Mechanics of Unsaturated Soils for the Design of Foundation Structures

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Abstract: - Design of foundations for unsaturated soils is based on conventional soil mechanics assuming saturated conditions for soils. In this paper, semi-empirical models for predicting variation of the bearing capacity and the modulus of elasticity with respect to matric suction for unsaturated soils are presented. The saturated soil properties (i.e. bearing capacity and modulus of elasticity) and the Soil-Water Characteristic Curve (i.e. SWCC) are required for using these models. In addition, a simple method is also detailed to estimate matric suction of as-compacted soils using a pocket penetrometer. The proposed techniques are simple and should encourage geotechnical engineers to implement the mechanics of unsaturated soils into practice.

Key-Words: - unsaturated soils, matric suction, bearing capacity, modulus of elasticity, immediate settlement, soil-water characteristic curve, pocket penetrometer

1 Introduction

Approximately one-third of the earth's surface constitutes of arid or semi-arid regions where the soils are typically in a state of unsaturated condition [1]. The natural ground water table in these regions is deep and the stresses associated with the constructed infrastructure such as shallow foundations are distributed in the zone above the ground water table, where the pore-water pressures are negative with respect to the atmospheric pressure (i.e. matric suction). Rational design of foundations in arid and semi-arid regions should therefore be based on the mechanics of unsaturated soils taking account of the influence of matric suction. Similar procedures can also be extended for the design of infrastructure such as pavements and other foundation structures placed in compacted soils or in natural soil cuttings as the soil stresses are typically negative in nature with respect to the atmospheric pressure.

The bearing capacity and settlement behavior are two key properties required in the design of shallow foundations. The bearing capacity and the modulus of elasticity of unsaturated soils are commonly interpreted in practice assuming the soil is in a state of saturated condition although they remain in a state of unsaturated condition during their entire service period. This approach is followed because of two reasons: (i) extending the approach used for saturated soils to soils that are in a state of unsaturated condition provides conservative analysis with bearing capacity values being lower and settlements higher; (ii) there is no framework available for practicing engineers to design geotechnical structures such as the foundations using the mechanics of unsaturated soils.

The bearing capacity of saturated soils are analyzed using two different approaches; effective (i.e. c', ϕ') and total stress (i.e. $\phi_u = 0$) approach using the Terzaghi [2] and the Skempton [3] bearing capacity theory, respectively taking account of drainage conditions. The bearing capacity of unsaturated soils is commonly interpreted extending effective stress approach used for saturated soils regardless of the type of soil (i.e. coarse- and fine-grained soils) and the drainage conditions.

The model footing test results by Mohamed and Vanapalli [4] and Vanapalli et al. [5] in unsaturated coarse- and fine-grained soils show that effective and total stress approach should be respectively used in interpretation of the bearing capacity of unsaturated soils taking account of the influence of matric suction.

The modulus of elasticity, E is a key parameter in estimating the immediate settlement of foundations. The E values are commonly considered to be constant for the soils below and above ground water table. In other words, the effect of matric suction on E is not taken into account. Oh et al. [6] studies showed that E in sands is significantly influenced by matric suction [4,7].

In this paper, semi-empirical models are provided for predicting the variation of bearing capacity and modulus of elasticity with respect to matric suction for both coarse- and fine-grained soils. These models use the Soil-Water Characteristic Curve (i.e. SWCC), which is defined as a relationship between water

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content (gravimetric or volumetric) or degree of saturation and soil suction as a tool along with saturated soil properties (i.e. bearing capacity and modulus of elasticity under saturated condition). The proposed models are simple and there is a smooth transition between unsaturated and saturated soil behavior. In other words, the proposed semi-empirical models take the same form as conventionally used equations in practice when the matric suction value is zero (i.e. saturated condition).

In addition to the above studies, a simple method is proposed to estimate matric suction of as-compacted soils using a pocket penetrometer.

The key objective of the studies presented in the paper is to provide simple techniques for the practicing engineers to implement the mechanics of unsaturated soils in the design of shallow foundations in coarse- and fine-grained soils.

2 Bearing Capacity of Unsaturated Soils

The contribution of matric suction towards bearing capacity of unsaturated sands [7,8] and fine-grained soils [9-11] has been studied by several investigators. The bearing capacities of coarse- and fine-grained unsaturated soils are significantly different and require different approaches. This section provides some background information along with the details of how to interpret the bearing capacity of coarse- and fine-grained unsaturated soils.

2.1 Effective stress approach (*ESA*)

2.1.1 Coarse-grained soils (I_p = 0)

Oloo [9] proposed a method to estimate the bearing capacity of surface footing on unsaturated fine-grained soils extending the effective stress approach (*ESA*) as given below:

$$q_{ult(unsat)} = \{c' + (u_a - u_w)_b \tan\phi' + [(u_a - u_w) - (u_a - u_w)_b]\tan\phi^b\} \times N_c + \frac{1}{2}B\gamma N_\gamma \quad (1)$$

where $q_{ult(unsat)}$ = ultimate bearing capacity of unsaturated soil, c', ϕ' = effective cohesion and internal friction angle, respectively, $(u_a - u_w)_b$ = air-entry value, $(u_a - u_w)$ = matric suction, B = width of footing, γ = soil unit weight, N_c, N_γ = bearing capacity factors, and ϕ^b = internal friction angle indicating the rate of increase in shear strength related to the suction.

Eq. (1) is based on the assumption that the shear strength failure envelope of unsaturated soils is bilinear for simplification purposes [12]. In other words, the bearing capacity of unsaturated soils with respect to matric suction linearly increases with different slopes for the matric suction values less and greater than the air-entry value (curve (iii) in Fig 1 (b)).

The equation proposed by Oloo et al. [9] (i.e. Eq. (1)) has limitations in interpreting the bearing capacity behavior of unsaturated soils over a large suction range. This is because the bearing capacity of unsaturated soils is nonlinear for matric suction values greater than the air-entry value, $(u_a - u_w)_b$ for both coarse- and fine-grained soils (see curves (i) and (ii) in Fig. 1). In case of fine-grained soils, the bearing capacity increases as the matric suction increases and likely converges to a certain value (curve (ii) in Fig. 1). This limitation becomes more predominant for coarse-grained soils since the net contribution of matric suction towards bearing capacity decreases as the matric suction approaches the residual suction value (curve (i) in Fig. 1).

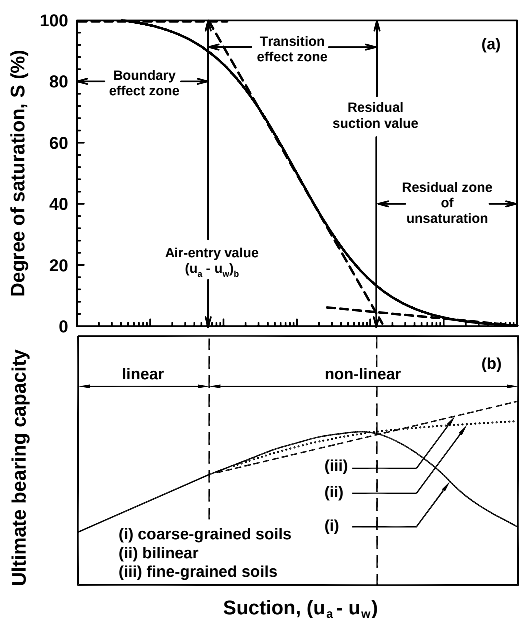

Fig. 1. Variation of bearing capacity with respect to suction for coarse- and fine-grained soils.

To overcome this limitation, Vanapalli and Mohamed [8] proposed a model to predict the nonlinear variation of bearing capacity of unsaturated soils with respect to matric suction for surface footings extending the *ESA* (Eq. (2).

$$q_{ult(unsat)} = [c' + (u_a - u_w)_b (1 - S^{\psi} \tan\phi') + (u_a - u_w)_{AVR} S^{\psi} \tan\phi'] \times N_c \xi_c + 0.5 B\gamma N_\gamma \xi_\gamma \quad (2)$$

where $(u_a - u_w)_{AVR}$ = average matric suction value, S = degree of saturation, ψ = fitting parameter with respect to bearing capacity, which is a function of I_p, L = length of footing, N_c = bearing capacity factor from Terzaghi [2], N_γ = bearing capacity factor from Kumbhokjar [13]; and ξ_c, ξ_γ = shape factors from Vesić [14].

The validity of Eq. (2) was verified using the model footing test results in sand [4]. The tests were performed using two different footing sizes (i.e. 100×100 mm and 150×150 mm) in a specially designed bearing capacity tank (900×900×750 mm) which has provisions to simulate saturated and unsaturated conditions. The matric suction value at the centroid of the matric suction distribution diagram from 0 to 1.5*B* (*B*: width of the model footing) depth region was considered as the average matric suction value in the analysis. This is the zone of depth in which the stresses due to shallow foundations loading is predominant [15,16]

Fig. 2 shows the variation of measured matric suction with depth and typical data from the test tank for an average matric suction value of 6 kPa. There was a reasonable agreement between the measured bearing capacity values and those estimated using Eq. (2) (Fig. 3).

Vanapalli and Mohamed [8] extended Eq. (2) towards the estimation of bearing capacity of unsaturated fine-grained soils (Fig. 4) and suggested that the fitting parameter, ψ is a function of plasticity index, I_p as shown in Eq. (3) ($\psi = 1$ for $I_p = 0$).

$$\psi = -0.0031(I_p)^2 + 0.3988(I_p) + 1 \quad (3)$$

Vanapalli and Oh [17] analyzed two more sets of in-situ plate load test results ([10,11]) and showed that ψ value is constant (i.e. $\psi = 3.5$) for the I_p values greater than 8%. The different behavioral trends of the fitting parameter, ψ can be explained offering the following two reasons. Firstly, the two open square points (□) in Fig. 4 were obtained using the result of only one data point, which means ψ values may not represent the variation of bearing capacity over a wide range of suction. Secondly, the ψ values obtained from both Vanapalli and Mohamed [8] and Vanapalli and Oh [17] may be considered to be reasonable as the parameter ψ may also be influenced by the drainage condition (or rate of loading). In other words, it is likely that different rate of loading can lead to different values of ψ even at the same I_p value. Hence, more studies are necessary to investigate the effect of rate of loading on the fitting parameter, ψ for unsaturated fine-grained soils.

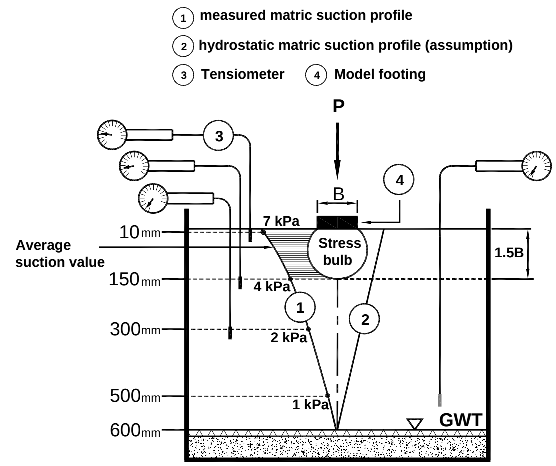

Fig. 2. Variation of measured matric suction with depth along with hydrostatic distribution for average matric suction of 6 kPa in the stress bulb zone.

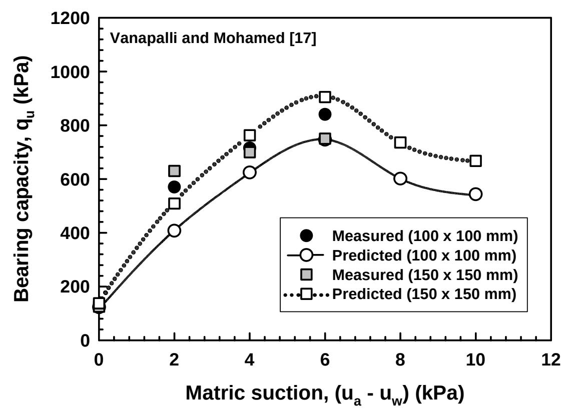

Fig. 3. Comparison between the measured and the predicted bearing capacity values.

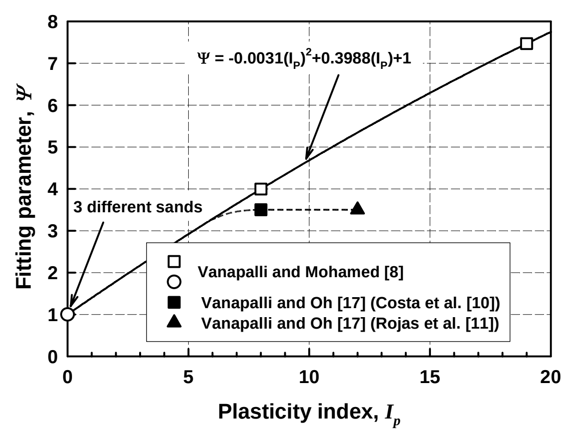

Fig. 4. Relationship between I_p and ψ in Eq. (2).

2.1.2 Fine-grained soils

Schnaid et al. [18] carried out in-situ plate (0.3, 0.45, 0.6, 0.7 and 1 m) load tests for unsaturated fine-grained soils. The bearing capacity values interpreted extending the *ESA* was 4 to 6 times greater than the measured values. Similar trends were also observed for the in-situ plate (Dia. = 0.8 m) load tests results by Costa et al. [10].

The discrepancy between the measured and the estimated bearing capacity values extending the *ESA* can be attributed to two key reasons. Firstly, the drainage conditions of pore-air and pore-water during loading stages in unsaturated fine-grained soils cannot be well defined (i.e. it may not be representing fully drained condition). Secondly, the bearing capacity equation originally proposed by Terzaghi [2] was based on the general shear failure (hereafter referred to as *GSF*) criteria assuming drained loading conditions; however in most cases, well-defined *GSF* modes are not observed for both model footing and in-situ plate load tests from the stress versus settlement relationships in unsaturated fine-grained soils [5,9,10,11,18,19].

Costa et al. [10], Schnaid et al. [18] and Consoli et al. [19] also estimated the bearing capacity values with reduced effective shear strength parameters (i.e. two-third of the initial values; Eqs. (4) and (5)), which is the conventional approach for interpreting local shear failure conditions [2]. Reasonably good agreement between the measured and the estimated bearing capacity values was observed using the reduction factors approach for the results by Schnaid et al. [18] and Consoli et al. [19]. However, the estimated bearing capacity values were still higher than the measured values by 3 to 5 times for the results by Costa et al. [10].

$$c^* = 0.67c' \tag{4}$$

$$\tan\phi^* = 0.67\tan\phi' \tag{5}$$

where c^*, ϕ^* = modified effective cohesion and effective internal friction angle, respectively.

Oloo [9] also extended a similar approach earlier to interpret the model footing (width and dia. = 30 mm) tests results for two different compacted unsaturated fine-grained soils (i.e. silt and till). The reduction factors approach provided good results for the glacial till, but not for the silt. The studies by Oloo [9], Costa et al. [10] and Schnaid et al. [18] indirectly suggest that using the reduced shear strength parameters should not be generalized or extended for all types of unsaturated fine-grained soils and suction values.

2.2 Total Stress Approach (*TSA*)

2.2.1 Unsaturated-fine grained soils behavior below a footing

The behavior of the unsaturated fine-grained (hereafter referred to as *UFG*) soils below footings can be more appropriately interpreted using punching shear failure (hereafter referred to as *PSF*) mechanism. For *PSF* conditions, the slip surfaces below footings are typically not extended to the ground surface but instead restrict to vertical planes as shown in Fig. 5.

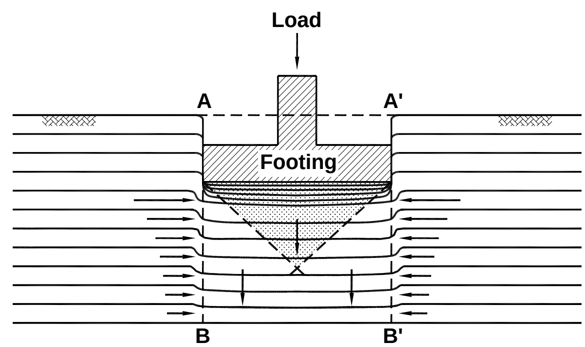

Fig. 5. Unsaturated fine-grained soils behavior below a footing.

This characteristic behavior indicates that the bearing capacity of the *UFG* soils is governed by the compressibility of the soil below a footing (i.e. soil A-A'-B-B' in Fig. 5; hereafter referred to as soil block). The soil around the soil block acts as confining pressure when the soil block is compressed due to the stress applied by a footing. In other words, the bearing capacity of the *UFG* soils can be represented as a function of a compressive strength of the soil block. Yamamoto et al. [20] proposed a model to predict the bearing capacity of compressible sand based on the fact that there is quasi-linear relationship between applied stress versus settlement (up to 10% of the diameter of footing). Their study also shows that even the bearing capacity of sand can be governed by the compressibility during loading stages.

Reasonable assumption can be made with respect to the pore-air to be under drained condition while the pore-water is under undrained condition during the loading stages of model footings or in-situ plate load tests in the *UFG* soils. This means that the pore-air is equal to atmospheric pressure and the water content in the soil is constant throughout the loading stage. Among the various methods available for estimating the shear strength of unsaturated soils, the constant water content (*CW*) test is regarded as the most reasonable technique for simulating this loading and drainage condition [21,22]. However, the *CW* test is time-consuming and needs elaborate testing equipments. Hence, unconfined compressive strength for the *UFG* soils can be used instead of the conventional *CW* test results. The use of unconfined compression test results can be justified based on reasonable assumptions.

i) The drainage condition for unconfined compression (*UC*) test for *UFG* soils is the same as the *CW* test (i.e. pore-air pressure is atmospheric pressure and the water content is constant throughout the test).

ii) The shear strength increases with increasing confining pressure for the same matric suction values for *CW* tests [21]. Therefore, the shear strength obtained from the unconfined compression tests typically provides conservative estimates.

2.2.2 Total stress approach (*TSA*)

Vanapalli et al. [5] extended the above concept and proposed a method to estimate the bearing capacity of the *UFG* soils using unconfined compression test results as shown in Eq. (6). This approach is conceptually the same as Skempton [3] bearing capacity theory (i.e. $\phi_u = 0$ approach) that is used to estimate the bearing capacity of saturated soils under undrained loading conditions.

$$q_{ult(unsat)} = \left[\frac{q_{u(unsat)}}{2}\right]\xi_{CW} N_{CW} \tag{6}$$

where $q_{ult(unsat)}$ = ultimate bearing capacity for unsaturated soil, $q_{u(unsat)}$ = unconfined compressive strength for unsaturated soil, N_{CW} = bearing capacity factor with respect to constant water content condition, and ξ_{CW} = shape factor with respect to constant water content condition.

Eq. (6) can be re-written as Eq. (7) with the shape factor, ξ_{CW} proposed by Meyerhof [23] and Vesić [14] for $\phi_u = 0$ condition.

$$q_{ult(unsat)} = \left[\frac{q_{u(unsat)}}{2}\right]\left[1 + 0.2\left(\frac{B}{L}\right)\right]N_{CW} \tag{7}$$

Vanapalli et al. [5] carried out model footing ($B \times L$ = 50 × 50 mm) tests in *UFG* soils for five different matric suction values to study the validity of Eq. (7) and to determine the bearing capacity factor, N_{CW}. Fig. 6 shows model footing test results for five different matric suction values (i.e. 0, 55, 100, 160, 205 kPa). The ultimate bearing capacity, q_{ult} was estimated as the stress corresponding to the intersection of elastic and plastic lines in the settlement range of 0 to 5 mm (10 % of the width of the footing).

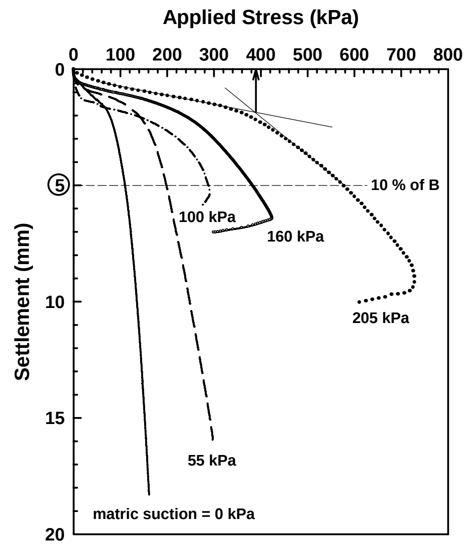

Fig. 6. Model footing test results in fine-grained soils for different matric suction values.

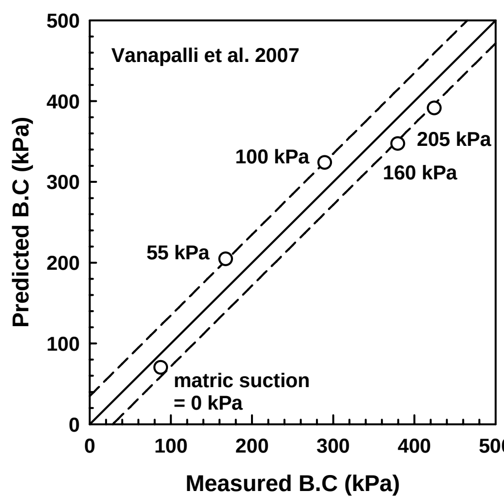

Fig. 7. Comparison between the measured and the predicted bearing capacity values using the *TSA*.

The N_{CW} values were back-calculated using the test results. The average of the back-calculated N_{CW} value was 5.23, which is close to the value of 5.14 that is used for Skempton [3] bearing capacity theory. Therefore, Eq. (7) can be justified and re-written as Eq. (8). There was good agreement between the measured bearing capacity values and those estimated using Eq. (8) (Fig. 7).

$$q_{ult(unsat)} = 5.14\left[\frac{q_{u(unsat)}}{2}\right]\left[1+0.2\left(\frac{B}{L}\right)\right] \tag{8}$$

2.2.3 Verification of the *TSA*

Fig. 8 shows the comparison between the measured bearing capacity values and those estimated extending both the *ESA* (i.e. Eq. (1) along with reduction factors approach) and the *TSA* (Eq. (8)) using the in-situ plate load tests results by Costa et al. [10]. The value of ϕ^b = 10.8° was used to calculate the contribution of matric suction towards the bearing capacity. The results shows that the bearing capacity values estimated extending the *TSA* are conservative and reasonable, whereas those estimated extending the *ESA* are significantly overestimated.

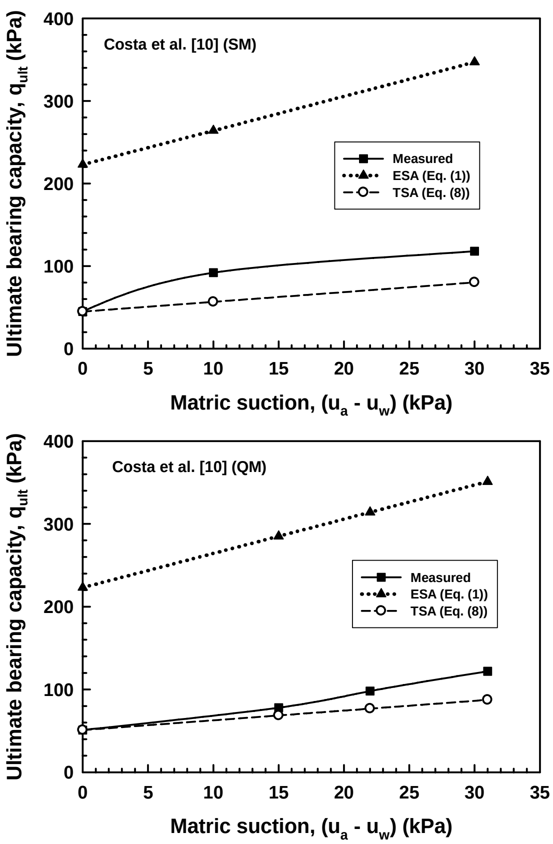

Fig. 8. Comparison between the measured and the estimated bearing capacity values using the *ESA* and the *TSA* (data from Costa et al. [10]).

Consoli et al. [19] conducted in-situ plate (1 m × 1 m) load tests in a residual homogeneous, cohesive soil (I_p = 20%). The bearing capacity values estimated using the *ESA* were overestimated by 1.5 – 2.5 times compared to the measured values. On the other hand, the bearing capacity calculated using the *TSA* along with the average unconfined compressive strength (i.e. 50.2 kPa) was 155 kPa, which is approximately the same as that of 1 m square concrete footing (i.e. 180 kPa) (Fig. 9).

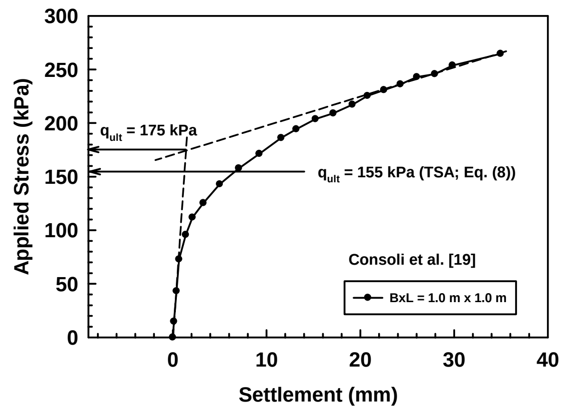

Fig. 9. Comparison between the measured and the estimated bearing capacity values using the *TSA* (data from Consoli et al. [19]).

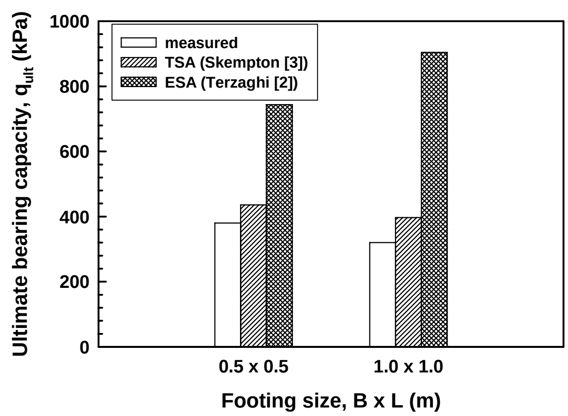

Fig. 10. Comparison between the measured and the estimated bearing capacity values using the *ESA* and the *TSA* (data from Larsson [24]).

Larsson [24] performed in-situ plate load tests on saturated silty soils (I_p = 6 ~ 12%) using three different sizes of square footings (i.e. width = 0.5, 1 and 2 m) under drained loading condition (i.e. the load was increased after the excess pore-water pressure due to the previous load was dissipated). The bearing capacity values estimated extending the *ESA* were overestimated by about 2 times whereas those estimated using average undrained shear strength according to Swedish building code (SBN-80) [25] showed a good agreement in comparison to the measured values (Fig. 10). The test results suggest the bearing capacity of *UFG* soils is governed by the compressibility of the soils below footings even under drained loading conditions.

2.2.4 Prediction of shear strength with respect to suction for unsaturated fine-grained soils

The total stress approach (*TSA*) shown in Eq. (8) suggests that the bearing capacity of *UFG* soils can be estimated from the unconfined compression tests results for unsaturated soils (i.e. $c_{u(unsat)}$). In other words, the variation of bearing capacity of *UFG* soils with respect to suction can be predicted by estimating the variation of shear strength, $c_{u(unsat)}$ (= $q_{u(unsat)}/2$) with respect to suction.

Oh and Vanapalli [26] extended this concept and proposed a model to predict the variation of shear strength of the *UFG* soils with respect to suction using the shear strength derived from unconfined compression test for the specimens at saturated condition and the *SWCC* as below.

$$c_{u(unsat)} = c_{u(sat)}\left[1+\frac{(u_a-u_w)}{(p_a/101.3)}\left(S^{\nu}\right)/\mu\right] \quad (9)$$

where $c_{u(sat)}$, $c_{u(unsat)}$ = shear strength under saturated and unsaturated condition, respectively, P_a = atmospheric pressure (= 101.3 kPa), and ν, μ = fitting parameters.

In Eq. (9), the terms, S^{ν} and μ control the nonlinear variation of the shear strength. The term, (P_a/101.3) is used for maintaining consistency with respect to dimensions and units on both sides of the equation. The fitting parameter, ν = 2 can be used in Eq. (9) for fine-grained soils. A relationship was developed between the fitting parameter, μ, and plasticity index, I_p analyzing six sets of unconfined compression tests results for *UFG* soils reported in the literature ((1) Chen [27]; (2) Ridley [28]; (3) Vanapalli et al. [29]; (4) Babu et al. [30]; (5) Pineda and Colmenares [31]; (6) Vanapalli et al. [5]). The basic soil properties and the *SWCC*s for the soils used for the analysis are shown in Table 1 and Fig. 11.

The fitting parameter, μ required for providing a good comparison between the measured and the estimated shear strength for each data set is summarized in Table 2. The relationship between the fitting parameter, μ, and plasticity index, I_p is summarized on a semi-logarithmic plot in Fig. 12. The fitting parameter, μ shows constant value of 9 for the I_p values between 8 and 15.5% (i.e. low plastic soils). The value of μ increases with increasing I_p following the relationship given in Eq. (10).

$$\begin{aligned} &\mu = 9 && \text{for } 8.0 \le I_p(\%) \le 15.5 \\ &\mu = 2.1088 \cdot e^{0.0903(I_p)} && \text{for } 15.5 < I_p(\%) \le 60.0 \end{aligned} \quad (10)$$

Table 1. Basic physical properties of the soils to determine the fitting parameter, μ in Eq. (9).

	(1)	(2)	(3)	(4)	(5)	(6)
G_s	2.88	2.61	2.68	2.7	2.61	2.72
I_p	38	32	8	60	38	15.5
OMC (%)	-	-	-	32.5	35	18.3
$\gamma_{d(max)}$ (kN/m^3)	-	-	-	15.4	12.2	17.3

Table 2. Fitting parameter, μ for the soils used in the study.

	I_p	μ
Vanapalli et al. [29]	8	9
Vanapalli et al. [5]	15.5	9
Ridley [28]	32	35
Chen [27]	38	60
Pineda and Colmenares [31]	38	65
Babu et al. [30]	60	490

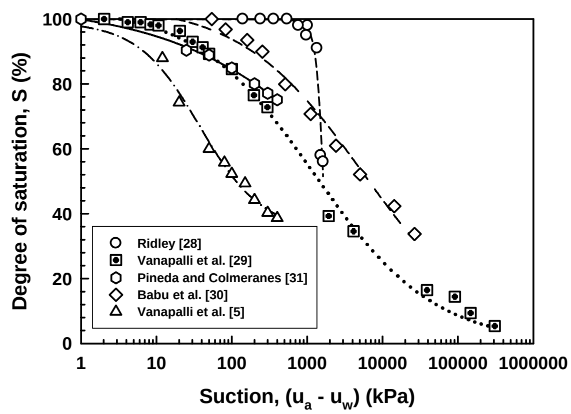

Fig. 11. *SWCC*s for the soils used to determine the fitting parameter, μ in Eq. (9).

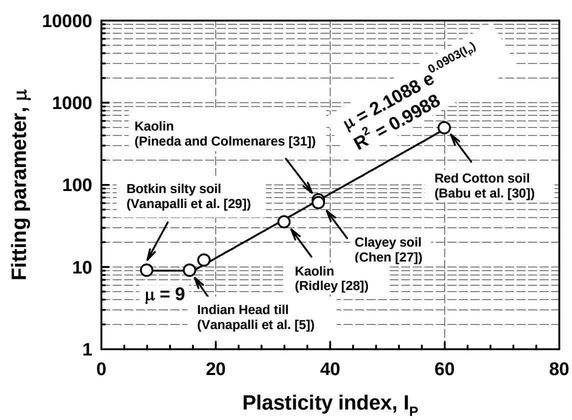

Fig. 12. Relationship between I_p and μ.

There are two main limitations of Eq. (9) in predicting the variation of the shear strength, $c_{u(unsat)}$ (= $q_{u(unsat)}/2$) with respect to suction as follows.

i) The relationship between μ and I_p shown in Fig. 12 and Eq. (10) are developed with limited data (i.e. six data sets) for a certain range of I_p values (i.e. $8 \leq I_p (\%) \leq 60$). Therefore, more supporting data would be valuable of this relationship to use with greater degree of confidence in geotechnical engineering practice applications.

ii) The results by Ridley [28] (see Fig. 13) shows that there is a discrepancy between the measured and the predicted shear strength values after a certain suction value (i.e. > 1,500 kPa). This behavior can be explained using the differential form of Eq. (9) as shown in Eq. (11).

$$\frac{d\left(c_{u(unsat)}\right)}{d\left(u_a - u_w\right)} = \frac{c_{u(sat)}}{\mu}\left[\left(S^{\nu}\right) + \left(u_a - u_w\right)\frac{d\left(S^{\nu}\right)}{d\left(u_a - u_w\right)}\right] \tag{11}$$

Eq. (11) indicates that at suction values close to the residual state conditions, the net contribution of matric suction towards shear strength decreases since the degree of saturation, S is small and the value of $[d(S^{\nu})]/[d(u_a - u_w)]$ is negative [32]. In other words, the predicted shear strength obtained using Eq. (9) starts decreasing at suction values close to residual suction value although the measured shear strength continues to increase. It can be seen that the *SWCC* for the soil used by Ridley [28] (Fig. 11) desaturates at a rapid rate, which leads to the fact that the suction values for the points (a) and (b) in Fig. 13 are close to the residual suction value. The residual suction value of the soil used by Ridley [28] can be estimated as about 1,500 kPa based on the *SWCC* in Fig. 11. The movement of water at this suction value is governed by vapor movement for several soils [33]. The test results for the Kaolin (the plasticity index, I_p for the material was not available in the literature) by Aitchison [34] also showed the similar trend as Ridley [28] data (see Fig. 14).

3 Immediate Settlement of Soils

The bearing capacity and the settlement are two key parameters that have a significant influence on the design of foundations. However, it is the settlement behavior that typically governs the design of a

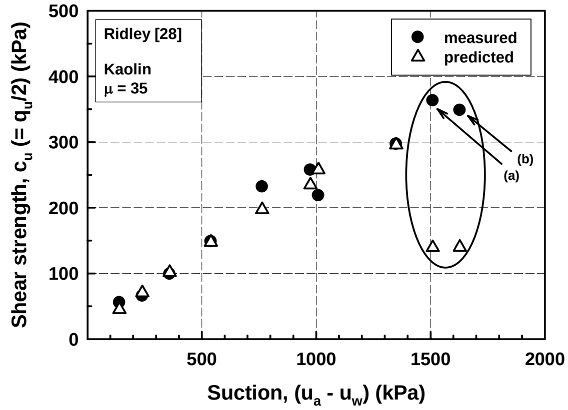

Fig. 13. Comparison between the measured and the predicted shear strength (data from Ridley [28]).

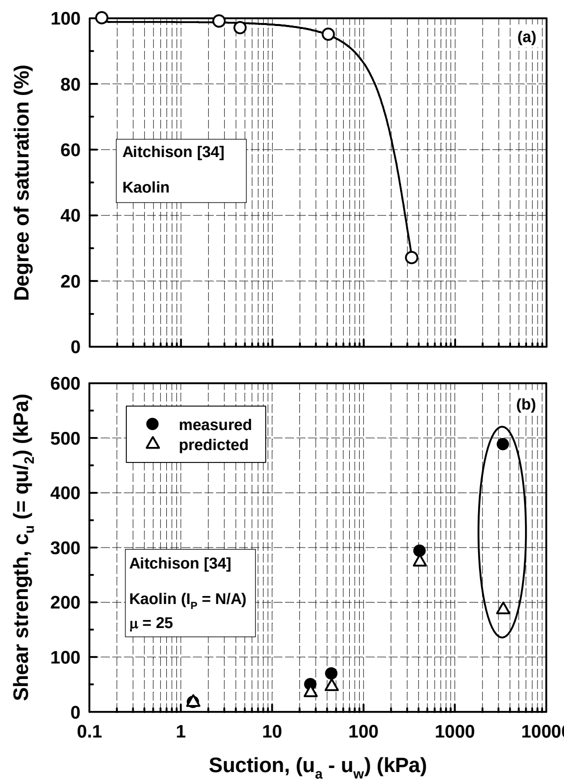

Fig. 14. (a) SWCC and (b) comparison between the measured and the predicted shear strength (data from Aitchison [34]).

foundation in comparison to the bearing capacity in several scenarios. This is particularly true for coarse-grained soils such as sands in which foundation settlements are immediate in nature. In sandy soils, settlement must be estimated or predicted reliably due to two main reasons. Firstly, the differential settlements in sandy soils are predominant in comparison to clayey soils because sand deposits are typically heterogeneous in nature. Secondly, the settlements in sandy soils occur quickly and may cause significant damages to the superstructures immediately after the construction [35].

The immediate settlement is estimated based on the modulus of elasticity, which is typically assumed to be constant both below and above the ground water table in homogeneous soil deposits. In other words, the influence of capillary or matric suction is not taken into account. A close examination of the experimental results of stress versus displacement relationships for model footing tests conducted on soils that are in a state of unsaturated condition showed that the modulus of elasticity is significantly influenced by matric suction [8].

Oh et al. [6] proposed a semi-empirical model for predicting the variation of modulus of elasticity of unsaturated soils using the *SWCC* and the modulus of elasticity under saturated condition, E_{sat} (Eq. (12))

$$E_{unsat} = E_{sat}\left[1+\alpha\frac{(u_a-u_w)}{(P_a/101.3)}\left(S^{\beta}\right)\right] \tag{12}$$

where E_{sat}, E_{unsat} = elastic modulus under saturated and unsaturated condition, respectively, and α, β = fitting parameters.

The details of the determination of the fitting parameters, α and β for coarse- and fine-grained soils are provided in the following sections.

3.1 Coarse-grained soils (I_p = 0)

Oh et al. [6] investigated five sets of model footing test results on three different sands ((1) Steensen-Bach et al. [7]; (2) Mohamed and Vanapalli [4]; (3) Li [36]); Table 3) and suggested that the fitting parameter, β = 1 is required to provide reasonable comparison between the measured and the predicted modulus of elastic of unsaturated coarse-grained soils.

Table 3. Details of the model footing tests for sands.

Author	Soil type	I_p	Plate base (mm)	Remark
(1)	Sollerod/ Lund sand	NP	22 × 22	Compacted soil (Laboratory test)
(2)	Unimin sand		100 × 100 150 × 150	
(3)	Unimin sand		37.5 × 37.5	

Fig. 15 shows the *SWCC*s, the variation of modulus of elasticity and elastic settlement with respect to matric suction for two model footing tests conducted on Unimin sand. Comparisons were provided between the measured and predicted values of elastic settlements for an applied stress of 40 kPa. At this applied stress value of 40 kPa, all sands studied in this paper exhibited elastic behavior.

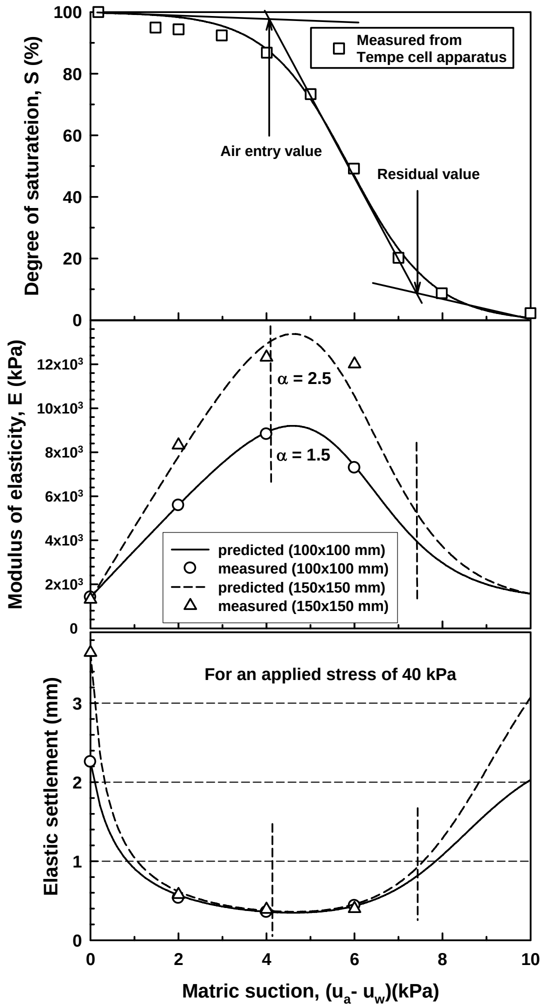

Fig. 15. Variation of elastic settlement of Unimin sand with respect to matric suction.

Table 4. Fitting parameter, α for the sands.

Soil	α
Unimin sand	
30 mm × 30 mm	0.5
100 mm × 100 mm	1.5
150 mm × 150 mm	2.5
Sollerod sand	
22 mm × 22 mm	2.5
Lund sand	
22 mm × 22 mm	0.5

The parameter, α from the analysis for the three sands is summarized in Table 4. These results show no defined trend or relationship between α and the size of the footing for the three different sands analyzed. Such a behavior can be attributed to the model footing tests carried out on Sollerod and Lund sands with relatively small size model footing in a relatively low suction range.

Figs 16 and 17 show the comparison between the measured and the predicted modulus of elasticity with respect to matric suction and the variation of the fitting parameter, α with respect to footing size, respectively for Unimin sand. The fitting parameter, α increases nonlinearly with increasing footing size. Oh et al. [6] explained this phenomenon using the degree of contribution of matric suction towards modulus of elasticity with respect to the ratio of footing size to soil particle sizes. In other words, the contribution of matric suction towards modulus of elasticity decreases as the ratio of footing size to soil particle sizes decreases. This is because when a footing size is relatively small, the load applied on the model footing is mostly carried by the individual soil particles rather than due to the friction arising at the contact points of the soil particles. Hence, it can be postulated that the parameter α value will not be less than 2.5 for field conditions since the ratio of footing size to soil particle sizes will be far greater than the laboratory model tests. Extending this concept, they concluded that the value of α between 1.5 and 2 can provide conservative elastic settlements in engineering practice.

The elastic settlements of model footing gradually decrease with an increase in modulus of elasticity values as matric suction increases in the boundary effect zone. In the transition zone, decreasing trends of elastic settlements can be observed in the lower suction region. However, elastic settlements gradually start increasing as the suction approaches the residual zone. Such a behavior can be attributed to the gradual decrease of modulus of elasticity in this zone. The elastic settlements in the residual zone are almost constant irrespective of increase in matric suction values. It is of interest to note that the elastic settlement at zero matric suction (i.e. saturated condition) and 10 kPa (i.e. residual condition) values are almost the same for the tested Coarse-grained sand (see Fig. 15).

3.2 Fine-grained soils

Vanapalli and Oh [37] extended the concept in Eq. (12) towards the estimated of modulus of elasticity of the *UFG* soils using model footing and in-situ plate load test results available in the literature ((1) Costa et al. [10]; (2) Rojas et al. [11]; Vanapalli et al. [5]; Table 5).

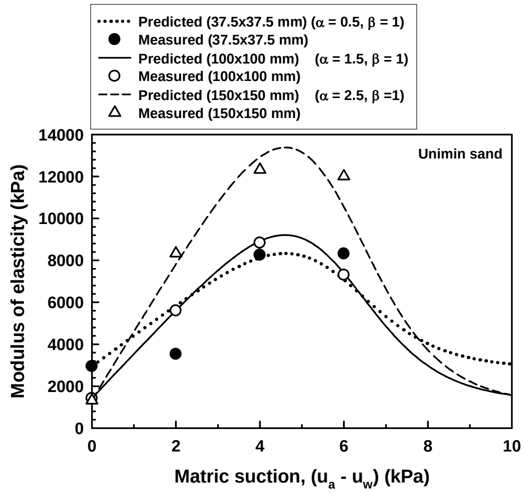

Fig. 16. Comparison between measured and predicted modulus of elasticity for Unimin sand (data from Mohamed and Vanapalli [4] and Li [36]).

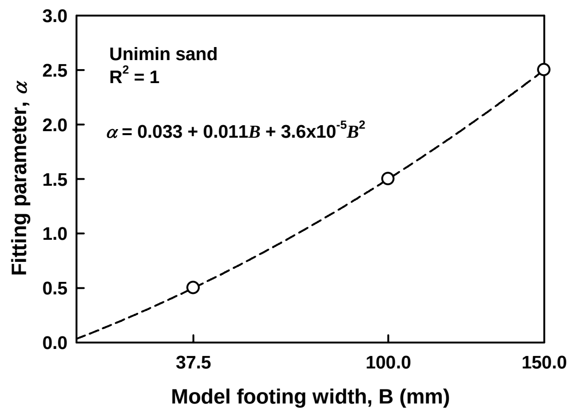

Fig. 17. Variation of the fitting parameter, α with respect to model footing width for Unimin sand.

Table 5. Details of the model footing tests for *UFG* soils

Author	Soil type	I_p	Plate base (mm)	Remark
(1)	Clayey sand	8	80 in Dia.	Natural soil (In-situ)
(2)	Lean clay	12	310 in Dia.	
(3)	Glacial till	15.5	50 × 50	Compacted soil (Laboratory test)

The comparison between the measured and the estimated modulus of elasticity showed that the fitting parameter, $\beta = 2$ is required for fine-grained soils regardless of I_p.

Fig. 18 provides a comparison between the measured and the predicted moduli of elasticity from test results by Costa et al. [10]. The fitting parameter,

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α was estimated between 1/10 and 1/3.2 depending on the matric suction values. Extending this technique, α was estimated as a value between 1/20 and 1/6.5 for the results by Rojas et al. 11] (Fig. 19). Fig. 20 shows the comparison for the results by Vanapalli et al. [5]) with the fitting parameter, α = 1/10. The comparisons for the results by Costa et al. [10] and Rojas et al. [11] indicate that lower values of α are required for relatively low matric suction values. This implies that the increment of modulus of elasticity is low when the matric suction values are low. This phenomenon was not observed for the results by Vanapalli et al. [5] since the lowest matric suction value used for the experiments was greater than 50 kPa.

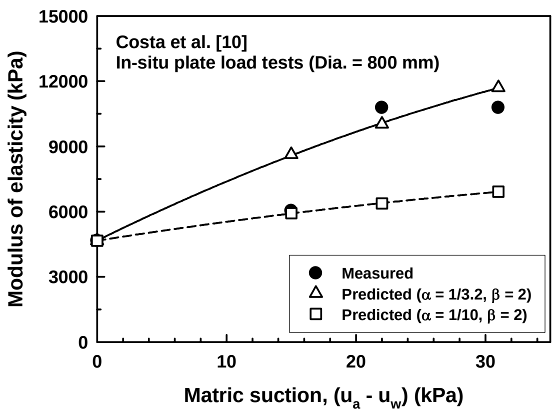

Fig. 18. Comparison between measured and predicted modulus of elasticity (date from Costa et al. [10]).

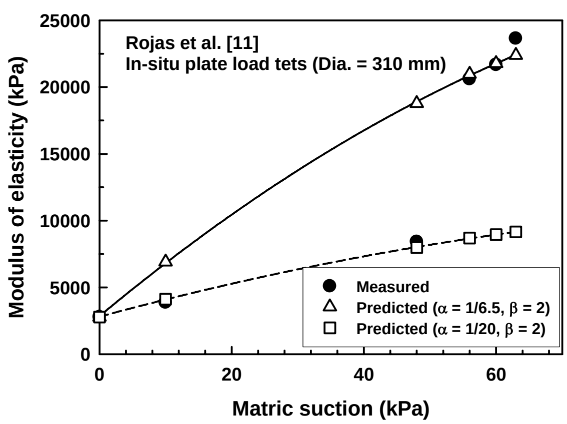

Fig. 19. Comparison between measured and predicted modulus of elasticity (date from Rojas et al. [11]).

Based on the analysis results, a relationship was developed between (*1/α*) and plasticity index, I_p using upper (Eq. (13)) and lower (Eq. (14)) boundary as shown in Fig. 21. The relationship shows that the inverse of α (i.e., *1/α*) nonlinearly increases with increasing I_p. The upper and lower boundary relationship can be used for low and high matric suction values respectively for soils with different plasticity index, I_p values.

$$(1/\alpha) = 0.5 + 0.312(I_p) + 0.109(I_p)^2 \tag{13}$$

$$(1/\alpha) = 0.5 + 0.063(I_p) + 0.036(I_p)^2 \tag{14}$$

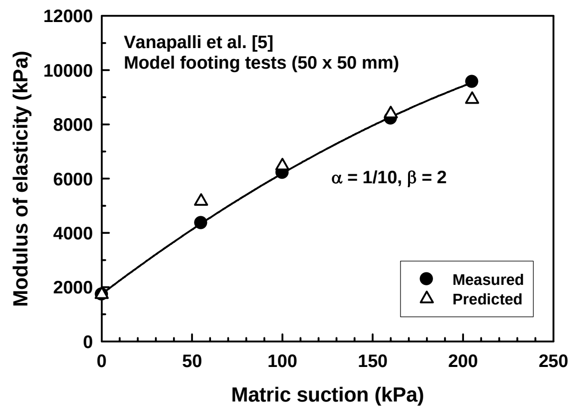

Fig. 20. Comparison between measured and predicted modulus of elasticity (data from Vanapalli et al. [5]).

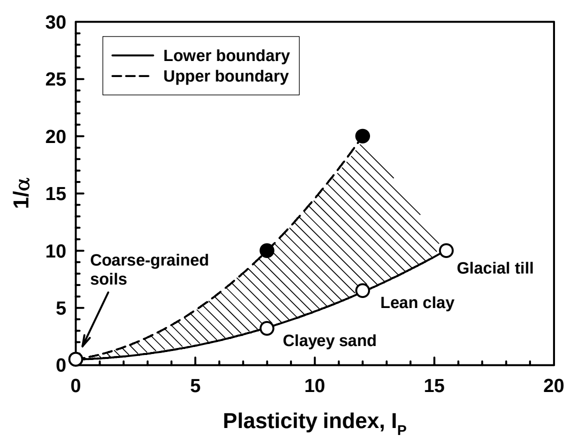

Fig. 21. The relationship between (*1/α*) and plasticity index, I_p

4 Estimation of Matric Suction of As-Compated Fine-Grained Soils Using a Pocket Penetrometer

There are limited applications in conventional geotechnical engineering practice in spite of the significant advancements made in our present understanding of the mechanics of unsaturated soils during the past 50 years [38]. One of the reasons for limited practical applications may be attributed to the lack of simple and reliable tools for determining or estimating matric suction quickly and economically. The conventional tensiometers (i.e., jet fill tensiometer or small tip) are regarded as the most reliable, simple and economical devices that can be used in the direct measurement of matric suction both in the laboratory and in the field. For coarse-grained soils, the conventional tensiometers can be used as a reliable instrument due to relatively low matric suction values. However, there are limitations to measure the matric suction of fine-grained soils using the conventional tensiometers since they are only suitable for measuring matric suction values lower than 90 kPa due to the problems associated with cavitation [39,40]. Other instruments such as high capacity tensiometers [41] and thermal conductivity sensors [42] can be used for measuring higher matric suction values. These instruments are not only expensive and cumbersome but also need highly qualified technical personnel to collect the data using them. Filter paper is another technique to measure suction values [43]. This technique is regarded as economical and simple; however, it is time consuming as long equilibration times are required prior to determining the matric suction values. In addition, the results are operator dependent unless tested by trained personnel and hence difficult to measure reproducible test results [44]. The most reliable technique for the measurement of matric suction in the range of 25 to 500 kPa is the axis-translation technique; however, it can be only used in laboratory environment [45,46].

Vanapalli and Oh [47] proposed a simple technique to estimate matric suction of as-compacted fine-grained soils using a pocket penetrometer (hereafter referred to as *PP*). This technique is based on the assumption that there is a strong relationship between the compression strength determined using a pocket penetrometer (hereafter referred to as PP_{cs}) and the matric suction of soils. To verify this assumption, they conducted a series of *PP* tests with matric suction measurements for statically compacted fine-grained soils

Based on the compaction curves (Fig. 22), three specimens with different water contents (i.e. dry of OMC, OMC and wet of OMC) that represent varying soil structures (Vanapalli et al. [48]) were first chosen from each compaction curve as shown in Figure 3 (i.e., specimens A, B, C for 1125 kPa, A', B', C' for 750 kPa, and A'', B'', C'' for 375 kPa). The *PP* tests were conducted on the prepared specimens along with the matric suction measurements using axis-translation technique [46].

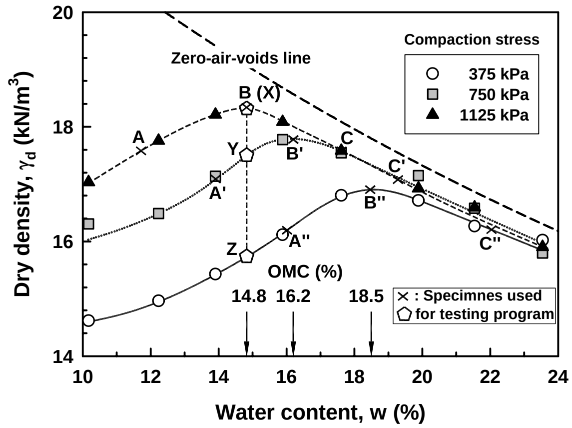

Fig. 22. Compaction curves for different compaction stresses and specimens used for the testing program.

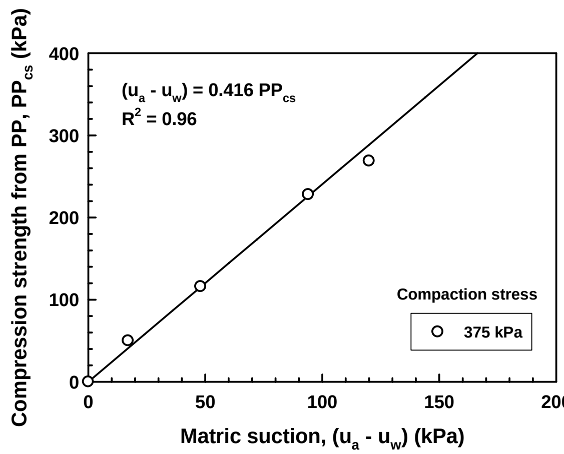

Fig. 23. Relationship between matric suction, (u_a – u_w) and PP_{cs} for the soil specimens compacted with the stress of 375 kPa.

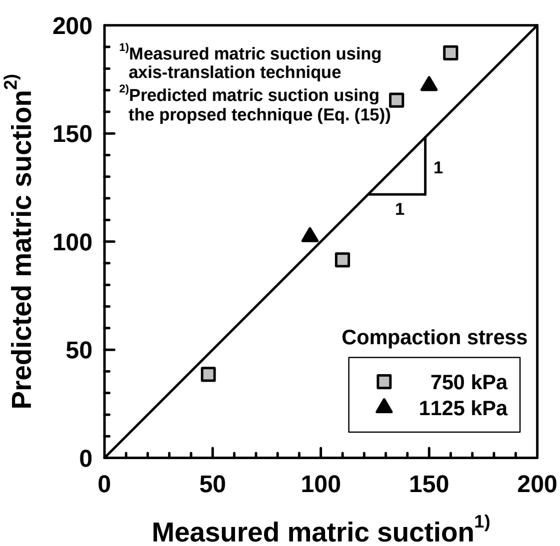

Fig. 24. Comparison between the measured and predicted matric suction values.

The *PP* tests were performed to determine the PP_{cs} on specimens compacted in the same metal rings that were used for compacting soil-water mixtures to establish compaction curves. The piston end of the *PP* is first located in the center of the compacted specimens. The specimen was then slowly and continuously loaded with the *PP* using both hands until the piston end penetrated to a depth of 6.35 mm. The tests were conducted on two identical specimens and the average PP_{cs}

Fig. 23 shows the relationship between the PP_{cs} and matric suction for the soil specimens prepared with the compaction stress of 375 kPa. The *PP* penetrated into the soil specimens by its own weight (or with low resistance) for the soil specimens compacted at higher water contents ($\geq$ 24%; S $\approx$ 100) since the matric suction values were close to 0 kPa with relatively low dry density values. Hence, the PP_{cs} values corresponding to matric suction, $(u_a - u_w)$ = 0 kPa were regarded as zero.

The results in Fig. 23 indicates that PP_{cs} linearly increases with increasing matric suction with the relationship shown in Eq. (15). The PP_{cs} for the specimen '*A*' could not be plotted on the figure since the value was beyond the maximum measurable capacity of the PP used (i.e. greater than 450 kPa).

$$(u_a - u_w) = 0.416\, PP_{cs} \tag{15}$$

To check the validity of Eq. (15), they tested the specimens prepared using different compaction stresses (i.e. 750 and 1125 kPa). There was a good comparison between the measured and the estimated matric suction values using Eq. (15) (Fig. 24).

5 Summary and Conclusions

In this paper, semi-empirical models are presented for predicting the variation of bearing capacity and modulus of elasticity of unsaturated soils with respect suction that are useful in the design of shallow of foundations. The models summarized in the paper use the Soil-Water Characteristic Curve (*SWCC*) as a tool along with soil properties under saturated condition (i.e. bearing capacity and modulus of elasticity under saturated condition). The key details of the study presented in this paper are summarized below:

i) Bearing capacity: the bearing capacity of unsaturated soils should be interpreted taking account of soil types (i.e. coarse- and fine-grained soils). The effective stress approach (*ESA*) extending Terzaghi [2] bearing capacity theory can be reliably used to interpret the bearing capacity of unsaturated coarse-grained soils; however, total stress approach (*TSA*) extending Skempton [3] bearing capacity theory can provide more reasonable estimates for unsaturated fine-grained soils. In this paper, two semi-empirical models for both coarse- (Vanapalli and Mohamed [8]) and fine-grained (Vanapalli et al. [5]) are provided. The bearing capacity of unsaturated fine-grained soils can be estimated using unconfined compression test results for unsaturated soils. A simple model is also proposed for predicting the variation of unconfined compressive strength of unsaturated fine-grained soils with respect to suction. This model can be used along with the bearing capacity equation for predicting the bearing capacity of *UFG* soils.

ii) Modulus of elasticity: the semi-empirical model provided in this paper to predict the variation of modulus of elasticity with respect to suction can be used for both coarse- and fine-grained soils. Summary of the details of the fitting parameter studies using model footing and in-situ plate load test results for coarse-grained are available in Oh et al., [6]) and fine-grained soils in Vanapalli and Oh [37].

Lastly, a simple method is provided to estimate matric suction of as-compacted soils using a pocket penetrometer.

The techniques presented in this paper are simple and are encouraging for practicing engineers to implement the mechanics of unsaturated soils in the design of shallow foundations.

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