Using Fuzzy Techniques for Students’ Evaluation

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Abstract: - The learning evaluation process tries to establish the degree of students’ knowledge and competence assimilation. There are many learning evaluation methods from both pedagogical and system approach point of view. In this work we present a students’ system evaluation based on fuzzy techniques. The proposed algorithm implementation is realized in Java. The importance and the role of fuzzy evaluation topics in the domain of evaluation of learning performance are discussed. Some connections and applications of these topics with the content of consecrated topics in a university environment examination are explored. A special emphasis is given to the applications of classical and fuzzy logic in the process of aggregation of specialist opinions.

Key-Words: - Fuzzy sets, fuzzy evaluation, learning performance, competence, students’ evaluation

1 Introduction
Students’ evaluation should be based on objective criteria. The evaluation has to reflect students’ learning performance, should be correct and fair. The evaluation objectives are determined by the degree of knowledge assimilation and by the establishment of the different levels of knowledge and competence exhibited.

An evaluation system may contain different forms of knowledge verification, quantified by different algorithms. This work, which utilizes fuzzy sets, takes into consideration experts’ opinion that is reflected by the allocation of importance coefficients to the tests, which constitute the evaluation object.

Dealing with imprecision and system complexity requires to rely on experts as a source of knowledge. Experts’ opinion and elicitation methods are important issues. The aggregation of the opinions can affect the quality of the final outcome. The expert elicitation process can be with or without consensus. If there are no consensuses then equal weights or non-equal weights can be assigned. If there is consensus, then experts agree that a particular probability distribution represents their view or each expert believes in same deterministic value or model.

2 Main Concepts
Fuzzy logic is a logical system that generalizes the classical (two-valued) logic and includes many-valued logic, i.e. it’s a body of concepts, constructs and techniques which relates to modes of reasoning which are approximate rather than exact in order to
deal with the imprecision of information [1], [4]. A proposition is viewed as a fuzzy constraint and membership in a set is a matter of degree [7]. Fuzzy logic plays a pivotal role in AI to knowledge representation and to make inferences from information that is imprecise, incomplete, uncertain or partially true [3], [5].

In order to study imprecise knowledge (not inexact), we can use certain membership functions. The values of these functions, contained within the interval [0, 1] will indicate the degree of membership of an element to a fuzzy set.

**The fuzzy sub-sets** of space X are in one-to-one correspondence with the richer category of functions \( \{\chi: X \rightarrow [0, 1]\} = [0, 1]^X \), which we call membership functions, and with which they will be identified and labeled A, B, C, ...

A fuzzy sub-set of X is a sub-set \( F \subseteq X \times [0, 1] \), such as:

(i) \( pr_{ox}(F) = X \);
(ii) \( (x, y_1) \in F \) and \( (x, y_2) \in F \rightarrow y_1 = y_2 \).

where \( pr_{ox} \) is the projection along axis 0x.

Therefore, \( F \in [0, 1]^X \), \( x \rightarrow F(x) \in [0, 1] \). We also write \( F = (x, F(x)) \), where F is called the membership function.

It is important to note the net difference between the theory of probability and the mathematical techniques that operate with fuzzy concepts [8]. The problematic “random element”, for example, results from incertitude as to its belonging or non-belonging to a classical set, whereas a fuzzy phenomenon typically shows the existence of various degrees of belonging. The notion of belonging then does not play the role here that it has traditionally played in the theories based on classical sets. It makes no sense, where a fuzzy set F is concerned, to state whether or not x belongs to F.

### 3 A Decision Support System Using Fuzzy Sets

The objective of this work is to develop a decision support system to find the best students’ results, which satisfied certain criteria.

The method based on fuzzy sets allows to order a set with \( n \) elements, taking into account simultaneously \( m \) decision criteria. Let be \( e(1), e(2), ..., e(n) \), the compared elements and \( c(1), c(2), ..., c(m) \) the decision criteria. We denote by \( a(i,j) \) the mark assigned to element \( e(i) \) applying criterion \( c(j) \). Therefore, for each element \( e(i) \) we can obtain a vector of marks \( v(i) = (a(i,1), a(i,2), ..., a(i,m)) \). Let’s consider an other element \( e(k) \) with its vector of marks \( v(k) = (a(k,1), a(k,2), ..., a(k,m)) \). The problem of ordering is reduced to compare the vectors \( v(i) \) and \( v(k) \) to be able to decide the priority order between the two vectors: either \( e(i) < e(k) \) or \( e(i) > e(k) \).

If this decision cannot be taken then we say that these vectors are in indifferent relation: \( e(i) I e(k) \). As a preliminary stage one has to establish the importance coefficients \( k(j) \) attached to each criterion \( c(j) \). In order to obtain these coefficients, a group of experts (professors, psychologists, managers, etc.) are consulted. To each expert one assigns a certain weight regarding its competence. One computes the weighted average (mean) of proposed coefficients by each expert. One obtains then the importance coefficients of each criterion, which are well substantiated.

In order to illustrate the method let’s consider the following example: suppose to be admitted to a certain university, there are three tests to pass regarding the disciplines \( c(j) \), \( j = 1, 3 \) with the importance coefficients \( k(j) (k(1) = 5, k(2) = 4, k(3) = 3) \). The candidates \( e(i) \), \( i=1,5 \) have obtain the marks, \( MAR(i,j) \), presented in Table 1.

<table>
<thead>
<tr>
<th>candidate</th>
<th>c(1)</th>
<th>c(2)</th>
<th>c(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>e(1)</td>
<td>6</td>
<td>10</td>
<td>8</td>
</tr>
<tr>
<td>e(2)</td>
<td>9</td>
<td>8</td>
<td>10</td>
</tr>
<tr>
<td>e(3)</td>
<td>10</td>
<td>5</td>
<td>7</td>
</tr>
<tr>
<td>e(4)</td>
<td>8</td>
<td>9</td>
<td>10</td>
</tr>
<tr>
<td>e(5)</td>
<td>8</td>
<td>10</td>
<td>9</td>
</tr>
</tbody>
</table>

The goals obviously are to obtain the highest possible marks.

In order to find the solution, the first step is to determine the relative values of the marks, REL(i, j). The relative values are defined as follows:

- when the optimal solution is provided by the smallest value of the indicator, one reports this value to the values of the other indicators;
- when the optimal solution is provided by the largest value of the indicator, the relative value is obtained as the relation between the value attached to each alternative and the largest value.

We are interested by the latest situation. The relative values will be obtained by the relation between the value of the student mark and the maximum value, which in our case is 10.

By interpreting the considered indicators as goals (the relative marks have to be as largest as possible) one obtains three fuzzy sets. For the fuzzyfication
process, in practice, one can consider a membership function of exponential type given by the formula:

$$\Omega(j) = \exp(-k(j) |X(i,j)|) \quad (1)$$

where $k(j)$ are the importance coefficients.

The variable $X(i,j)$ is given by the formula:

$$X(i,j) = |Y(i,j) - Y1(j)|/Y1(j) \quad (2)$$

Here, $Y(i,j)$ is the value of the indicator $c(j)$ for the candidate $e(i)$ and $Y1(j)$ the optimal values for the candidates $e(i)$.

To obtain the decision regarding the best student’s results, we proposed an algorithm that was implemented in Java [2], [6]. We can extend the number of candidates, tests and importance coefficients to how many are needed.

The input data are:

- MAR[i,j] //The marks table
- k [i] //The importance coefficients
- nrCrit //Number of decision criteria
- nrCand //Number of elements to be compared

The data was imported from a text file.

The output data are:

- REL[i,j] //The relative values of the marks
- max //The maximum value
- Y1[i] //The optimal values for the candidates $e(i)$
- X[i,j]
- $\Omega[i,j]$ //The fuzzy values
- D[i] //The decision set

The implementation of the algorithm is based on the next sequence:

- read input data;
- calculate the maximum value;
- calculate the relative values;
- calculate the optimal value for each candidate;
- calculate $X(i,j)$;
- calculate the omega values and
- calculate the decision set.

The interface after the data was imported is presented in Figure 1.

Fuzzy decision is by definition the intersection of goals and constrains described by fuzzy sets [3]. In our case one has only goals.

Finally, the decision is the fuzzy set which contains the minimum omega values for each element:

```
for (int i = 0; i < nrCrit; i++)
    for (int j = 0; j < nrCand; j++)
        if (i==1) // the decision set is:
            D[i] = Double.min(D[i], REL[i,j]);
```

The results corresponding to our example, obtained in each step, are presented in Figure 2.
Therefore, the decision will be the fuzzy set:

\[ D = \{(e(1), 0.135352832), (e(2), 0.4493289641), \]
\[ (e(3), 0.135352832), (e(4), 0.3678794412), (e(5), \]
\[ 0.3678794412)\} \quad (3) \]

One element, which maximizes \( D \), will be called the maximal decision. In our case that element will be \( e(2) \) and the hierarchy of candidate will be:

\[ e(2) > (e(4) \text{ I } e(5)) > (e(1) \text{ I } e(3)). \]  

(4)

4 Conclusions

The increase of exactingness concerning the students' preparation level leads to the study of more sensitive criteria of appreciation of their knowledge. This student's classification (ordering) is necessary in many stages of university evaluation: admission, exams (learning performance), specialization option, awards, hiring, job selection, etc. Compared with the refinement and fineness of today's decision models the weighted average is far to be satisfying. Using fuzzy sets techniques represents one of the alternatives of modern decision approaches.

For future work we propose the improvement of the program such that the data input (students, tests, marks, etc.) to be realized by the help of some forms. The obtained results will be compared with the results proposed by other evaluation methods.

References:


