Inversion of a Three-stage Full-Range Stress-Strain Relation for Stainless Steel Alloys

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Abstract—Presented in this paper is a new stress-strain relation for stainless steel alloys that provides the stress as an explicit function of the strain. The relation is an approximate inversion of a recently proposed three-stage stress-strain relation based on a modified Ramberg-Osgood equation. The new relation is derived by making a rational function assumption on the fractional deviation of the actual stress-strain curve from an idealized linear elastic behaviour. The new expression is valid over the full-range of the stress well beyond the elastic region. The validity of the inverted expression is tested over a wide range of material parameters. These tests demonstrate that, the new expression results in stress-strain curves which are both qualitatively and quantitatively in excellent agreement with experimental results and the fully iterated numerical solution of the full-range stress-strain relation.

Index Terms—Three-stage, Stainless steel, Ramberg-Osgood, Plastic, Elastic, Stress-Strain

I. INTRODUCTION

Due to its ability to retain strength at high temperatures and to resist corrosion as well as its attractive appearance and structural and architectural qualities, stainless steel has become increasingly popular in structural and architectural applications. Therefore, the proper description of the mechanical properties of stainless-steel such as the constitutive equation relating the stress and the strain is needed in the design and numerical modelling of engineering structures [1], [2]. The constitutive equation that relates stress to strain for stainless steel alloys and many other metals, is often expressed by the nonlinear Ramberg-Osgood equation which expresses the stress as an implicit function of the total strain [3]. The Ramberg-Osgood relation has been successfully employed in various applications including the development of moment-curvature relationship [4], the prediction of cyclic deformation [5], the determination of structural deflection [6], numerical modeling and new design methods [1],[2].

For stress levels below the yield stress value, it is well known that the Ramberg-Osgood equation provides stress-strain curves that are consistent with experimental results. However, for stress levels above the yield point, the Ramberg-Osgood relation highly overestimates the stress leading to serious inaccuracy. Having recognized this difficulty Rasmussen [7] and Mirambel and Real [6] independently developed a two-stage full-range relation based on the Ramberg-Osgood. Later, a more accurate two-stage relation that is applicable to both tensile and compressive stress-strain responses was proposed by Gardner and Nethercot [8] who used the 1% proof stress as a calibrating stress value. Gardner and Nethercot demonstrated that their agrees with the experimental observations for strain values up to the 2%. A highly accurate three-stage relation which is applicable to both tensile and compressive stress-strain responses was recently proposed by Quach et.al.[9]. They showed that, their new model is accurate for the full range of stress-strain response up to the ultimate stress. The common drawback of the Ramberg-Osgood based full-range stress-strain relations is that the stress is expressed as an implicit function of a highly nonlinear relation. However, a closed-form inversion of this nonlinear equation describing the stress as an explicit function of the strain is highly desired since it would considerably simplify its use in most applications. Moreover, an explicit relation would lead to computational efficiency and eliminate problems associated with numerical convergence of nonlinear equations. This has motivated several recent attempts in order to obtain an approximate inversion for the basic Ramberg Osgood relation including the works of Mostaghel and Byrd [10], who used high order expansion, Chryssanthopoulos and Low [4] who used polynomial approximation methods, and Abdella [11]who used approximation method based on a power law assumption for the fractional deviation of the full-range stress-strain curve from the linear elastic rule. More recently, Abdella [12] derived a more accurate inversions based on a rational function assumption rather than the power law assumption. While, Abdella’s inversions are based on the two-stage full-range formulation, all other inversion approaches have focussed on the standard Ramberg-Osgood which is not valid beyond the yield stress values.

The research of this paper is motivated by the 2009 work of Quach et.al. [9] who developed a new and highly accurate three-stage full-range stress-strain model for stainless steel alloys applicable to both tensile and compression stresses and
to small and intermediate strains. This model can be used for austenitic, duplex, and ferritic alloys - the three most commonly used alloys for constructing stainless steel structures. Although based on the Ramberg-Osgood parameters, the Quach model used experimental stress-strain curves in order to demonstrate that their three-stage full-range relation is much more accurate than any of the existing relations. In some cases, they were able to show that existing relations and their inversions, including those of Abdella [11], could overestimate the stress by up to 50%. However, currently there are no inversion formulations for this highly accurate three-stage full-range relation.

In this paper, a highly accurate approximate inversion of the three-stage full-range stress-strain relation will be proposed. The derivation is based on a rational function assumption on the fractional deviation of the full-range stress-strain curve from the linear elastic rule. The inversion formula is compared with experimental results as well as the fully iterated numerical solution of the full-range stress-strain relation. The comparison will demonstrate that the inversion formula is a highly accurate approximation which is valid for the full range of the stress levels up to the ultimate stress. Moreover, the inversion is applicable for tensile as well as compressive stress-strain responses.

II. THE THREE-STAGE STRESS-STRAIN RELATIONSHIP

For stainless steel alloys and many other metals, the relationship between the stress $\sigma$ and the total strain $\epsilon$ is often described by the Ramberg-Osgood nonlinear equation which expresses $\epsilon$ as the sum of the elastic strain $\epsilon_e$ and the plastic strain $\epsilon_p$ (Ramberg and Osgood [3]):

$$ \epsilon = \epsilon_e + \epsilon_p, \quad \epsilon_e = \frac{\sigma}{E_0}, \quad \epsilon_p = \left( \frac{\sigma}{K} \right)^{\frac{1}{n'}} $$

where $E_0$ is the modulus of elasticity, $K$ is the strain-hardening coefficient and $n'$ is the strain-hardening exponent signifying the degree of nonlinearity in the stress-strain curve. Then eliminating $K$ from equation (1), the Ramberg-Osgood equation becomes:

$$ \epsilon = \frac{\sigma}{E_0} + \epsilon_{pp} \left( \frac{\sigma}{\sigma_{pp}} \right)^{n} $$

where $n = \frac{1}{n'}$ is a nonlinearity index, and $\sigma_{pp}$ is the equivalent yield stress called the proof stress corresponding to a plastic strain level of $\epsilon_{pp}$ which has commonly used value of $\epsilon_{pp} = 0.2\%$ so that:

$$ \epsilon = \frac{\sigma}{E_0} + 0.002 \left( \frac{\sigma}{\sigma_{0.2}} \right)^{n} $$

where $\sigma_{0.2}$ represents the 0.2% proof stress. The parameter $n$ is usually determined by substituting the 0.2% and the 0.01% proof stresses into the $\epsilon_p$ expression in equation (1) and solving for $n$ to obtain the commonly used formulation

$$ n = \frac{\ln(20)}{\ln(\sigma_{0.2}/\sigma_{0.01})} $$

where $\sigma_{0.01}$ is the 0.01% proof stress corresponding to $\epsilon_p = 0.01\%$.

For stress levels below $\sigma_{0.2}$, equation (3) accurately represents the experimentally observed stress-strain curves for stainless steel and other metal alloys. However, experimental evidence demonstrates that, for stress levels beyond $\sigma_{0.2}$, equation (3) tends to overestimate the stress values significantly. Therefore, for $\sigma > \sigma_{0.2}$, the usual Ramberg-Osgood relation expressed in Equation (3) is inadequate for most applications.

In order to improve the stress-strain relation for stress levels above the 0.2% proof stress and below the ultimate tensile strength $\sigma_u$, several approaches have been proposed recently. These include the formulation proposed by Ramussen [7] and Mirambel and Real [6] who proposed a modified Ramberg-Osgood relation based on a new reference system in which the origin of the stress-strain curve is linearly translated to the point $(\epsilon_{0.2}, \sigma_{0.2})$ where $\epsilon_{0.2}$ is the total strain corresponding to $\sigma_{0.2}$. Therefore, the modified Ramberg-Osgood relation is a two-stage relation which is valid over the interval $0.2 < \sigma < \sigma_u$, is given by:

$$ \epsilon_s = \frac{\sigma_s}{E_s} + \epsilon_{sup} \left( \frac{\sigma_s}{\sigma_{sup}} \right)^{n_s} $$

where $\epsilon_s = \epsilon - \epsilon_{0.2}$, $\sigma_s = \sigma - \sigma_{0.2}$, $\sigma_{sup} = \sigma_u - \sigma_{0.2}$, $E_s = E_{0.2}$ is the tangent modulus of the stress-strain curve at the new origin $(\epsilon_{0.2}, \sigma_{0.2})$, $n_s = m$ is a new index and $\epsilon_{sup}$ is the ultimate plastic strain in the transformed reference system corresponding to $\sigma_{sup}$. It can be also shown that, $\epsilon_{sup} = \epsilon_u - \epsilon_{0.2} - \frac{\sigma_u - \sigma_{0.2}}{E_{0.2}}$ and the tangent modulus

$$ E_{0.2} = \frac{E_0}{1 + 0.002n'/\epsilon} $$

where $\epsilon$ is a nondimensional proof stress given by $\epsilon = \frac{\sigma_{0.2}}{E_0}$.

Therefore, their formulation becomes, the following differentiable full-range expression valid for any stress level under the ultimate tensile strength $\sigma_u$ :

$$ \epsilon = \begin{cases} \frac{\sigma}{E_0} + 0.002 \left( \frac{\sigma}{\sigma_{0.2}} \right)^{n} & 0 \leq \sigma \leq \sigma_{0.2} \\ \frac{\sigma_{0.2}}{E_0} + \frac{\sigma}{E_2} + \epsilon_{sup} \left( \frac{\sigma}{\sigma_{sup}} \right)^{m} & \sigma_{0.2} < \sigma \leq \sigma_u. \end{cases} $$

While Eq. (7) is clearly a great improvement over the basic Ramberg-Osgood relation, Gardner and Nethercot [8] recently proposed the use of the 1% proof stress instead of the ultimate stress $\sigma_u$. This has resulted in a more accurate approximation for stress values up to $\sigma_2$, the 2% proof stress. Moreover, unlike the previous formulations, their formulation is applicable for modelling compressive stress-strain behaviors. More recently, Quach et.al. [9] proposed a three-stage stress-strain model that is highly accurate for the full range of stress values beyond $\sigma_2$ and the corresponding strain of $\epsilon_2$. For the first stage covering up to the yield stress, the standard Ramberg-Osgood expression is adopted while, for the second stage covering the stress values beyond the yield stress and
up to $\sigma_2$, a modified version of the Gardner and Nethercot [8] formulation is adopted. The third stage Quach et al. is based on the observation of Olsson [13] who found that the slope variation of the stress-strain curve beyond $\sigma_2$ is quite negligible. With this assumption, Quach et al.[9] adopt a linear approximation for the third stage ranging from $\sigma_2$ to $\sigma_u$ which leads to the following three-stage full-range relation:

$$\varepsilon = \begin{cases} \frac{\sigma}{E_0} + 0.002 \left( \frac{\sigma}{\sigma_{0.2}} \right)^n & 0 \leq \sigma \leq \sigma_{0.2} \\ \varepsilon_{0.2} + \frac{\sigma}{E_2} + \varepsilon_{2p} \left( \frac{\sigma}{\sigma_{s1}} \right)^{n_2} & \sigma_{0.2} < \sigma \leq \sigma_2 \\ \frac{\sigma - a}{b - \sigma} & \sigma_2 < \sigma \leq \sigma_u. \end{cases} \quad (8)$$

where

$$a = \sigma_2(1 + \varepsilon_2) - b \varepsilon_2, \quad b = \frac{\sigma_u(1 + \varepsilon_u) - \sigma_2(1 + \varepsilon_2)}{\varepsilon_u - \varepsilon_2}, \quad (9)$$

and $n_2$ represents a strain-hardening exponent representing a curve that passes through $\sigma_{0.2}$ and $\sigma_1(1\%$ proof stress), and its value can be determined from measured stress-strain curves. In addition to its accuracy, this model is applicable to both tensile and compressive stress-strain responses. In their inves-
tigations Quach et al.[9] show that, in some cases, existing relations and their inversions, including those of Abdella [11], could overestimate the stress by up to 50%. However, currently there are no inversion formulations for this highly accurate three-stage full-range relation.

Using linear regression of the experimental data for stainless steel alloys, Quach et al. [9] also developed the following expression for their model parameters as functions of $n$, $E_0$, and $\sigma_{0.2}$:

$$n_2 = c_1 \frac{E_{0.2}}{E_0} \frac{\sigma_1}{\sigma_{0.2}} + c_2, \quad (11)$$

$$\sigma_1 = \sigma_{0.2} \left( \frac{d_1}{n} + d_2 \right), \quad (12)$$

$$\sigma_{u}^t = \sigma_{0.2}^t \left( \frac{1 - 0.0375(n^t - 5)}{0.2 + 185\varepsilon^t} \right), \quad (13)$$

$$\varepsilon_u^t = 1 - \frac{\sigma_{u}^t}{\sigma_{0.2}^t}, \quad (14)$$

$$\sigma_c^C = \sigma_{u}^t \left( 1 + \varepsilon_u^t \right)^2, \quad (15)$$

$$\varepsilon_c^C = 1 - \frac{1}{1 + \varepsilon_u^t}, \quad (16)$$

where the superscript "$t$" and "$c$" respectively indicate tensile and compressive stresses and the constants corresponding to the tensile and the compressive cases are $c_1^t = 12.255$, $c_2^t = 6.399$, $c_1^c = 1.037$, $c_2^c = 1.145$, $d_1^t = 0.542$, $d_1^c = 0.662$, $d_2^t = 1.072$, $d_2^c = 1.085$. Quach et al.[9] also proposed an approximate expression for $\sigma_2$. However, this parameter can be estimated numerically as well.

Due to nonlinearity it is not possible to obtain closed form inversion of Equation (8) describing the stress $\sigma$ as an explicit function of the strain $\varepsilon$. As a result, graphical or iterative numerical procedures are typically employed to compute the stress corresponding to a given value of a strain. However, a closed form inversion would greatly simplify the analysis and evaluation of various quantities of interests arising from a wide range of applications. Moreover, inversion using iterative procedures is highly undesirable, since this will introduce computational costs and difficulties associated with numerical convergence. Therefore, the purpose of this paper is to present an accurate approximation to the closed form inversion for the above three-stage full-range stress-strain relationship which explicitly expresses the stress in terms of the strain. The derivation of the approximate inversion is presented in the next section.

III. THE NEW APPROXIMATE INVERSION

In this section the approximate inversion which expresses the stress $\sigma$ as an explicit function of the strain $\varepsilon$ is presented. Recall that the three-stage stress-strain expressions are formulated differently for stress values over the three stages. Therefore, the inversions for the $\varepsilon \leq \varepsilon_{0.2}$ case, $\varepsilon_{0.2} \leq \varepsilon \leq \varepsilon_2$ and the $\varepsilon > \varepsilon_2$ case will be derived separately.

A. The $\varepsilon \leq \varepsilon_{0.2}$ Case (First Stage)

Let $f_1(\varepsilon, \sigma)$ represent the fractional deviation of the stress-strain curve from the linear elastic behaviour:

$$f_1(\varepsilon, \sigma) = \frac{\sigma E_0 - \sigma_0}{\sigma} = \frac{0.002}{n} \left( \frac{\sigma}{\sigma_{0.2}} \right)^n. \quad (17)$$

where $\sigma$ is the stress satisfying the three-stage stress-strain relation. Recall that the closed-form approximation of Abdella ([11], [14]) is derived by making the power law assumption for the function $f_1(\varepsilon, \sigma)$ which was later modified in Abdella [12] in order to reduce the underestimation of stress values that was observed for small strain values. In both Abdella[11] and Abdella[12], the model parameters of the proposed approximations were derived by matching the approximations with the two-stage stress-strain relation.

In this paper, we consider the approximation of $f_1(\varepsilon, \sigma)$ for the three-stage stress-strain relation with a more general rational function assumption of the form:

$$f_{1a}(\varepsilon, \sigma) = \frac{C\varepsilon_F^{P} + Q}{1 + D\sigma_c^Q} \quad (18)$$

where $f_{1a}$ represents the approximation to $f_1(\varepsilon, \sigma)$ and $\varepsilon_r = \frac{\varepsilon_{0.2}}{\varepsilon}$. The constants $C$, $P$, $D$ and $Q$ are positive parameters which need to be determined. Solving for $\sigma$ from equation (17) and using the approximated $f_{1a}$, we obtain the following approximate inversion of the stress-strain curve:

$$\sigma_{1a} = \frac{E_0\varepsilon \left( 1 + D\sigma_c^Q \right)}{1 + Ce_F^{P} + D\sigma_c^Q} \quad (19)$$
where \( \sigma_{1a}(\epsilon) \) represents, the inverted approximation of the stress \( \sigma \). In order to determine the model parameters, the values of \( \sigma_{1a}, \frac{d\sigma_{1a}}{d\epsilon}, \) and \( \frac{d^2\sigma_{1a}}{d\epsilon^2} \) are matched with the actual three-stage stress-strain curve at the two endpoints of the first stage interval. First, the matching at left end point of the interval which is the origin is considered. Note that, since 
\[
\lim_{\epsilon \rightarrow 0} \sigma_{1a} = 0, \quad \text{and} \quad \lim_{\epsilon \rightarrow 0} \frac{d\sigma_{1a}}{d\epsilon} = \sigma_0,
\]
the inversion as well as its slope automatically match their respective values of the actual three-stage inversion at the origin. However, it can be shown that matching \( \frac{d^2\sigma_{1a}}{d\epsilon^2} \) leads to the condition \( P > 1 \) and \( Q > 1 \). Hence, for a proper matching of the inversion at the origin, the parameters \( P \) and \( Q \) must be chosen to be above unity.

In order to match the inversion with the three-stage formulation at \((\epsilon_{02}, \sigma_{02})\), we set 
\[
f_1(\epsilon_{02}, \sigma_{02}) = f_{1a}(\epsilon_{02}, \sigma_{02}), \\
f'_1(\epsilon_{02}, \sigma_{02}) = f'_{1a}(\epsilon_{02}, \sigma_{02}), \text{ and } f''_1(\epsilon_{02}, \sigma_{02}) = f''_{1a}(\epsilon_{02}, \sigma_{02}),
\]
which yields the following expressions for the model parameters:

\[
C = G_0(1 + D), \quad G_0 = \frac{\epsilon_{02}E_0}{\sigma_{02}} - 1 = \frac{0.002E_0}{\sigma_{02}} > 0 \quad (20)
\]
\[
P = \Delta + G_1, \quad G_1 = \frac{\epsilon_{02}E_{02}(n - 1)}{\sigma_{02}} \quad (21)
\]
\[
Q = 1 + \frac{B_1}{\Delta} \quad (22)
\]
\[
D = \frac{\Delta}{Q - 1} \quad (23)
\]
where
\[
B_1 = \frac{G_1E_2}{E_0} (n + G_0) > 0 \quad (24)
\]
\[
\Delta = 1 + \sqrt{1 + 4B_1} > 1. \quad (25)
\]

B. The \( \epsilon_{02} \leq \epsilon \leq \epsilon_2 \) Case (Second Stage)

Recall that, in the translated reference system with \((\epsilon_{02}, \sigma_{02})\) as its origin, the behaviors of the second stage stress-strain curve part is similar to that of the first stage that is derived above. Therefore, the following assumption for the approximation of the inversion will be used:

\[
f_2(\epsilon_, \sigma) = \frac{\epsilon E_0 - \sigma}{\sigma} \quad (26)
\]
As in the first stage, the following rational function assumption is made for the function \( f \):

\[
f_{2a}(\epsilon) = \frac{ce^{p}}{1 + ce^{p}} \quad (27)
\]
where \( \epsilon_{sr} = \frac{\epsilon}{\epsilon_1 - \epsilon_{02}} \), and positive constant \( c, d, p, \) and \( q \) are model parameters that must be determined through the matching process similar to the stage one case. Therefore, the approximated stress from Equation (33) is given by,

\[
\sigma_{2a} = \frac{E_{02} \epsilon_{sr} (1 + de^{p}_{sr})}{1 + ce^{p}_{sr} + de^{q}_{sr}} + \sigma_{02} \quad (28)
\]

where \( \sigma_{2a}(\epsilon) \) represents, the inverted approximation of the stress \( \sigma \) for the second stage interval.

As in the previous case, the matching of \( \sigma_{*} \) and its derivative at the left end point of the current interval, \((\epsilon_{02}, \sigma_{02})\), are automatically satisfied. Similarly, it can be shown that, the matching of the second derivative at the left end point leads to the requirement that both \( p \) and \( q \) must be above unity.

Recall that, in the first stage derivation, the constants were determined by matching the values of \( f_{1a}, f'_{1a} \) as well as \( f''_{1a} \) with their respective values of the actual three-stage stress-strain curve at \((\epsilon_{02}, \sigma_{02})\), the right end point of the second stage. However, for the second stage, the situation is different since the approximation must pass through \((\epsilon_1, \sigma_1)\) as well as \((\epsilon_2, \sigma_2)\). Hence, in order to determine the four parameters, the values of the function \( f_2 \) as well as its first derivative are matched with their respective three-stage curve values at \((\epsilon_1, \sigma_1)\) as well as at \((\epsilon_2, \sigma_2)\).

This leads to the following expressions for the model parameters:

\[
c = H_0(1 + d), \quad H_0 = \frac{\epsilon_1 E_{02}}{\sigma_{*}} - 1 = \frac{\epsilon_2 E_{02}}{\sigma_{*}} > 0. \quad (29)
\]
\[
p = 1 + H_1, \quad H_1 = \frac{(n_2 - 1)(H_2 + 1)}{1 + n_2H_0}. \quad (30)
\]
\[
q = p + \frac{1}{\ln(\epsilon_2) + \ln(H_2/H_0)} \left[ \ln (1 + A_2) + \ln \left( \frac{H_0}{H_2} \right) \right] \quad (31)
\]
\[
d = \frac{1}{q - 1} \quad (32)
\]
where
\[
A_2 = \frac{(n_2 - 1)^2(H_2 - H_0)}{(1 + n_2H_0)(1 + n_2H_2)}. \quad (33)
\]
and
\[
H_2 = \frac{\epsilon_2 E_{02}}{\sigma_{*}} - 1 > 0. \quad (34)
\]

C. The \( \epsilon_2 \leq \epsilon \leq \epsilon_u \) Case (Third Stage)

The final stage of the three-stage formulation is easy to invert as it is a linear equation. The linear assumption was a result of the observation made by Olson [13], who found that there is little variation on the slope of the stress-strain curve beyond \( \sigma_2 \). Therefore, solving the simple linear equation, we obtain the following exact inversion formulation for the third stage:

\[
\sigma = \frac{a + b\epsilon}{1 + \epsilon}. \quad (35)
\]

IV. RESULTS AND COMPARISONS

The proposed three-stage inversion formula is now given by:

\[
\sigma (\epsilon) = \begin{cases} 
E_{02}(\epsilon + D\epsilon) & 0 \leq \epsilon \leq \epsilon_{02} \\
E_{02}(\epsilon + D\epsilon) + \sigma_{02} & \epsilon_{02} < \epsilon \leq \epsilon_2 \\
\frac{a + b\epsilon}{1 + \epsilon} & \epsilon_2 < \epsilon \leq \epsilon_u.
\end{cases} \quad (36)
\]
where the model parameters are expressed in terms of the material properties as given in the previous section. In this section, the accuracy of the inversion in representing the full-range stress-strain formulation is tested. Although all the 15 experimental cases considered in Rasmussen [7] have been used in this study, only the results related to the four cases presented on Table 1 are reported here. All the other cases lead to very similar results.

Table 1

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The stress-strain curves for the four cases in Table 1 are depicted in Figures 1-4. The solid lines represent the stress-strain curves obtained by numerically solving the three-stage stress-strain relation of equation (8). The dashed lines represent the stress-strain curves obtained using the proposed full-range inversion formulation of equation (36). The figures demonstrate the excellent performance of the proposed inversion. In all the cases considered in this study, the computed maximum deviation from the actual full-range stress-strain relation was relatively low. Therefore, the proposed inversion formula is applicable for full-range stress levels.
V. CONCLUSION

An approximate inversion of a full-range stress-strain relation has been presented. The inversion is derived using a rational function assumption on the deviation of the three-stage stress-strain curve from the elastic linear law. The parameters arising from the assumption are determined by a matching process of the stress and its derivatives with their respective values of the three-stage stress-strain curve.

The performance of the proposed inversion formula is tested by comparing its stress value with the stress obtained using the fully iterated numerical solution of the full-range stress-strain relation. The comparison demonstrates that the inversion formula is in good agreement with the full numerical solution for a wide range of material properties.

The application of the new inversion formula in the analysis and evaluation of quantities of interest such as the construction of analytic smooth hysteresis will be investigated in later work.

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REFERENCES