Forced convection heat transfer in the transition region between laminar and turbulent flow for a vertical circular tube

D. Huber and H. Walter

***Abstract*—**In this study, first results of the heat transfer characteristic of a vertical double tube heat exchanger were determined. The heat exchanger was operated under co-current-flow conditions. The Reynolds-number was varied in the transition region between laminar and turbulent of a single phase flow. A detailed description of the test rig and the data reduction procedure is given. A throughout uncertainty analysis was performed to validate the experimental data. The experimental data are compared with three correlations from open literature, namely with Petukhov [6], Gnielinski [2] and Churchill [12]. Finally, a linear regression of the experimental data is compared with the three given. The equation of Gnielinski and Churchill shows a good accordance with the measured data.

***Keywords*—**Forced convection, Heat transfer, Transition region, circular tube

I. Introduction

In many industrial applications heat exchangers are designed in such a way that the gas which should be cooled down flows inside a tube and the mass flow of the cooling medium flows around the tube e.g. smoke tube boilers or stoves. In some of these facilities the velocity of the gas inside the tube is small, so that the flow falls into the transition region between turbulent and laminar flow.

In the open literature, e.g. [1]-[4], it is generally stated that the Reynolds-number Re must be > 10^4 for a full developed turbulent fluid flow in a pipe. In the region between 2300 < Re < 10^4 the transition from turbulent to laminar flow takes place. As mentioned in [2] and [5] the heat transfer in the transition region is affected by the history of the of the fluid, e.g. the type of inlet configuration or a unheated entrance region, which is used for the fluid dynamic development of the fluid flow.

Ghajar and Tam [5] have made measurements for local forced and mixed convective heat transfer in a horizontal circular straight tube with three different inlet configurations, namely reentrant, square-edged, and bell-mouth inlets to determine the influence of the entrance region to the heat transfer. The tube was heated electrically. The measurement range of Re was 280 up to 49000, the Prandtl-number Pr was varied from 4 to 158 and the Grashof-number was ranged between 1000 and $2.5 \cdot 10^5$. They have recommended correlations for the turbulent, laminar and the transition regions for each of the three inlets.

It is well known and described in the literature, e.g. [1], [2], [5] or [7], that the heat transfer in the transition region drops down compared to that of the full developed turbulent fluid flow. Therefore it is necessary to have an heat transfer correlation which include this phenomenon. Based on a suggestion of [8] the factor (Re - 1000) was included from Gnielinski [1] in his Nusselt-correlation to reproduce the measured data in the transition region with his equation for a full developed turbulent fluid flow in a pipe.

In a following work [2] Gnielinski has changed the calculation method in the transition region because the values for the Nusselt-number Nu at the validity boundary of the equations for laminar and transition heat transfer be different with a decreasing d_i/L ratio (tube length L and inner tube diameter d_i). To compensate this effect a new equation for the Nusselt-number for the transition region are used from [2]. This equation includes an interpolation function between the correlations for the heat transfer of turbulent and laminar fluid flow. This interpolation function was suggested the first time by [9].

In the present study the heat transfer characteristics for a single phase fluid flow in a vertical tube were investigated. All measurements are done in the transition region between 4000 < Re < 10000. The Prandtl-number of the fluid was 0.71 and the air in the circular tube was cooled down. First results will be presented from this ongoing study.

II. Experimental Apparatus

The experimental test rig consists of a double tube heat exchanger (tube length 3000 mm) where heated air flows in the inner pipe with the dimension of 32.8 mm and is cooled by

Manuscript received June 30, 2010. This work was supported within the framework of the program ENERGIE DER ZUKUNFT. This program will be transacted on behalf of the Federal Ministry for Transport, Innovation and Technology and the Federal Ministry of Economy and Work by the Austrian Research Promotion Agency (Project number FFG 815699).

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water, flowing in the annulus between the inner and the outer pipe with the dimension of 3.9 mm (see Fig. 1). A fan sucks air at ambient condition into the test rig through an orifice plate which is arranged 2400 mm downstream of the fan. The orifice plate is used to determine the airflow rate according to VDI 2041 [14]. The fan is driven by a voltage controller, which allows setting an air mass flow rate between 0 and 0.015 kg/s. After the orifice plate the air is passing through an electric heater where the air is heated up to a temperature of approx. 100 °C. The air temperature is controlled via the electric power of the heater by using the thyristor controller ESGT-3PH/SP60. The heated air enters downstream of the electrical heater the vertical arranged double tube heat exchanger of the test rig. During the experiments the dimensions of the double tube heat exchanger are fixed.

The water cycle consists of a water tank, an electric flow heater with a maximum power of 28 kW, a pump and a thermostat mixer. Water is pumped from the tank through the electric heater, where the water is heated up to a exit temperature of approx. 60 °C. The hot water, which leaves the heater, is mixed in the thermostat mixer with cold city water. The water temperature at the outlet of the thermostat mixer is adjusted directly at the thermostat. After the thermostat mixer the water passes the cooling part of the heat exchanger and returns to the tank. The water mass flow is controlled with a valve arranged behind the thermostat mixer.

The double tube heat exchanger is isolated against the surrounding area. The heat exchanger is operated under co-current-flow conditions.

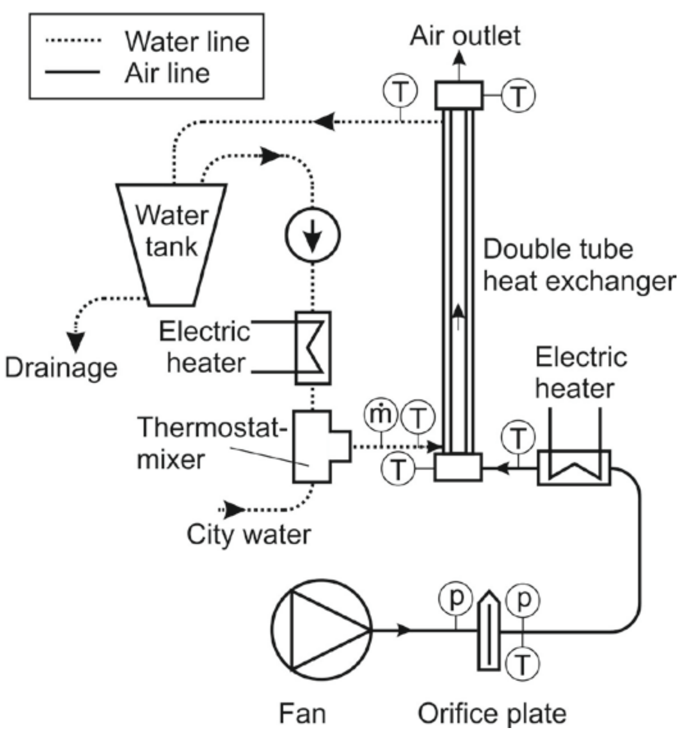

Fig. 1 schematic diagram of experimental apparatus

III. Procedure

The temperatures of air and water are measured simultaneously using calibrated Pt-100 RTD's (resistance temperature detector) which were placed near the inlet respectively the outlet of the double tube heat exchanger (see Fig. 1). After the electric air heater and the orifice plate, the air temperature was measured with a FeCu-Ni (J-type) thermocouple. The mass flow of cooling water was measured with a calibrated paddle wheel flow meter type DPL-1-P25 and the air mass flow was measured with a orifice plate according to DIN-EN-ISO 5167-1 [13] in connection with VDI/VDE 2041 [14]. The differential pressure between the inlet and outlet of the orifice plate as well as to the ambient pressure were measured with Honeywell micro-switch differential pressure sensors of the type 163PC01D75. The barometric pressure is measured using a digital barometer of the type GTD 1100. For the calculation of the air density, the absolute pressure was calculated with the help of the differential pressure and the ambient pressure. The uncertainties of the measuring sensors are summarized in table I.

TABLE I
Uncertainties of measuring sensors

Sensor	Uncertainty
Pt100 RTD	0.03 K
Type J thermocouple	3 K
Differential pressure sensor	1%
Paddle wheel mass flow sensor	2.5 %
Barometer	1.5 mbar

Data acquisition was done with the modular data acquisition system NI-cDAQ-9172 from National Instruments which consists of a chassis for different input modules. The input modules used in the test rig are:

- NI-9205 analog input module for the voltage signals of the pressure sensors and the mass flow sensor,
- NI-9217 analog input module for Pt-100 RTD’s and
- NI-9211 analog input module for the thermocouples.

The supply voltage of all analog pressure transducers was supplied using two constant-voltage sources (K.E.R.T Mod. AT 4 VD), with either 8V or 24V.

The measured values are transmitted to the process computer using the measurement value periphery by National Instruments and the LabView 8.5 program system.

The measurements were performed for different Reynolds numbers as follows: An air mass flow, corresponding to a desired Reynolds number, was applied to the test rig, and the cooling water temperature was adjusted. After achieving steady state conditions, the measuring was started. Every approx. 20 s a complete loop was performed by the generated program. This contains the time for the sampling rate of the signals and a time delay. For each measuring point, 100 samples were taken. The accuracy of each measuring point was calculated according to DIN 1319 [15] and [16].

IV. Data reduction

To determine the overall heat balance of the double tube heat exchanger the measured air and water side temperatures

and mass flows are used. With the help of the first law of thermodynamics for stationary flow processes (the kinetic and potential energy of the fluid flow was neglected) the exchanged heat flow $\dot{Q}$ between both fluids must be equal and can be written in the following form:
for the air side

$$\dot{Q} = \dot{m}_{air}\, c_{p,air}\, (T_{air,out} - T_{air,in}) \tag{1}$$

and for the water side

$$\dot{Q} = \dot{m}_{water}\, c_{p,water}\, (T_{water,in} - T_{water,out}) \tag{2}$$

with the mass flow for the air $\dot{m}_{air}$ and the water $\dot{m}_{waterr}$, the spec. isobaric heat capacity for air $c_{p,air}$ and water $c_{p,water}$, the heat exchanger outlet temperature for air $T_{air,out}$ and water $T_{water,out}$ and the heat exchanger entrance temperature of air $T_{air,in}$ and water $T_{water,in}$. The spec. isobaric heat capacities are determined at an average temperature between the inlet and the outlet temperature of the heat exchanger.

The low heat exchange rate between air and water results in a very low water temperature difference between the heat exchanger inlet and outlet. Therefore the uncertainty for the transferred heat calculated according to DIN 1319 is very high, as shown in table II. Thus (2) was neglected, and only the results of (1) are used for the following calculations.

TABLE II
UNCERTAINTIES OF CALCULATED VALUES

Value	Uncertainty range (%)	average Uncertainty
Q_{air}	1.53 – 2.02	1.73
Q_{water}	80.01 – 139.68	94.78
α	1.47 – 2.26	1.8

For the overall analysis it was assumed that the heat exchanger is adiabatic to the surrounding area. For the heat flow through the wall

$$\dot{Q} = 2\pi\, d_a\, L\, k\, \Delta\vartheta_{\log} \tag{3}$$

was used in combination with the heat conduction through the wall

$$k = \frac{1}{\dfrac{1}{\alpha_a} + \dfrac{d_a}{2\lambda}\ln\left(\dfrac{d_a}{d_i}\right) + \dfrac{d_a}{d_i}\dfrac{1}{\alpha_i}} \tag{4}$$

and the logarithmic mean temperature difference for a co-current-flow heat exchanger

$$\Delta\vartheta_{\log} = \frac{\left(T_{air,in} - T_{water,in}\right) - \left(T_{air,out} - T_{water,out}\right)}{\ln\left(\dfrac{T_{air,in} - T_{water,in}}{T_{air,out} - T_{water,out}}\right)}. \tag{5}$$

α_a indicates the heat transfer coefficient between tube wall and water, α_i the heat transfer coefficient between the tube wall and air, d_a the outer tube diameter of the inner tube and λ the heat conduction of the tube material.

For the calculation of the heat transfer coefficient value α_i for every measuring point, the arithmetic mean of all 100 samples of each measuring point is used. All measuring values are acquired at the same time, thus a correlation between the different signals has to be considered while calculating the uncertainties. The Nusselt-number was calculated with the heat transfer coefficient and the properties of matter of dry air as shown in [17]. The thermal conductivity for the tube material was considered as constant because of the low temperature differences along the tube.

V. Measured data

Fig. 2 shows the measured data and a curve for the linear regression of the data (full line). The measured range of the Reynolds-number is between about 4095 and about 10400. For this Reynolds-numbers, the Nusselt-number varies between about approx. 11 and 33. The full line shows the linear regression of the data with an area of +/- 10% overall relative uncertainty represented by the dashed lines.

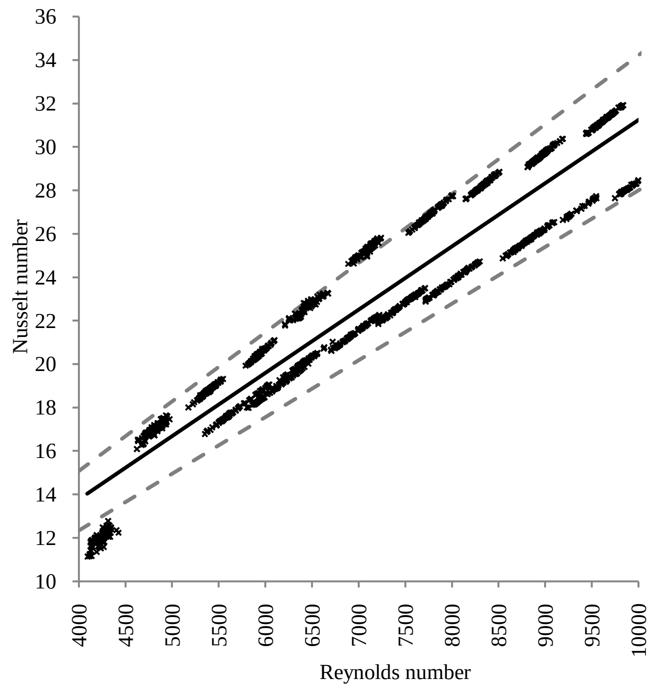

Fig. 2 measured Nusselt-numbers (cross), linear regression (full line), and +/- 10% overall relative uncertainty of the regression (dashed line)

It can be seen in Fig. 2 that 90.6% of all the measured data are within the lines with +/- 10% overall relative uncertainty.

VI. Comparison with literature

In the present study the measured data are compared with three different Nu-correlations which are described below in this paragraph.

For the comparison the semi-empirical equation of Petukhov [6] for the heat transfer of a turbulent fluid flow

$$Nu = \frac{(\xi/8)\mathrm{Re}\,\mathrm{Pr}}{1.07 + 12.7\sqrt{\xi/8}\left(\mathrm{Pr}^{2/3} - 1\right)} \tag{6}$$

with the pressure loss coefficient according to [10]

$$\xi = \left[1.82\log_{10}(\mathrm{Re}) - 1.64\right]^{-2}, \tag{7}$$

which is regarded by many authors as being most accurate, was used, because (6) was also used for some other authors e.g. [1], [2] as basis for their Nu-correlation. The definition range of (6) is $4 \cdot 10^3 \le \mathrm{Re} \le 5 \cdot 10^5$ and $0.7 \le \mathrm{Pr} \le 60$. This definition range includes also the analyzed transition region for single phase fluid flow.

The equation of Petukhov [6] was used from [2] as basis for the development of his Nu-correlation (8).

$$Nu_t = \frac{(\xi/8)\mathrm{Re}\,\mathrm{Pr}}{1+12.7\sqrt{\xi/8}\left(\mathrm{Pr}^{2/3}-1\right)}\left\{1+\left(\frac{d_i}{L}\right)^{2/3}\right\} \tag{8}$$

with the equation of Konakov [11] for the pressure loss coefficient.

$$\xi = \left(1.8\log_{10}(\mathrm{Re})-1.5\right)^{-2} \tag{9}$$

Equation (8) is expanded with an element to describe the dependency of the heat transfer from the tube length according to [8] and is valid in the range of $10^4 \le \mathrm{Re} \le 10^6$ and $0.1 \le \mathrm{Pr} \le 1000$.

For the transition region of $2300 < \mathrm{Re} < 10^4$ the following interpolation function should be used according to [2].

$$Nu_m = (1-\gamma)Nu_{L,2300} + \gamma\, Nu_{t,10000} \tag{10}$$

with

$$\gamma = \frac{\mathrm{Re}-2300}{10^4-2300} \text{ and } 0 \le \gamma \le 1 \tag{11}$$

$Nu_{t,10000}$ is calculated from (8) with a Re-number of 10^4. For the Nu-number of the laminar tube flow $Nu_{L,2300}$, the following equations, which are valid for a constant wall temperature, are used.

$$Nu_{L,2300} = \left[49.371 + \left(Nu_{L,1,2300}-0.7\right)^3 + \left(Nu_{L,2,2300}\right)^3\right]^{1/3} \tag{12}$$

with

$$Nu_{L,1,2300} = 1.615\left(2300\,\mathrm{Pr}\,d_i/L\right)^{1/3} \tag{13}$$

and

$$Nu_{L,2,2300} = \left(\frac{2}{1+22\,\mathrm{Pr}}\right)^{1/6}\left(2300\,\mathrm{Pr}\,d_i/L\right)^{1/2} \tag{14}$$

According to [2] the Nu-number should be corrected by multiplying of Nu_m with a factor K^n which takes into account the influence of temperature dependency of the fluid properties. In case of a cooled gas the exponent n should be set to zero. Therefore no correction of Nu_m is necessary in our case.

Churchill [12] has developed a single correlating equation for the mean Nusselt-number for all Reynolds and Prandtl-numbers and for either uniform wall temperature or a uniform heat flux density. The shift from laminar to transitional convection in the equation from Churchill is at Re = 2100. The overall Nu-equation of [12] reduces to

$$Nu = \left(\frac{1}{Nu_t^2}+\frac{1}{Nu_{tr}^2}\right)^{-1/2} \tag{15}$$

for the turbulent and transition region. Nu_t indicates the Nusselt-number for turbulent flow and Nu_{tr} for the transition region. The applicability of the equation is limited to fully developed flow in smooth tubes.

The proposed Nusselt-number by [12] for the transition region is given with

$$Nu_{tr} = Nu_l \exp\left(\frac{\mathrm{Re}-2200}{730}\right). \tag{16}$$

For a uniform wall temperature the Nusselt-number for the laminar flow regime Nu_l attains to the constant value of 3.657.

The Nusselt-number for the turbulent flow regime and a constant wall temperature can be calculated with

$$Nu_t = 5.67 + \left(\frac{0.079\,\mathrm{Re}\sqrt{\xi}\,\mathrm{Pr}}{\left(1+\mathrm{Pr}^{4/5}\right)^{5/6}}\right). \tag{17}$$

The pressure loss coefficient for all Reynolds-numbers is given by Churchill with

$$\frac{1}{\xi} = \left[\frac{1}{\sqrt{\left(\frac{8}{\mathrm{Re}}\right)^{10}+\left(\frac{\mathrm{Re}}{36500}\right)^{20}}}+\left(2.21\ln\frac{\mathrm{Re}}{7}\right)^{10}\right]^{1/5}. \tag{18}$$

In Fig. 3 a comparison of the experimental data with the correlations from the open literature described above is presented.

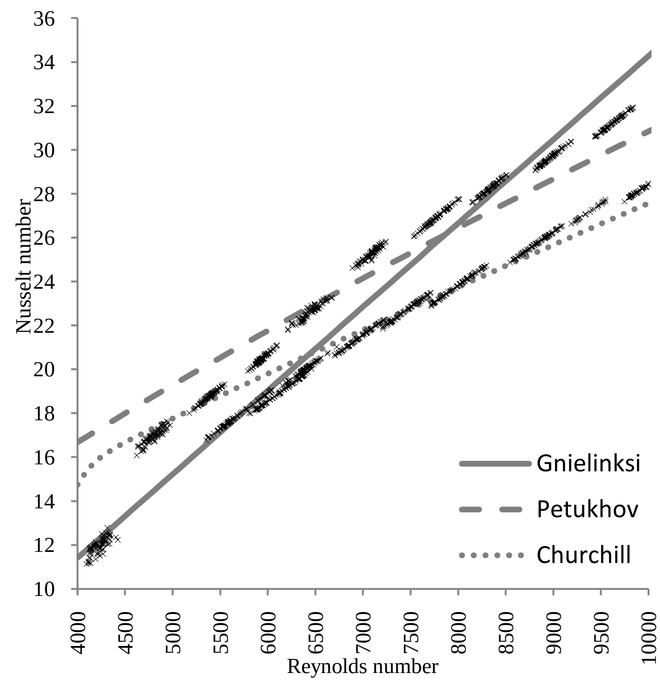

Fig. 3 comparison of the correlations of Gnielinski (full line), Petukhov (dashed line) and Churchill (dotted line) with the experimental data

It can be seen that the Nusselt-number varies depending on the correlation used for its calculation. Furthermore it can be seen also, that all measured values lie in a region that is covered by one of those three curves. The correlations of Petukhov [6] and Churchill [12] have a lower gradient in the transition region compared to the equation of Gnielinski [2]. This is a result of the d_i/L ratio in the equation of [2] for the laminar tube flow, which is included in the correlation for the transition region. With increasing of this geometrical ratio the gradient decreases.

The maximum deviations from the linear regression curve of the measured data (see Fig. 2) are as follows:

- correlation of Petukhov [6]: maximum deviation =

17.75% at Re = 4000,

- correlation of Gnielinski [2]: maximum deviation = 20.19% at Re = 4000,
- correlation of Churchill [12]: maximum deviation = 11.4% at Re = 10000.

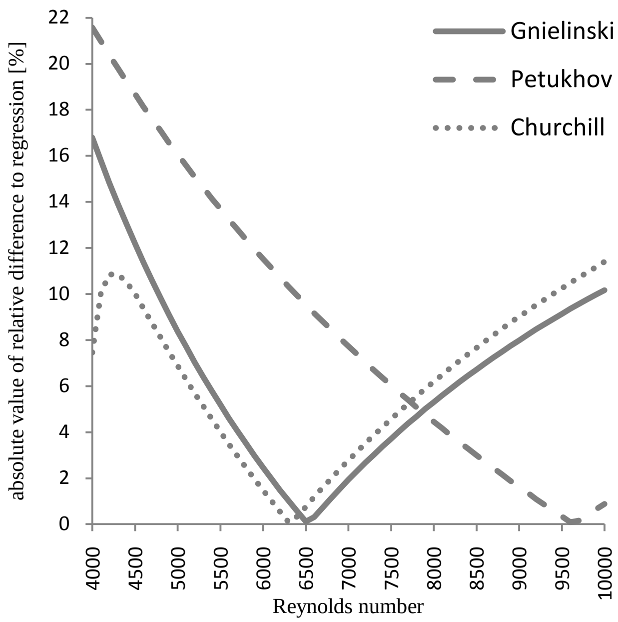

Fig. 4 comparison of the difference of the linear regression of the experimental data with the correlations of Gnielinski (full line), Petukhov (dashed line) and Churchill (dotted line)

In Fig. 4 a comparison of the absolute values of the relative difference between the linear regression of the experimental data and the values calculated with the correlation of Gnielinski [2], Petukhov [6] and Churchill [12] are presented. The relative difference between the linear regression of the experimental data and the values calculated with the correlations (6), (10) and (15) are calculated as (experimental data - data from correlation) / experimental data.

The correlation of Gnielinski and Churchill represent 85.2% respectively 80.3% of the linear regression curve of the measured data within a relative difference of +-10%. At low Reynolds-numbers the equation of Petukhov shows a relative high difference to the measured data.

VII. Conclusion

In the present study first results of the heat transfer characteristic of a vertical double tube heat exchanger were determined. The heat exchanger was operated under co-current-flow conditions. The Reynolds-number was parametrically varied in the transition region between laminar and turbulent flow of a single phase flow. The experimental data are compared with three correlations from open literature, namely the correlation of Petukhov [6], Gnielinski [2] and Churchill [12]. The equation of Gnielinski and Churchill shows a good accordance with the measured data within a relative difference of +-10% to the linear regression curve of the measured data. At low Reynolds-numbers the equation of Petukhov shows a relative high difference to the measured data.

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