A Fuzzy Representation of CBR Systems

Michael Gr. Voskoglou
Professor of Mathematical Sciences
Graduate Technological Educational Institute (T.E.I.)
School of Technological Applications
263 34 Patras – Greece
e-mail: voskoglou@teipat.gr

Abstract: Case-Based Reasoning (CBR), which provides a foundation for a new technology of intelligent computer systems that can solve problems and adapt to new situations, is the process of solving new problems based on the solutions of similar past problems. In the present paper we give a fuzzy representation of a CBR system and we use the total possibilistic uncertainty as a measure of its effectiveness in solving new related problems. Examples are also given to illustrate our results.

Key words: Case-Based Reasoning, Fuzzy sets, Uncertainty, Possibility theory

1 Introduction
Case-Based Reasoning (CBR) is a general paradigm for problem-solving and learning from expertise, which is not only a psychological theory of human cognition, but it also provides a foundation for a new technology of intelligent computer systems that can solve problems and adapt to new situations.

Broadly construed CBR is the process of solving new problems based on the solutions of similar past problems. Its coupling to learning occurs as a natural by-product of problem solving. When a problem is successfully solved, the experience is retained in order to solve similar problems in future. When an attempt to solve a problem fails, the reason for the failure is identified and remembered in order to avoid the same mistake in future. Thus CBR is a cyclic and integrated process of solving a problem, learning from this experience, solving a new problem, etc. It must be noticed that the term problem-solving is used here in a wide sense, which means that it is not necessarily the finding of a concrete solution to an application problem, it may be any problem put forth by the user. For example, to justify or criticize a proposed solution, to interpret a problem situation, to generate a set of possible solutions, or generate explanations in observable data, are also problem solving situations. A lawyer, who advocates a particular outcome in a trial based on legal precedents, or an auto mechanic, who fixes an engine by recalling another car that exhibited similar symptoms, are using CBR; in other words CBR is a prominent kind of analogy making.

All inductive reasoning, where data is too scarce for statistical relevance, is inherently based on anecdotal evidence. Critics of CBR argue that it is an approach that accepts anecdotal evidence as its main operating principle, but without statistically relevant data for backing an implicit generalization, there is no guarantee that the generalization is correct. This criticism has only a theoretical base, because in practice CBR methods give satisfactory results in most cases. For special facts on CBR we refer freely to [1].

CBR traces its roots in Artificial Intelligence to the work of Roger Schank and his students at Yale University, U.S.A. in early 1980’s. Schank’s model of dynamic memory was the basis of the earliest (1983) computer intelligent systems that can be viewed as prototypes for CBR systems, the Kolodner’s CYRUS and Lebowitz’s IPP. An alternative approach is the category and exemplar model.
applied first to the PROTOS system of Porter and Bareiss (1986), while some other types of memory models, developed later on.

The CBR systems expertise is embodied in general in a collection (library) of past cases rather, than being encoded in classical rules. Each case typically contains a description of the problem plus a solution and/or the outcomes. The knowledge and reasoning process used by an expert to solve the problem is not recorded, but is implicit in the solution.

As an intelligent-systems method CBR has got a lot of attention over the last few years, because it enables the information managers to increase efficiency and reduce cost by substantially automating processes. CBR first appeared in commercial systems in the early 1990’s and since then has been sued to create numerous applications in a wide range of domains including diagnosis, help-desk, assessment, decision support, design, etc. Organizations as diverse as IBM, VISA International, Volkswagen, British Airways and NASA have already made use of CBR in applications such as customer support, quality assurance, aircraft maintenance, process planning and many more applications that are easily imaginable.

As a general problem-solving methodology intended to cover a wide range of real-world applications, CBR must face the challenge to deal with uncertain, incomplete and vague information. In fact, uncertainty is already inherent in the basic CBR hypothesis demanding that similar problems have similar solutions. Correspondingly recent years have witnessed an increased interest in formalizing parts of the CBR methodology within different frameworks of reasoning under uncertainty, and in building hybrid approaches by combining CBR with methods of uncertain and approximate reasoning.

Fuzzy sets theory can be mentioned as a particularly interesting example. In fact, even though both CBR and fuzzy systems are intended as cognitively more plausible approaches to reasoning and problem-solving, the two corresponding fields have emphasized different aspects that complement each other in a reasonable way. Thus fuzzy set-based concepts and methods can support various aspects of CBR including: Case and knowledge representation, acquisition and modeling, maintenance and management of CBR systems, case indexing and retrieval, similarity assessment and adaptation, instance-based and case-based learning, solution explanation and confidence, and representation of context. On the other way round ideas and techniques for CBR can contribute to fuzzy set-based approximate reasoning. For special facts on fuzzy sets and on uncertainty theory we refer freely to [2].

2 The steps of the CBR process
CBR has been formalized for purposes of computer and human reasoning as a four steps process. These steps involve:

R₁: Retrieve the most similar to the new problem past case.

R₂: Reuse the information and knowledge of the retrieved case for the solution of the new problem.

R₃: Revise the proposed solution.

R₄: Retain the part of this experience likely to be useful for future problem-solving.

More specifically, the retrieve task starts with the description of the new problem, and ends when a best matching previous case has been found. The subtasks of the retrieving procedure involve: Identifying a set of relevant problem descriptors, matching the case and returning a set of sufficiently similar cases given a similarity threshold of some kind, and selecting the best case from the set of cases returned. Some systems retrieve cases based largely on superficial syntactic similarities, while advanced systems use semantic similarities.

The reuse of the solution of the retrieved case in the context of the new problem focuses on two aspects: The differences between the past and the current case, and what part of the retrieved case can be transferred to the new case. Usually in non trivial situations part of the solution of the retrieved case cannot be directly transferred to the new case, but requires an adaptation process that takes into account the above differences.
Through the revision the solution generated by reuse is tested for success – e.g. by being applied to the real world environment, or to a simulation of it, or evaluated by a specialist – and repaired, if failed. When a failure is encountered, the system can then get a reminding of a previous similar failure and use the failure case in order to improve its understanding of the present failure, and correct it. The revised task can then be retained directly (if the revision process assures its correctness), or it can be evaluated and repaired again.

The final step $R_4$ involves selecting which information from the new case to retain, in what form to retain it, how to index the case for better retrieval in future for similar problems, and how to integrate the new case in the memory structure.

Notice that Slade ([6]; Figure 1), Lei et al ([4]; Figure 1), Aamodt and Plaza ([1]; Figures 1 and 2) and others have presented detailed flowcharts illustrating the basic steps of the CBR process. In an earlier paper [13] we have also presented a detailed analysis of the CBR methodology.

The general knowledge usually plays a part in the CBR cycle by supporting the CBR process. This support however may range from very weak (or none) to very strong, depending on the type of the CBR method. By general knowledge we here mean general, domain-dependent knowledge, as opposed to specific knowledge embodied by cases. For example, in the case of a lawyer, mentioned in our introduction, who advocates a particular outcome in a trial based on legal precedents, the general knowledge is expressed through the knowledge of the existing relevant laws and the correlations among them and the case of the trial. A set of rules may have the same role in other CBR cases.

3 The fuzzy model

Let us consider a CBR system whose library contains $n$ past cases, $n \geq 2$. We denote by $R_i$, $i=1,2,3$, the steps of retrieval, reuse and revision respectively, and by $a$, $b$, $c$, $d$, and $e$ the linguistic labels of negligible, low, intermediate, high and complete degree of success respectively for each of the $R_i$'s. Set $U=\{a,d,e\}$; then we are going to represent the $R_i$'s as fuzzy sets in $U$. For this, if $n_a$, $n_b$, $n_c$, $n_d$ and $n_e$ denote the number of cases where it has been achieved negligible, low, intermediate, high and complete degree of success for the state $R_i$ respectively, $i=1,2,3$, we define the membership function $m_R$ in terms of the frequencies, i.e. by $m_R(x)=\frac{n_x}{n}$, for each $x$ in $U$. Thus we can write

$$R_i = \{(x, \frac{n_x}{n}) : x \in U\}, i=1,2,3.$$

Notice that there is no need to include step $R_4$ of CBR process in our fuzzy representation, because all the past cases, either successful, or not, are retained in the system’s library and therefore there is no fuzziness in this case. In other words, keeping the same notation, we have that $n_{4a}=n_{4b}=n_{4c}=n_{4d}=0$ and $n_{4e}=1$.

In order to represent all possible profiles (overall states) of a case during the CBR process, we shall consider a fuzzy relation, say $R$, in $U^3$ of the form

$$R=\{(s, m_R(s)) : s=(x,y,z) \in U^3\}.$$

In order to determine properly the membership function $m_R$ we give the following definition: A tuple $s=(x,y,z)$, with $x,y,z$ in $U$, is said to be well ordered if $x$ corresponds to a degree of success equal or greater than $y$, and $y$ corresponds to a degree of success equal or greater than $z$; e.g. the tuple $(c,c,a)$ is well ordered, while the tuple $(b,a,c)$ is not.

We define now the membership function $m_R$ to be $m_R=s=m_{R_1}(x)m_{R_2}(y)m_{R_3}(z)$, if $s$ is a well ordered tuple, and 0 otherwise. In this way we block the possibility for profiles like $(b,a,c)$ to possess non zero membership degrees, which is absurd. In fact, if for a case the degree of success at the step of reuse is negligible, how the proposed solution could be revised?

In order to simplify our notation we shall write $m_i$ instead of $m_R(s)$. Then the possibility $r_s$ of the profile $s$ is given by $r_s=\frac{m_s}{\max\{m_i\}}$, where $\max\{m_i\}$ denotes the maximal value of $m_i$, for all $s$ in $U^3$. 
Notice that during the CBR process it might be used reasoning that involves ampliative inferences, whose content is beyond the available evidence and hence obtain conclusions not entailed in the given premises. The appearance of conflict in conclusions requires that the conclusions be appropriately adjusted so that the resulting generalization is free of conflict. The value of the total conflict during the CBR process can be measured by the strife function on the ordered possibility distribution \( r = r_1 \geq r_2 \geq \ldots \geq r_n \geq r_{n+1} \) of the profiles defined by:

\[
S(r) = \frac{1}{\log 2} \left[ \sum_{i=1}^{n} (r_i - r_{i+1}) \log \frac{i}{\sum r_j} \right].
\]

In general, the amount of information obtained by an action can be measured by the reduction of uncertainty that results from the action. Thus the total possibilistic uncertainty \( T(r) \) during the CBR process can be used as a measure for the system’s effectiveness in solving new related problems. The value of \( T(r) \) is measured by the sum of the strife \( S(r) \) and nonspecificity \( N(r) \) ([3] ; p.28), defined by:

\[
N(r) = \frac{1}{\log 2} \left[ \sum_{i=2}^{n} (r_i - r_{i+1}) \log i \right]
\]

In contrast to strife, which, as we have already seen, expresses conflicts among the various sets of alternatives, nonspecificity is connected with the sizes (cardinalities) of relevant sets of alternatives. The lower is the value of \( T(r) \), the higher is the effectiveness of the CBR system.

Assume now that one wants to study the combined results of the behaviour of \( k \) different systems, \( k \geq 2 \), designed for the solution of the same type of problems via the CBR process. Then it becomes necessary to introduce the fuzzy variables \( R_i(s) \), with \( i=1,2,3 \) and \( t=1,2,\ldots,k \), and determine the possibilities \( r(s) \) of the profiles \( s(t) \) through the pseudofrequencies \( f(s) = \sum_{i=1}^{k} m_{ijs} \).

Namely, \( r(s) = \frac{f(s)}{\max \{ f(s) \}} \), where \( \max \{ f(s) \} \) denotes the maximal pseudofrequency. The possibilities \( r(s) \) of all the profiles \( s(t) \) measure the degree of evidence of the combined results of the \( k \) CBR systems.

### 4 An application of the fuzzy model

We consider a CBR system with an existing library of 105 past cases, where in no case there was a failure at the step of retrieval of a past case for the solution of the corresponding problem. In fact, in 51 cases we had a complete success in retrieving a suitable past case, in 24 cases high, and in 30 cases we had a complete success respectively.

Thus the state of retrieval is represented as a fuzzy set in \( U \) as

\[
R_1 = \{(a,0),(b,0),(c,\frac{51}{105}),(d,\frac{24}{105}),(e,\frac{30}{105})\}.
\]

In the same way we obtain that

\[
R_2 = \{(a,\frac{18}{105}),(b,\frac{18}{105}),(c,\frac{48}{105}),(d,\frac{21}{105}),(e,0)\},
\]

and

\[
R_3 = \{(a,\frac{36}{105}),(b,\frac{30}{105}),(c,\frac{19}{105}),(d,0),(e,0)\}.
\]

It is a straightforward process now to calculate the membership degrees of all the possible profiles (see column of \( m_i(1) \) in Table 1). For example, if \( s=(c,b,a) \), then

\[
m_r(c) = \sum_r m_{rj}(c) m_{rj}(b) m_{rj}(a) = \frac{51}{105} \times \frac{18}{105} \approx 0.029.
\]

It turns out that \((c,c,c)\) is the profile with the maximal membership degree 0.082 and therefore the possibility of each \( s \) in \( U^2 \) is given by \( r_s = \frac{m_s}{0.082} \). For example the possibility of \((c,b,a)\) is \( \frac{0.029}{0.082} \approx 0.353 \), while the possibility of \((c,c,c)\) is of course equal to 1.

Calculating the possibilities of the \( 5^3=125 \) in total profiles (see column of \( r_i(1) \) in Table 1) one finds that the ordered possibility distribution \( r \) of the profiles is: \( r_1 = 1, r_2 = 0.927, r_3 = 0.768, r_4 = 0.512, r_5 = 0.476, r_7 = 0.415, r_8 = 0.402, r_9 = 0.378, r_10 = 0.341, r_11 = 0.329, r_12 = 0.317, r_13 = 0.305, r_14 = 0.293, r_15 = 0.256, r_17 = 0.207, r_18 = 0.195, r_19 = 0.171, r_20 = r_21 = r_22 = 0.159, r_23 = 0.134, r_24 = r_25 = \ldots = r_{125} = 0 \). Therefore the total possibilistic uncertainty is

\[
T(r) = S(r) + N(r) = 0.565 + 2.405 = 2.97.
\]

Next we want to study the combined results of the behaviour of the above system and of another one designed for the solution of the same type of problems via the CBR process, with an existing library of 90 past cases.
Working as before we find for the second system that
\[ R_1 = \{(a, 0,0), (b, 45, 90), (c, 45, 45), (d, 90), (e, 0)\}, \]
\[ R_2 = \{(a, 18, 90), (b, 90), (c, 90, 45), (d, 0), (e, 0)\}, \]
\[ R_3 = \{(a, 36, 90), (b, 27, 90), (c, 27, 60), (d, 0), (e, 0)\}. \]

The calculation of all possible profiles gives the results shown in column of \( m_s(2) \) in Table 1. It turns out that (c,c,a) is the profile possessing the maximal membership degree 0,107 and therefore the possibility of each s is given by \( r_s = \frac{m_s}{0.107} \) (see column of \( r_s \) in Table 1). Finally, in the same way as above, one finds that \( T(r) = S(r) + N(r) = 0.452 + 1.87 = 2.322 \). Thus, since 2.322<2.97, the effectiveness of the second system in solving new related problems is better than that of the first one. This happens despite the fact that the profile (c,c,c) with the maximal possibility of appearance in the first system is a more satisfactory profile than the corresponding profile (c,c,a) of the second system.

We introduce now the fuzzy variables \( R_i(t) \), \( i=1,2,3 \) and \( t=1,2 \). Then the pseudo-frequency of each profile \( s \) is given by \( f(s) = m_s(1) + m_s(2) \) (see the corresponding column of Table 1). It turns out that (c,c,a) is the profile with the highest pseudo-frequency 0,183 and therefore the possibility of each profile is given by \( r(s) = \frac{f(s)}{0.183} \). The possibilities of all profiles having nonzero pseudo-frequencies are given in the last column of Table 1.

<table>
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Note: The outcomes of Table 1 are calculated with accuracy up to the third decimal point.

5. Conclusions and remarks

The following conclusions can be drawn from the discussion presented in the paper:

- Although both CBR and fuzzy systems are intended as cognitively more plausible approaches to reasoning and problem-solving, the two corresponding fields have emphasized different aspects that complement each other in a reasonable way.

- Our fuzzy representation of a CBR system is based on the formalization of CBR as a four steps process (retrieve, reuse, revise, retain).

- Our fuzzy model is not restricted only to quantitative information (possibilities, value of \( T(r) \), etc), but it also gives a qualitative view of the behaviour of a CBR system. In fact, through it one studies all the possible profiles of the stored cases, and gets – in terms of the linguistic labels – a comprehensive idea about the degree of success of each step of the CBR process.

- Another advantage of our model is that it gives the possibility to study the combined results of behaviour of two, or more, CBR systems designed for the solution of the same type of problems.
An analogous to the above model has been constructed by the author for a fuzzy representation of the process of learning a subject matter by a group of students in the classroom [14]. Analogous efforts, but with different methodologies, to use the fuzzy sets logic in the area of student modelling and student diagnosis in particular and in education in general have been attempted by other researchers as well ([5], [7], etc).

Another important approach towards the same direction is the use of stochastic methods (Markov chain models), for example see [8],[9], [10], [11], [12] etc. An attempt by the author to introduce a Markov model for the description of the CBR process is being now in progress, and it is hoped to be published in the near future.

References