Evaluation and Control of Forces Acting on Isolated Friction Pendulum

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Abstract: - In this paper are studied different aspects of isolated structures using the friction pendulum system and are subjected to earthquake ground motion. Friction pendulum isolators are structural joints that are installed between a structure and its columns. It is simple in design but with many properties and the flexibility to select any isolator period makes the approach suitable for a wide range of applications. In this case the vertical dynamics, large isolator deformation and bidirectional input motion are considered. In spite of the work already done to understand the dynamic behavior of structures isolated with the friction pendulum system, there are still important aspects regarding the behavior of the structures that need further investigation and the purpose is to minimize damage caused by high level of forces developed during earthquakes.

Key-words: friction pendulum, vibration, normal forces, statistical methods, base isolation

1 Introduction

Base isolation systems have become a significant element of a structural system to enhance reliability during an earthquake, and the research conducted on the characteristics of the Friction Pendulum System isolator has been published by Fenz and Constantinou in 2008 [1-2].

Through extensive experimental and numerical studies, the FPS isolator has been proven to be an efficient device for reduction of the seismic responses of structures [3–5]. One type of base isolation system is Friction Pendulum Bearings (Fig 1).

Fig.1 Friction pendulum system
Over the past 10 years the friction pendulum system has been studied analytically and experimentally by a number of researchers. Earlier studies developed simplified models capable of representing the behavior of these isolators. These models led to results in agreement with measured global responses of shake table tests.

The Friction Pendulum System consists of a spherical stainless steel surface and a lentil-shaped articulated slider covered by a Teflon-based high bearing capacity composite material. One of the most relevant properties of the friction pendulum system is that residual displacements in the isolation are reduced due to the self-centering action induced by the concave spherical surface. Residual displacements are an important drawback of other sliders in general. The contact pressure among isolators due to overturning or vertical input motion may induce coupled lateral-tensional vibrations and uplift that need to be better understood and given the powerful computational tools available today it is possible to formulate an exact solution of the motion of system isolated with the help of friction pendulum system.

2 Principles of operation of friction pendulum system

Friction Pendulum systems are based on achieving a pendulum motion where geometry and gravity achieve the desired seismic isolation properties and the result is a simple and stable seismic response. The Fig. 2 [6] presents a body diagram of the isolation system with the force \( F \) that is needed to produce a displacement \( u \).

The force \( F \) is given by the equations of vertical force (1) and horizontal force (2):

\[
G - N \cos \theta + F_j \sin \theta = 0 \quad (1)
\]
\[
F - N \sin \theta - F_j \cos \theta = 0 \quad (2)
\]

By multiplying equations (1) and (2) with \(- \sin \theta\) respectively with \( \cos \theta \) we obtain:

\[
-G \sin \theta + N \sin \theta \cos \theta - F_j \sin^2 \theta = 0 \quad (3)
\]
\[
F \cos \theta - N \sin \theta \cos \theta - F_j \cos^2 \theta = 0 \quad (4)
\]

These two equations can be simplified:

\[
-G \sin \theta + F \cos \theta - F_j (\sin^2 \theta + \cos^2 \theta) = 0 \quad (5)
\]

And so, we obtain the formula that defines \( F \):

\[
F = \frac{G}{R \cos \theta} u + \frac{F_j}{\cos \theta} \quad (6)
\]

For small displacements \( F \) can be defined by the below equation:

\[
F = \frac{G}{R} \cdot u + \text{sgh}(\ddot{u}) \cdot G \cdot \mu \quad (7)
\]

where

\( \ddot{u} \) is the vertical load
\( \mu \) is the coefficient of friction.

From geometrical reasons the vertical displacement is:

\[
v = (R - h) \left( 1 - \sqrt{1 - \frac{u^2}{(R - h)^2}} \right) \quad (8)
\]

or

\[
v = (R - h) - \sqrt{(R - h)^2 - u^2} \quad (9)
\]

The potential energy \( E_p \) accumulated during the lift-up can be expressed as the product of weight \( G \) and vertical displacement \( v \):

\[
E_p = G \left[ (R - h) - \sqrt{(R - h)^2 - u^2} \right] \quad (10)
\]

the energy \( E_{DE} \) dissipated for one cycle of sliding is estimated as:

\[
E_{DE} = 4 \mu Gd \quad (11)
\]
The isolator period is controlled by the selection of the radius of curvature, $R$, of the concave surface. The natural period of vibration of a rigid structure supported on Friction Pendulum bearings [6] is determined from the pendulum equation:

$$T = 2\pi \sqrt{\frac{R}{g}}$$

(12)

where $g$ is the acceleration of gravity.

When the earthquake forces are below the friction force level, a Friction Pendulum supported structure responds like a conventionally supported structure, at its non-isolated period of vibration. Once the friction force level is exceeded, the structure responds at its isolated period, with the dynamic response and damping controlled by the bearing properties. The operation of the bearing is the same whether the concave surface is facing up or down.

3 Mathematical formulation

The mathematical formulation has been given below for an N-storey shear building isolated by Friction Pendulum System, although the formulation can be easily extended for general 3-dimensional structures. Due to the action of frictional forces at the sliding surface, the motion consists of two phases namely, non-sliding phase and sliding phase. The equations [7] of motion are different in the two phases and the overall behavior consisting of a random series of sliding and non-sliding phases is highly non-linear. Depending on the phase of motion, the corresponding equations govern the response of structure and equipment.

3.1 Non-sliding phase

In this phase the structure behaves as a normal fixed base structure. Because of the frictional resistance [8] there is no relative movement between the base mass and the ground and the equations of motion governing this phase are:

$$M\ddot{x} + C\dot{x} + Kx = -M\dot{x}_g$$

(13)

and

$$\dot{x}_b = \dot{x}_g = 0 \text{ and } x_b = \text{constant}$$

(14)

with

$$\left| \sum_{i=1}^{N} m_i (\ddot{x}_i + \dot{x}_g) + m_b \ddot{x}_g \right| < m_b \mu g$$

(15)

3.2 Sliding phase

In this case the structure will start sliding when the forces on the system exceed the static frictional force leading to motion of the mass base, and so the structure has one additional degree-of-freedom [9]. The equations of motion are given by:

$$M\ddot{x} + C\dot{x} + Kx = -M\dot{x}_g$$

(16)

for the structure, and

$$\sum_{i=1}^{N} m_i (\ddot{x}_i + \dot{x}_g) + m_b \ddot{x}_g + m_b \dot{x}_g \text{sgn}(x_b) = 0$$

(17)

for the mass base.

4 The response of isolated structure

Many structures support sub-structures or secondary systems and equipments whose safety and functional integrity during earthquake ground motions is essential. A low-mass secondary structure responds to the acceleration of its supporting floor similar to response of primary structure to ground excitations. The floor accelerations at the support points of the secondary system are typically narrow-banded due to filtering effect of the structure. As a result, the response characteristics of secondary systems are strongly influenced by their dynamic properties relative to those of the primary structure, such as frequency ratio (ratio of frequency of secondary system to the fundamental frequency of the primary system), mass ratio (ratio of mass of the secondary system to the mass of the primary system) and damping in the secondary system. The secondary systems exhibit highly amplified responses when the frequency ratio is nearly equal to unity. Isolating the primary system changes the dominant frequency of excitation at the base of the secondary system as well as reduces excitation amplitude. In this paper the response of light equipment mounted on top of an example shear structure has been considered. The structure consists of a five-storey shear building (excluding base degree of freedom), 5m square in plan and the different story’s, including the base mass, have equal mass. The equipment has been modeled by a mass with linear spring and a damper attached to the top floor of the structure. The structural and equipment damping are both assumed to be equal to 5% of critical damping to eliminate the influence of non-classical damping from this evaluation. The coefficient of friction has been taken as 0.02. [11]
4.2 Time-history response

The response of structure-equipment system has been obtained by solving Eq. (13) or Eq. (16) depending on the phase of motion. The equipment frequency is chosen as 3.85 Hz since it corresponds to tuning with the second natural frequency of the isolated structure (first frequency is the isolator frequency) and represents the most severe case of tuning. The typical time-history plot of absolute acceleration and displacement of the equipment relative to the floor are given in Fig 3(a) and (b) (developed by Mural Pranesh). It can be observed from the time-history responses that there is considerable reduction in the peak response of the equipment in comparison with both the equipment on fixed-base structure and structure isolated by Friction Pendulum System [12].

![Time-history response](image)

Fig. 3 Typical response and energy time-histories of light equipment mounted on example structure

4.2 Floor response

The maximum response of single-degree-of-freedom equipment can be conveniently studied in terms of its floor response spectra. The floor response spectra enable one to evaluate the effectiveness of various isolation systems for secondary systems with different properties. The displacement spectra are normalized with respect to the peak displacement of equipment mounted on fixed-base structure (equal to 0.28 m). In case of a fixed-base structure the equipment acceleration response is maximum when the equipment frequency tunes with fundamental frequency of the structure (approximately 1.85 Hz) [13].

![Floor response spectra](image)

Fig. 4 Floor response spectra for light equipment on example structure developed by Mural Pranesh[14]
5 Testing and results
The first sequence in the experiment testing was to get the response of the structures under free vibration. An equal drift was applied to the top of both structures with and without base isolation system. The force was released and the structures allowed to oscillate until the natural damping of the structures brought the system to stop. The accelerometers recorded the acceleration that each structure experienced until they stopped oscillating. Fig. 4 [2] shows the responses of these two structures.

The second sequence of experiment was the forced vibration of the structures. The shake table was loaded with an increasing acceleration (sweep) for a period of 10 seconds. The acceleration of each structure was recorded and the responses of both structures were recorded, as shown in Fig. (developed by Constantinou and Mokha) [2].

The damping ratio for the system is estimated using the logarithmic decrement method which is the ratio of two consecutive peaks of acceleration-time history over one period. Fig. 6 [18] shows a general response of a SDOF system with two successive peak values to calculate the damping:

\[ m\ddot{u}(t) + c\dot{u}(t) + ku(t) = 0 \]  

where \( m \) is the mass of the system, \( u(t) \) is displacement, \( c \) is damping coefficient, and \( k \) represents the stiffness of the system. The damping ratio is estimated by combining the differential equation and the logarithmic decrement method, as follows:

\[ \xi = ln\left(y_1/y_2\right)/2\pi \]  

where \( y_1 \) and \( y_2 \) are the amplitudes of two consecutive peaks.

![Fig. 4. Responses of the model structures under free vibration](image)

![Fig. 5. Responses of the model structures under forced vibration](image)

![Fig. 6. Response of a SDOF system with free vibration](image)

6 Conclusion
The objective of this study was to model the concept of base isolation systems and to verify the forces that appear and their behavior due to a seismic event. The difference in responses was measured by comparing two identical model structures, one with a Friction Pendulum Bearing base isolation system and one without any base isolation. After extensive testing on a Shake Table, the structure with the Friction Pendulum Bearing base isolation showed a significant decrease in lateral acceleration due to varying lateral forces, as expected. Simulation results indicate that responses of the isolated bridges without constraint can be drastically reduced with acceptable bearing displacements. Responses of the isolated bridge can be amplified and the bearings displace excessively, if the bridge superstructure stays entirely unconstrained.
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