Performance of RANS turbulence models in predicting strained flows in a curved duct

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Abstract— In this paper, we assess the performance of four RANS turbulence models in computing the averaged flow field through a curved duct. The Models studied are the standard k-ε, the standard k-ω, The Reynolds Stress Model, and the $v^2 f$ model. Although the four models showed similar quantitative agreement for the tangential velocity profile with experimental measurements, it was only the $v^2 f$ model that correctly predicted the expected secondary flow pattern. Moreover, the V^2f model was the only model to capture the zero turbulence kinetic energy areas in which laminarization of the flow occur.

Keywords— *CFD, Turbulence modeling, RANS, Curved duct, Secondary flow, $v^2 f$ model.*

I. Introduction

Turbulent flow in curved ducts is a fundamental problem in fluid dynamics that affects many engineering applications such as cooling ducts, roto-dynamic machines, and aircraft engines intakes. The accurate numerical prediction of such flows is challenging due to the occurrence of secondary flow that is highly dependent on the turbulent field. Such fundamental problem has received a strong attention from the fluid dynamics community that can be revealed from previous experimental [1-5] and numerical [6-11] studies.

Several computational aspects of the problem in hand have been studied previously. Besserman and Tanrikut [12], and Xia and Taylor [13] emphasized the necessity of employing a near wall treatment that takes into account the low Reynolds number effects and the inadequacy of using wall functions. Bo et al. [14] demonstrated that it is essential to use high order discretization schemes for the convection of mean flow variables as well as the turbulent field. In the present study, the authors evaluate the performance of four RANS turbulence models in predicting the flow inside an 1800 curved duct. The performance of the turbulence models is assessed through qualitative analysis of both the velocity and turbulence fields. Moreover, the tangential velocity profile predicted by the different models is compared with experimentally measured data obtained by [15].

RANS models are by far the most widely used turbulence models especially in application oriented simulations. This is due their relatively low computational requirements comparing to deterministic and semi-deterministic approaches. However, this relative simplicity comes at the expenses of various physical assumptions in the models' formulation that may compromise the solution accuracy. Among the most severe assumptions in RANS models is the turbulence isotropy approximation. The inadequacy of such assumption is more evident in flow problems that exhibit highly anisotropic effects such as streamline curvature, high strain rates, and secondary flows. Since the problem in hand is one such case, and since RANS models are the most widely used models for this type of flows considering their simplicity, an evaluation of the performance of such models is of great significance.

II. Formulation of the turbulence models

A. *The Standard k-ε Model*

The standard k-ε model, developed by Launder and Spalding, [16, 17] is a two equation eddy viscosity turbulence model. In this model, the eddy viscosity is computed based on the turbulence kinetic energy k, and the turbulence dissipation rate ε via:Final Stage

$$\nu_t = C_\mu \frac{k^2}{\varepsilon} \tag{1}$$

Each of these two turbulence scales has its transport equation. The turbulence kinetic energy equation k is derived from the exact momentum equation by taking the trace of the Reynolds stress. This equation can be expressed as:

$$\frac{\partial k}{\partial t} + \overline{u}_i \frac{\partial k}{\partial x_i} = \nu_t \left[\frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i} \right] \frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial}{\partial x_i} \left(\frac{\nu_t}{\sigma_k} \frac{\partial k}{\partial x_i} \right) - \varepsilon \tag{2}$$

The dissipation rate equation, on the other hand, is obtained using physical reasoning. The equation is:

$$\frac{\partial \varepsilon}{\partial t} + \overline{u}_i \frac{\partial \varepsilon}{\partial x_i} = C_{\varepsilon 1} \frac{\varepsilon}{k} \nu_t \left[\frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial \overline{u_j}}{\partial x_i} \right] \frac{\partial \overline{u_i}}{\partial x_j} + \frac{\partial}{\partial x_i} \left(\frac{\nu_t}{\sigma_\varepsilon} \frac{\partial \varepsilon}{\partial x_i} \right) - C_{\varepsilon 2} \frac{\varepsilon^2}{k} \tag{3}$$

The standard k-ε has five empirical constants C_μ, σ_k, σ_ε, $C_{\varepsilon 1}$, $C_{\varepsilon 2}$ with values of 0.09, 1.0, 1.3, 1.44, 1.92,

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respectively. These values were obtained using experiments and computer optimization. It is worthy noting that these values are not universal and the k-ε model requires some amount of fine tuning in order to obtain correct results.

B. *The Standard k-ω Model*

The standard k-ω model [18] is an empirical model based on model transport equations for the turbulence kinetic energy (k) and the specific dissipation rate (ω), which can also be thought of as the ratio of ε to k.

The transport equation for the turbulence dissipation rate (ω) is:

$$\frac{\partial \omega}{\partial t}+\overline{u_i}\frac{\partial \omega}{\partial x_i}=2\alpha S_{ij}S_{ij}-\beta\omega^2+\frac{\partial}{\partial x_i}\left|\left(\nu+\frac{\nu_t}{\sigma_\omega}\right)\frac{\partial \omega}{\partial x_i}\right| \quad (4)$$

And the eddy viscosity relation

$$\nu_t=\frac{k}{\omega} \quad (5)$$

The model constants are:

C_μ =0.09, σ_k =2.0 , σ_ω =2.0, α =0.56 , β =0.075.

The Wilcox k-ω model [18] incorporates modifications for low-Reynolds-number effects, compressibility, and shear flow spreading. It also predicts free shear flow spreading rates that are in close agreement with measurements for far wakes, mixing layers, and plane, round, and radial jets, and is thus applicable to wall-bounded flows and free shear flows.

C. *Reynolds Stress Model*

The Reynolds stress model [19, 20], involves calculation of the individual Reynolds stresses $\overline{u_i'u_j'}$, using differential transport equations. The individual Reynolds stresses are then used to obtain closure of the Reynolds-averaged momentum equation, thus, avoiding the use of the eddy viscosity approximation. The exact form of the Reynolds stress transport equations may be derived by taking moments of the exact momentum equation. This is a process wherein the exact momentum equations are multiplied by a fluctuating property, the product then being Reynolds-averaged. Unfortunately, several of the terms in the exact equation are unknown and modeling assumptions are required in order to close the equations.

The exact transport equations for the transport of the Reynolds stresses, $\overline{u_i'u_j'}$ is:

$$\frac{\partial}{\partial t}\left(\rho\overline{u_i'u_j'}\right)+\frac{\partial}{\partial x_k}\left(\rho u_k\overline{u_i'u_j'}\right)=-\frac{\partial}{\partial x_k}\left[\rho\overline{u_i'u_j'u_k'}+\overline{p(\delta_{kj}u_i'+\delta_{ik}u_j')}\right]+$$
$$\frac{\partial}{\partial x_k}\left[\mu\frac{\partial}{\partial x_k}(\overline{u_i'u_j'})\right]-\rho\left(\overline{u_i'u_k'}\frac{\partial u_j}{\partial x_k}-\overline{u_j'u_k'}\frac{\partial u_i}{\partial x_k}\right)-\rho\beta\left(g_i\overline{u_j'\theta}+g_j\overline{u_i'\theta}\right)+$$
$$\overline{p\left(\frac{\partial u_i'}{\partial x_j}+\frac{\partial u_j'}{\partial x_i}\right)}-2\mu\overline{\frac{\partial u_i'}{\partial x_k}\frac{\partial u_j'}{\partial x_k}}2\rho\Omega_k\left(\overline{u_j'u_m'}\varepsilon_{ikm}+\overline{u_i'u_m'}\varepsilon_{jkm}\right) \quad (6)$$

Since the RSM model avoids the eddy viscosity approximation, instead it solves the exact equations of the Reynolds stress tensor, it has a great potential in accurately predicting complex flows in which anisotropic effects are evident. However, solving the equations of the Reynolds stresses is computationally complex and may result in convergence difficulties. Moreover, several terms in Equ (6) are not known and closure approximations are essential for solving the equations, which may affect the accuracy of predicting the anisotropic effects.

D. *The v^2 f model*

The v2 f model [21] is similar to the Standard k-ε model in that it computes the turbulence kinetic energy k and it dissipation ε from equations (2) and (3), respectively. However, it incorporates also some near-wall turbulence anisotropy as well as non-local pressure-strain effects. In contrast to the Standard k- ε, the v2 f model uses a velocity scale v2 for the evaluation of the eddy viscosity instead of the turbulence kinetic energy k. v2 can be thought of as the velocity fluctuation normal to the streamlines. It can provide the right scaling for the representation of the damping of turbulent transport close to the wall. The anisotropic wall effects are modeled through the elliptic relaxation function f, by solving a separate elliptic equation of the Helmholtz type.

The turbulent viscosity is defined as

$$\nu^{v^2}{}_t=C_\mu\overline{\upsilon^2}T \quad (7)$$

And the turbulent quantities, in addition to k and ε, are obtained from two more equations: the transport equation for v2

$$\frac{\partial\overline{\upsilon^2}}{\partial t}+U_j\frac{\partial\overline{\upsilon^2}}{\partial x_j}=kf-\frac{\overline{\upsilon^2}}{k}\varepsilon+\frac{\partial}{\partial x_j}\left[\left(\nu+\frac{\nu_t}{\sigma_{\overline{\upsilon^2}}}\right)\frac{\partial\overline{\upsilon^2}}{\partial x_j}\right] \quad (8)$$

and the elliptic equation for the relaxation function f

$$L^2\nabla^2 f-f=\frac{C_1-1}{T}\left(\frac{\overline{\upsilon^2}}{k}-\frac{2}{3}\right)-C_2\frac{P_k}{\varepsilon} \quad (9)$$

where the turbulence length scale L

$$L=C_L\max\left[\frac{k^{3/2}}{\varepsilon},C_\eta\left(\frac{\nu^3}{\varepsilon}\right)^{1/4}\right] \quad (10)$$

and the turbulence time scale T

$$T=\max\left[\frac{k}{\varepsilon},C_T\left(\frac{\nu}{\varepsilon}\right)^{1/2}\right] \quad (11)$$

The coefficients used read: C_μ =0.22, $\sigma_{\overline{\upsilon^2}}$ =1, C_1 =1.4, C_2 =0.45, C_T =6, C_L =0.25, and C_η =85.

III. Numerical scheme

The computational domain is assumed to be 2D. The flow field modeled as isothermal incompressible flow field. The computational domain dimensions as per figure (1), which is discretized to 155840 constant density quadrilateral grid cells. Flow enters the domain from the lift side with a mass flux 9.4325 kg/m^2.s and 7% turbulence intensity [15]. The outlet is set to atmospheric back pressure. No-slip condition is applied to the inner and outer wall.

A general-purpose finite volume pressure based solver is used. The SIMPLE algorithm is applied for pressure velocity coupling.

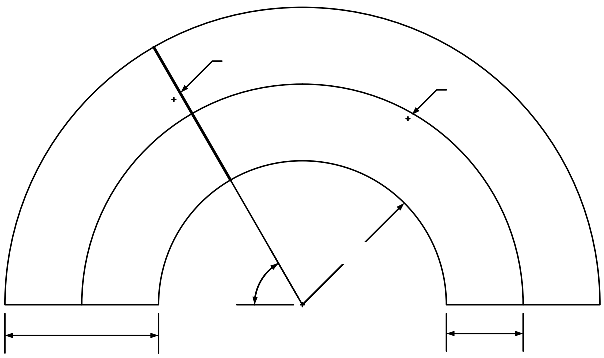


Figure (1) Computational domain

IV. Results and discussion

A. Tangential velocity

In order to validate the numerical model, the predicted tangential velocity is compared against tangential velocity measurements reported by [15]. Figure (2) shows predictions of tangential velocity against tangential measurements versus the radial distance alone inclined line (60^o with inlet). Concerning the three dimensionality, fuel jet effects and the experimental errors, calculated tangential velocity shows good agreement with experimental measurements. It is obvious from figure (2) that the predicted tangential velocity by all turbulence models almost identical.

B. Velocity contours

Although the tangential velocities are almost identical for all turbulence models, the velocity fields exhibit minor differences for different models, especially in the separation zone. Figure (3) show the velocity contours for each model. V2f model predicts a larger separation zone at the trailing edge of the inner wall which not predicted by k-ε model. The RSM model results are very close to such of the v^2f , however, RSM needs more computational resource, since it involves the computations of the Reynolds stresses.

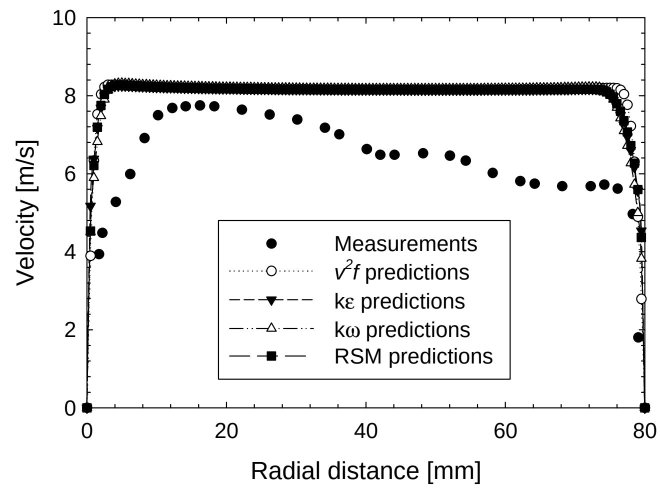


Figure (2) Tangential velocity a long line A

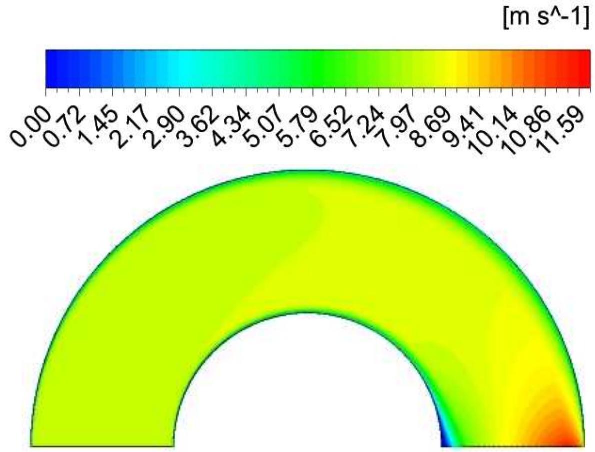


Figure (3.a) Velocity contours for kε

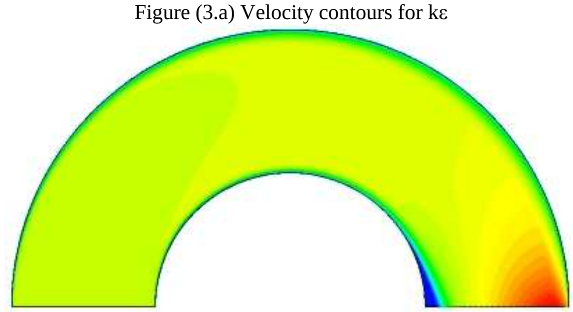


Figure (3.b) Velocity contours for kω

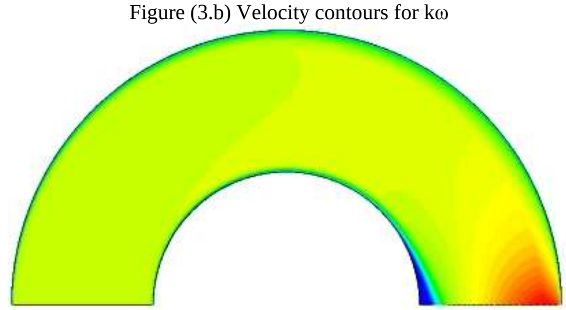


Figure (3.c) Velocity contours for RSM

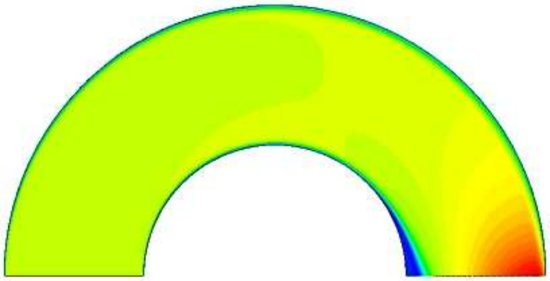
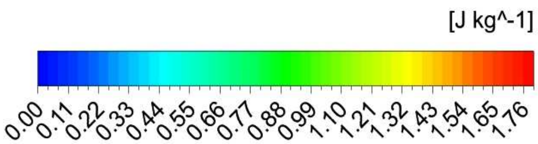


Figure (3.d) Velocity contours for v^2f

C. *Turbulence kinetic energy*

Figure (4) shows the turbulence kinetic energy contours, Also v2f model shows distinct results than other turbulence models. v^2f predicts the turbulence damping more than other models which leads to very small values of turbulence kinetic energy (almost zero) see figure(5). Here RSM model fail to give a close results to v^2f.



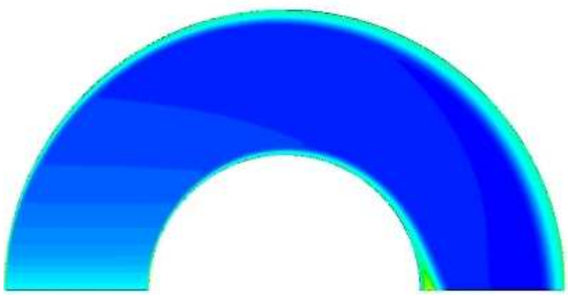


Figure (4.a) Turbulence kinetic energy contours for kε

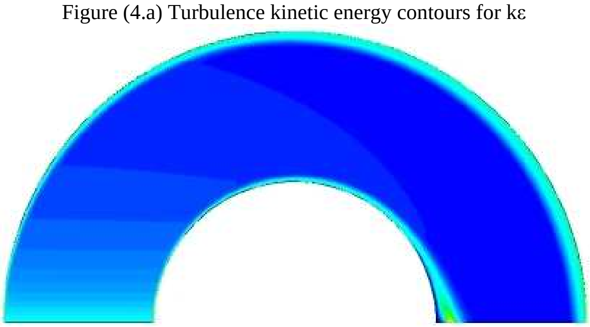


Figure (4.b) Turbulence kinetic energy contours for kω

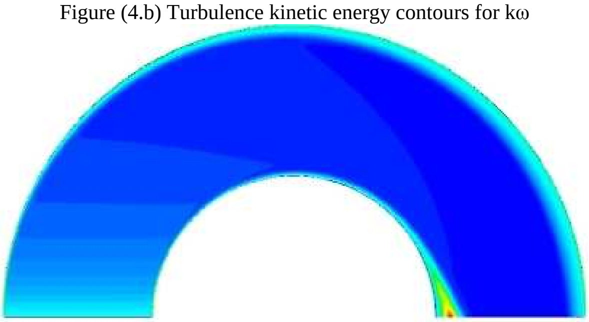


Figure (4.c) Turbulence kinetic energy contours for RSM

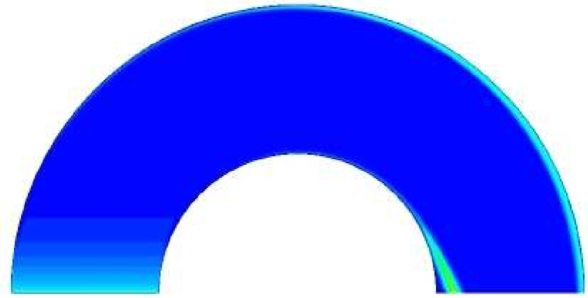


Figure (4.d) Turbulence kinetic energy contours for v^2f

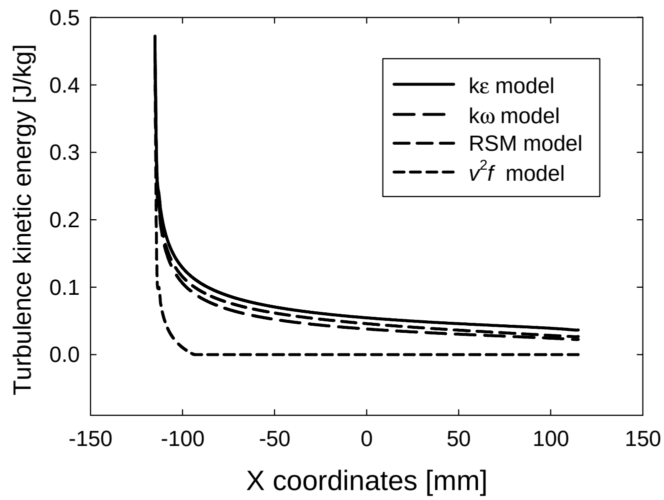


Figure (5) Turbulence kinetic energy along curve B

V. CONCLUSION

All RANS turbulence models gives almost identical tangential velocity, however the flow field results show many differences. V2f model shows a unique results and its far different than other models. The model is the best in predicting the separation zone. Also v2f model predicts the turbulence damping which lead to zero turbulence kinetic energy. The above results show the advances of using velocity scale v2 for the evaluation of the eddy viscosity instead of the turbulence kinetic energy k in the standard kε. It is makes the model able to predict turbulent to laminar transition and vice versa.

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