_T, \tilde{u}\right) - K_4\left(\xi_{4c} - \xi_4\right)\right\}, \\ \delta_{Tc} &= \delta_T - \left(\frac{\partial F_{21}}{\partial \delta_T}\right)^{-1}\left\{F_{21}\left(\xi, \delta_T, \tilde{u}\right) - K_{21}(V_{TASc} - V_{TAS})\right\}. \end{aligned} \tag{20}$$

4 Aircraft numerical application of the hierarchical dynamic inversion

One considers the lateral movement of an aircraft described by equations [1]

$$\begin{bmatrix} F_1^{lat} \\ F_2^{lat} \\ F_3^{lat} \\ F_4^{lat} \end{bmatrix} = \begin{bmatrix} \dot{e}_y \\ \dot{\psi}_a \\ \begin{bmatrix} \dot{\varphi} \\ \dot{\beta} \end{bmatrix} \\ \begin{bmatrix} \dot{P} \\ \dot{R} \end{bmatrix} \end{bmatrix} = A^{lat} \cdot \begin{bmatrix} e_y \\ \psi_a \\ \begin{bmatrix} \varphi \\ \beta \end{bmatrix} \\ \begin{bmatrix} P \\ R \end{bmatrix} \end{bmatrix} + B^{lat} \cdot \begin{bmatrix} b_{11}^{lat} & b_{21}^{lat} \\ b_{21}^{lat} & b_{22}^{lat} \\ b_{31}^{lat} & b_{32}^{lat} \\ b_{41}^{lat} & b_{42}^{lat} \\ b_{51}^{lat} & b_{52}^{lat} \\ b_{61}^{lat} & b_{62}^{lat} \end{bmatrix} \begin{bmatrix} \delta_e \\ \delta_d \end{bmatrix},$$

$$A^{lat} = \begin{bmatrix} a_{11}^{lat} & a_{12}^{lat} & a_{13}^{lat} & a_{14}^{lat} & a_{15}^{lat} & a_{16}^{lat} \\ a_{21}^{lat} & a_{22}^{lat} & a_{23}^{lat} & a_{24}^{lat} & a_{25}^{lat} & a_{26}^{lat} \\ a_{31}^{lat} & a_{32}^{lat} & a_{33}^{lat} & a_{34}^{lat} & a_{35}^{lat} & a_{36}^{lat} \\ a_{41}^{lat} & a_{42}^{lat} & a_{43}^{lat} & a_{44}^{lat} & a_{45}^{lat} & a_{46}^{lat} \\ a_{51}^{lat} & a_{52}^{lat} & a_{53}^{lat} & a_{54}^{lat} & a_{55}^{lat} & a_{56}^{lat} \\ a_{61}^{lat} & a_{62}^{lat} & a_{63}^{lat} & a_{64}^{lat} & a_{65}^{lat} & a_{66}^{lat} \end{bmatrix}, B^{lat} = \begin{bmatrix} b_{11}^{lat} & b_{21}^{lat} \\ b_{21}^{lat} & b_{22}^{lat} \\ b_{31}^{lat} & b_{32}^{lat} \\ b_{41}^{lat} & b_{42}^{lat} \\ b_{51}^{lat} & b_{52}^{lat} \\ b_{61}^{lat} & b_{62}^{lat} \end{bmatrix}, \tag{21}$$

$$\dot{\xi}_{lat} = A^{lat}\xi_{lat} + B^{lat}u_{lat}.$$

One customizes the relations (20) for variables $\xi_i\ \left(i = \overline{1,4}\right)$ defined by equation (13). Thus, for the lateral movement of the aircrafts, $i = 1, \xi_{2c}$ has components ψ_{ac}, γ_{ac} and V_{TASc}

$$\xi_2^{lon} = \gamma_{ac} = \gamma_a - \left(\frac{\partial F_1^{lon}}{\partial \gamma_a}\right)^{-1}\left[F_1^{lon}\left(\xi_1, \xi_2, u, \tilde{\xi}_1\right) - v_1^{lon}\right], \tag{22}$$

Similar equations are obtained for the lateral movements

$$\begin{aligned} \psi_{ac} &= \psi_a - \left(\frac{\partial F_1^{lat}}{\partial \psi_a}\right)^{-1}\left[F_1^{lat}\left(\xi_1, \xi_2, u, \tilde{\xi}_1\right) - v_1^{lat}\right], \\ \dot{e}_y &= F_1^{lat}\left(\psi_a\right) = a_{11}^{lat}e_y + a_{12}^{lat}\psi_a + a_{13}^{lat}\varphi + a_{14}^{lat}\beta + \\ &\quad + a_{15}^{lat}P + a_{16}^{lat}R + b_{11}^{lat}\delta_e + b_{12}^{lat}\delta_d \end{aligned} \tag{23}$$

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$$\psi_{ac} = -\left(a_{12}^{lat}\right)^{-1}\left(a_{11}^{lat} e_y + a_{13}^{lat}\varphi + a_{14}^{lat}\beta + a_{15}^{lat} P + a_{16}^{lat} R\right) - \left(a_{12}^{lat}\right)^{-1}\left[b_{11}^{lat}\delta_e + b_{12}^{lat}\delta_d - K_1^{lat}(e_{yc} - e_y)\right]. \quad (24)$$

For $i = 2$ one yields [1]

$$\beta_c = 0, \varphi_c = \varphi - \left(\frac{\partial F_2^{lat}}{\partial \varphi}\right)^{-1}\left[F_2^{lat}\left(\xi_2, \xi_3, u, \tilde{\xi}_2\right) - v_2^{lat}\right], \quad (25)$$

where F_2^{lat} is expressed wit equation (21) as follows

$$F_2^{lat} = \dot{\psi}_a = a_{21}^{lat} e_y + a_{22}^{lat}\psi_a + a_{23}^{lat}\varphi + a_{24}^{lat}\beta + a_{25}^{lat} P + a_{26}^{lat} R + b_{21}^{lat}\delta_e + b_{22}^{lat}\delta_d , \quad (26)$$

$$\varphi_c = -\left(a_{23}^{lat}\right)^{-1}\left(a_{21}^{lat} e_y + a_{22}^{lat}\psi_a + a_{24}^{lat}\beta + a_{25}^{lat} p\right) - \left(a_{23}^{lat}\right)^{-1}\left[a_{26}^{lat} R + b_{21}^{lat}\delta_e + b_{22}^{lat}\delta_d - K_2^{lat}\left(\psi_{ac} - \psi_a\right)\right] \quad (27)$$

and for $i = 3$ one gets

$$\begin{bmatrix} P_c \\ R_c \end{bmatrix} = \begin{bmatrix} P \\ R \end{bmatrix} - \left(\frac{\partial F_3^{lat}}{\partial \begin{bmatrix} P \\ R \end{bmatrix}}\right)^{-1}\left[F_3^{lat}\left(\xi_3, \xi_4, u, \tilde{\xi}_3\right) - v_3^{lat}\right], \quad (28)$$

$$F_3^{lat} = \begin{bmatrix} \dot{\varphi} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} a_{31}^{lat} & a_{32}^{lat} & a_{33}^{lat} & a_{34}^{lat} & a_{35}^{lat} & a_{36}^{lat} \\ a_{41}^{lat} & a_{42}^{lat} & a_{43}^{lat} & a_{44}^{lat} & a_{45}^{lat} & a_{46}^{lat} \end{bmatrix} \cdot x + \begin{bmatrix} b_{31}^{lat} & b_{32}^{lat} \\ b_{41}^{lat} & b_{42}^{lat} \end{bmatrix}\begin{bmatrix} \delta_e \\ \delta_d \end{bmatrix}, \quad (29)$$

$$\begin{bmatrix} P_c \\ R_c \end{bmatrix} = -\begin{bmatrix} a_{35}^{lat} & a_{36}^{lat} \\ a_{45}^{lat} & a_{46}^{lat} \end{bmatrix}^{-1}\left\{C \cdot x - \begin{bmatrix} K_{31}^{lat} & 0 \\ 0 & K_{32}^{lat} \end{bmatrix}\left\{\begin{bmatrix} \varphi_c \\ \beta_c \end{bmatrix} - \begin{bmatrix} \varphi \\ \beta \end{bmatrix}\right\}\right\}, \quad (30)$$

where x is the state vector and matrix C has the form

$$C = \begin{bmatrix} a_{31}^{lat} & a_{32}^{lat} & a_{33}^{lat} & a_{34}^{lat} & b_{31}^{lat} & b_{32}^{lat} \\ a_{41}^{lat} & a_{42}^{lat} & a_{43}^{lat} & a_{44}^{lat} & b_{41}^{lat} & b_{42}^{lat} \end{bmatrix}. \quad (31)$$

To calculate $\tilde{u}_c$ the authors use second equation (20) and take into account that $\tilde{u} = \left[\delta_e \; \delta_p \; \delta_d\right]^T$. Thus, for the lateral movement of the aircrafts, one obtains

$$F_4^{lat} = \begin{bmatrix} \dot{P} \\ \dot{R} \end{bmatrix} = D \cdot x + \begin{bmatrix} b_{51}^{lat} & b_{52}^{lat} \\ b_{61}^{lat} & b_{62}^{lat} \end{bmatrix}\begin{bmatrix} \delta_e \\ \delta_d \end{bmatrix}, \quad (32)$$

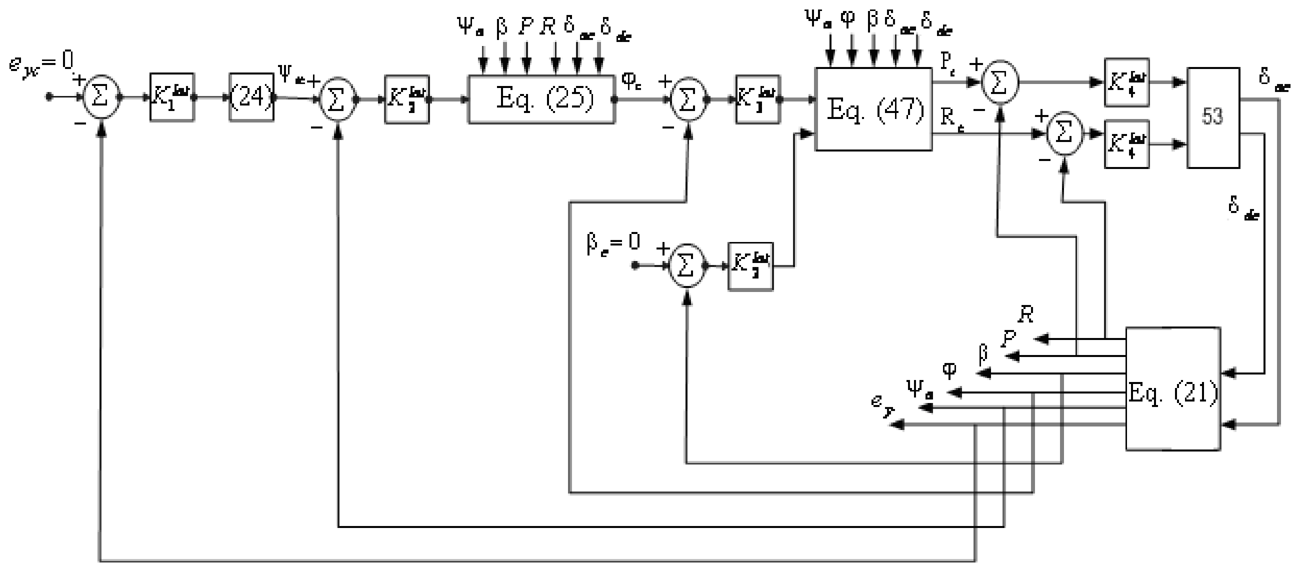

Fig.2 Block diagram of the system

$$\tilde{u}_c^{lat} = \begin{bmatrix} \delta_{ec} \\ \delta_{dc} \end{bmatrix} = \begin{bmatrix} \delta_e \\ \delta_d \end{bmatrix} - \left(\frac{\partial F_4^{lat}}{\partial \begin{bmatrix} \delta_e \\ \delta_d \end{bmatrix}}\right)^{-1}\left[F_4^{lat}\left(\xi, \delta_T, \begin{bmatrix} \delta_e \\ \delta_d \end{bmatrix}\right) - V_4^{lat}\right], \quad (33)$$

with

$$D = \begin{bmatrix} a_{51}^{lat} & a_{52}^{lat} & a_{53}^{lat} & a_{54}^{lat} & a_{55}^{lat} & a_{56}^{lat} \\ a_{61}^{lat} & a_{62}^{lat} & a_{63}^{lat} & a_{64}^{lat} & a_{65}^{lat} & a_{66}^{lat} \end{bmatrix}. \quad (34)$$

In general, feedback gains of exterior loop should be smaller than those in the inner loop. As the gains ratio between inner and outer loop is smaller, interference have less effect and stability is increased in expense of performance.

Therefore the most efficient gain ratio between inner and outer loop is approximately 0.3 to 0.4 [1]. The authors of this paper have increased this ratio to 0.5. This way they increased the stability of the aircraft and its dynamic characteristics. Thus, the loop's gains are

$$K_1^{lat} = \left(0.5^3\right) \cdot 1.3 \cdot \pi , K_2^{lat} = \left(0.5^2\right) \cdot 1.3 \cdot \pi , \quad K_3^{lat} = \left(0.5^1\right) \cdot 1.3 \cdot \pi , K_4^{lat} = \left(0.5^0\right) \cdot 1.3 \cdot \pi . \quad (35)$$

In order to apply the liniarised system obtained in the previous section, one uses an ALFLEX aircraft model presented in [1]. In fig.2 one presents the block diagram that models equations (21), (24), (25), (30) and (34), associated to the lateral movement of aircrafts.

Based on this block diagram one obtains the Matlab/ Simulink model of lateral motion (fig.3) and will obtain conclusions about the reliability and performance

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of the control method presented in this paper. The Matlab/Simulink model from fig.3 has three subsystems: *Eq. (25)*, *Eq. (30)* and *Eq. (34)*. In figures 4 - 6 one presents their Matlab/Simulink models.

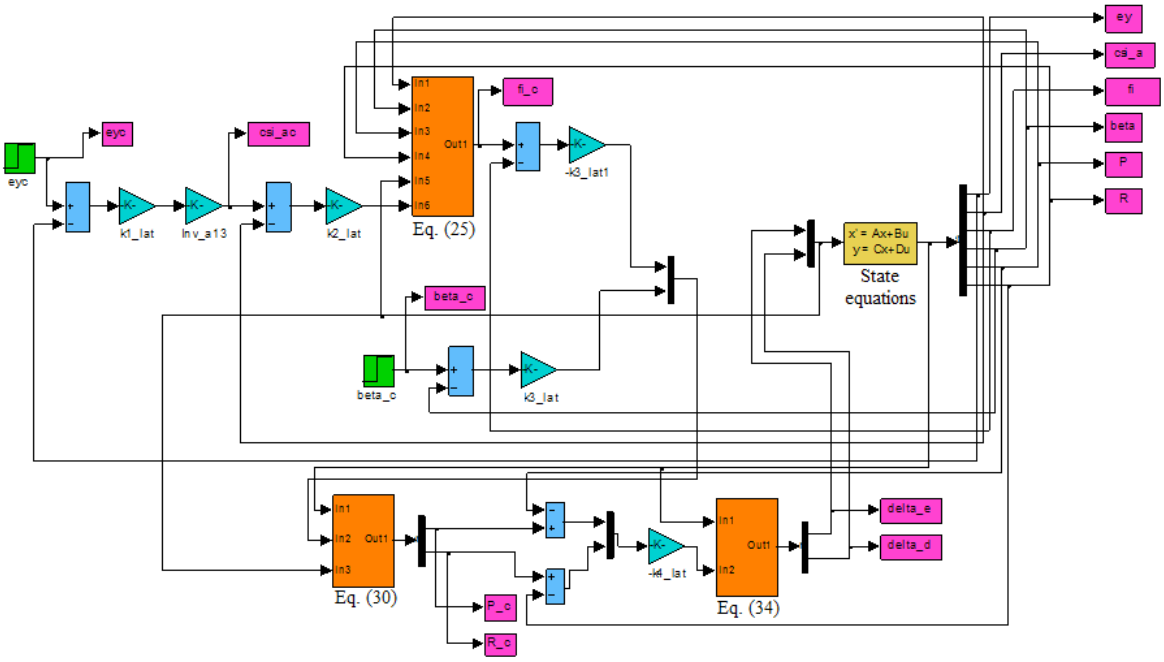

Fig.3 Matlab/Simulink model of the block diagram from fig. 2

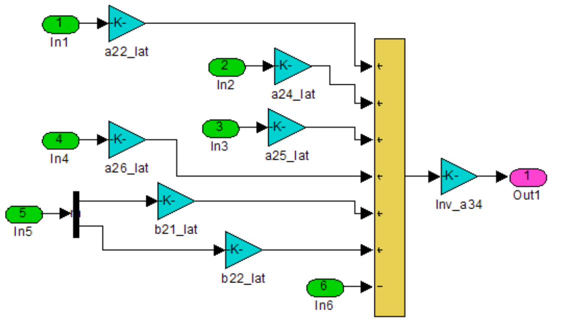

Fig.4 Matlab/Simulink model of the subsystem *Eq.(25)*

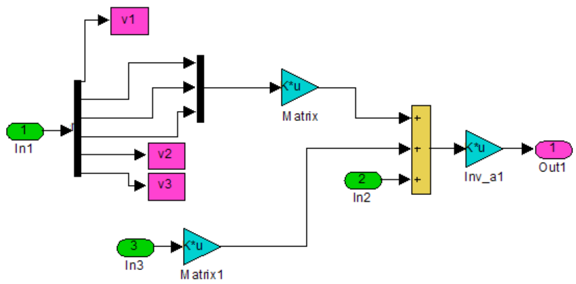

Fig.5 Matlab/Simulink model of the subsystem *Eq.(30)*

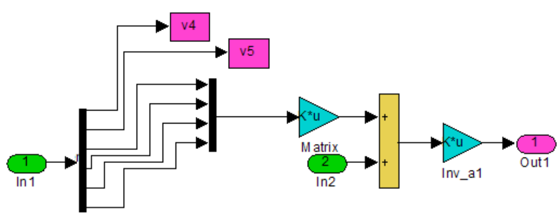

Fig.6 Matlab/Simulink model of the subsystem *Eq.(34)*

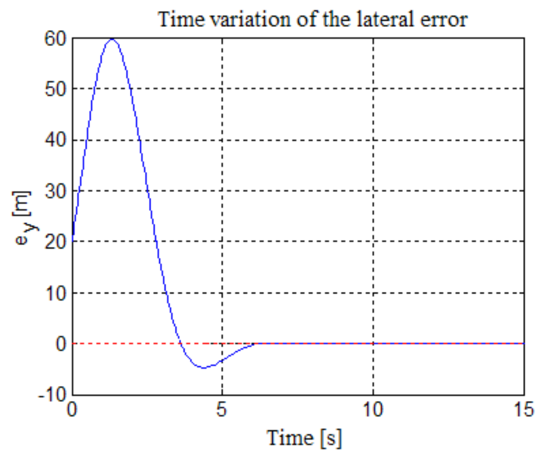

Fig.7 Time variation of the lateral error

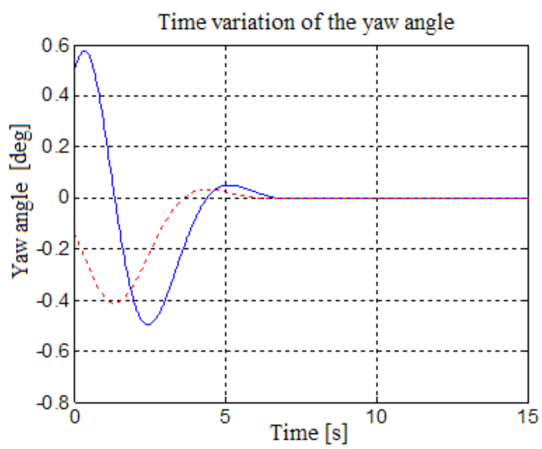

Fig.8 Time variation of the yaw angle

Next, using data for the lateral motion, one obtains graphic characteristics representing time variations of the lateral error (fig.7), yaw angle (fig.8), roll angle (fig.9), aileron deflection (fig.10) and direction deflection (fig.11). In figures 8 and 9 the command variable is represented with red dashed line while the variable is represented with blue continuous line.

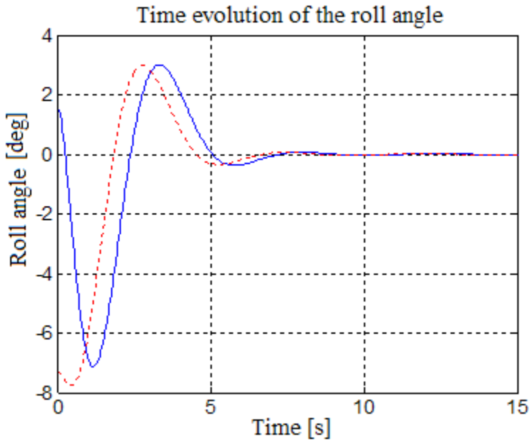

Fig.9 Time variation of the roll angle

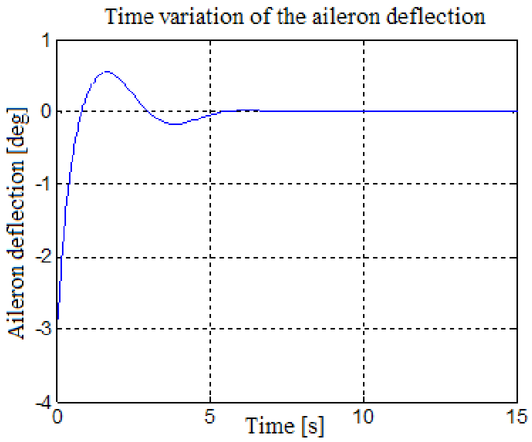

Fig.10 Time variation of the aileron deflection

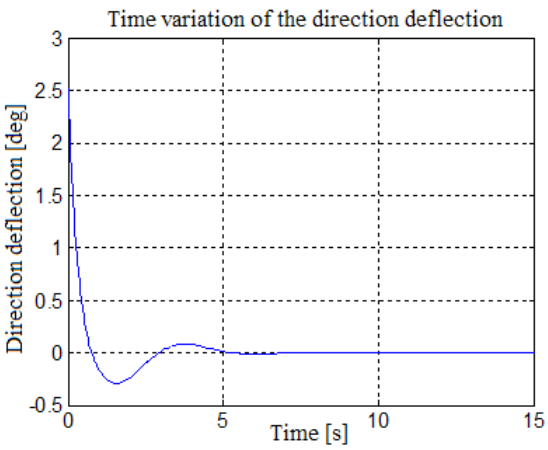

Fig.11 Time variation of the direction deflection

4 Conclusion

The paper presents a methodology for the flight control law's design for the trajectory pursuit using hierarchical dynamic inversion; this is based on separation of multi-time-scale and multi-loop closing method. The authors made the analysis of the lateral movement of aircrafts and obtained graphic characteristics which demonstrate the effectiveness of the proposed method.

The most efficient gain ratio between inner and outer loop is approximately 0.3 to 0.4 [1]. The authors of this paper have increased this ratio to 0.5, increasing the stability of the aircraft and its dynamic characteristics.

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