Model of Atmospheric Optical Channel with Scattering

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Abstract: The paper evaluates the effect of multipath propagation on broadband terrestrial free-space optical links. At 10 Gbs the pulse duration is 0.1 ns for binary signaling and multipath propagation during fog events may affect the channel bandwidth and cause intersymbol interference in the receiver. The analysis is based on single-scattering approximation and semi-analytical solution, which is computationally less expensive in comparison with Monte-Carlo methods. Simulation results are provided for typical continental fog parameters and a typical scenario for municipal links.

Key-Words: Free-space optical links, Atmospheric scattering, ISI

1 Introduction
The growing demand for multi-gigabit-class wireless broadband terrestrial communication leads to the utilization of radio frequency (RF) bands above 40 GHz for radio systems and infrared bands for Free-Space Optical (FSO) systems. Many propagation studies have shown that near carrier-grade availability can be obtained by combining FSO links with matched-rate millimeter RF links [1]. Phenomena that degrade RF link performance and FSO link performance are (almost) mutually exclusive near the ground.

Technology for 1.25 Gbs (Ethernet) hybrid FSO/RF municipal systems operating at 850 nm or 1550 nm optical windows and 60 GHz to 100 GHz RF bands for paths of several kilometers is now available from a few vendors. The next technology step is characterized by data rates of 10 Gbs and above, which requires careful characterization of the atmospheric communication channel.

Short-haul municipal FSO systems are characterized by a sufficient link margin to accommodate large attenuating effects of clouds and fog. Therefore the effect of atmospheric scintillation during clear-sky periods is not so strong. For low data rates the atmospheric channel can be modeled as a channel with slowly-varying attenuation, which can reach values of the order of hundreds of decibels per kilometer [1], [2].

Besides increased attenuation, scattering due to fog also imposes multiple propagation paths, which causes temporal broadening (dispersion) of transmitted pulses, resulting in intersymbol interference (ISI). The ISI introduces a certain power penalty, i.e. the real BER performance of an FSO link differs from laboratory experiments, where the signal is simply attenuated by grey filters. The effect significance grows with data rate.

The effect of time-domain dispersion has been studied for ground-to-air/satellite communication through clouds [4], [5] mainly for military purposes, where the analyzed scenario expected the utilization of multi-kilowatt pulses. Unfortunately, only few papers deal with an analysis of realistic scenarios for terrestrial FSO systems, where the transmitted power is limited by eye-safety regulation and affordable laser sources.

The present paper deals with an analysis based on single-scattering approximation and semi-analytical solution, which is computationally less expensive in comparison with Monte-Carlo methods [8]. Section 2 describes the model of the atmospheric channel. Section 3 presents simulation results.

2 Model of terrestrial FSO system
2.1 Power budget
Let us consider the FSO terrestrial link in Fig. 1. The mean optical power $P_{m,\text{RXA}}$ on the receiving aperture is given by the power budget equation (in decibels)

\begin{equation}
P_{m,\text{RXA}} = P_{m,\text{TXA}} - \alpha_{\text{sys}} - \alpha_{\text{atm}}
\end{equation}

where $P_{m,\text{TXA}}$ is the mean optical power on the transmitting aperture. The system attenuation $\alpha_{\text{sys}}$ includes all constant losses and gains, which depend only on transceiver design and path length $L_{12}$. Attenuation $\alpha_{\text{atm}}$ represents all random losses caused by atmospheric phenomena.
Fig. 1 FSO link in scattering media.

The dominant part of atmospheric attenuation can be expressed as a sum of scattering on hydrometeors (fog) $\alpha_{sc}$ and power penalties of turbulence $\alpha_{turb}$ and ISI $\alpha_{ISI}$ at the receiver

$$\alpha_{atm} \approx \alpha_{1,sc}L_{12} + \alpha_{turb} + \alpha_{ISI}(\alpha_{1,sc})$$

(2)

where $\alpha_{1,sc}$ is the specific attenuation coefficient, which is 0.5dB/km for the standard clear atmosphere and might reach 400dB/km during fog [2].

The effects of turbulence and ISI are modeled by introducing additional attenuation $\alpha_{turb}$ and $\alpha_{ISI}$ representing how much the transmitted power should be increased to get an error performance similar to the ideal environment [6], [9]. As a consequence, the occurrence of ISI, which depends on $\alpha_{1,sc}$ and link parameters, decreases the link margin.

### 2.2 Fog properties

Light propagating through fog is scattered on water droplets. As the droplet diameter is comparable to wavelength, the process is described by the Mie theory. The type of fog is characterized by particle size distribution (number of particles per unit volume ($\text{cm}^{-3}$) per unit increment of radius ($\mu$m)), which is usually approximated by the modified gamma distribution [2]

$$n(r) = a r^b e^{-a r}$$

(3)

where $r$ is the particle radius, and $a$, $b$, and $a$ are coefficients, Table I. Radiation (continental) fog generally appears during the night and at the end of the day, particularly in valleys. Advection (maritime) fog is formed by the movement of wet and warm air masses above the colder maritime or terrestrial surfaces. The table gives typical values. Actual fog parameters may vary significantly for individual fog events [2].

The optical pulse consists of a number of photons (packets of energy) that interact with water droplets of fog. When a photon interacts with a droplet, it is absorbed or randomly deflected from the original direction. The fog density is characterized by the mean distance $\bar{d}$ between two interactions. The optical thickness of fog is defined as

$$\tau = L_{12} / \bar{d}$$

(4)

#### Table I Fog model parameters [3]

<table>
<thead>
<tr>
<th>Type</th>
<th>$a$</th>
<th>$b$</th>
<th>$\alpha$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dense advection fog</td>
<td>0.027</td>
<td>0.3</td>
<td>3</td>
</tr>
<tr>
<td>Moderate advection fog</td>
<td>0.066</td>
<td>0.38</td>
<td>3</td>
</tr>
<tr>
<td>Dense radiation fog</td>
<td>2.37</td>
<td>1.5</td>
<td>6</td>
</tr>
<tr>
<td>Moderate radiation fog</td>
<td>607.5</td>
<td>3</td>
<td>6</td>
</tr>
</tbody>
</table>

The distance between two scatterings is an exponential random variable whose PDF and CDF are

$$f_d(d) = \exp(-d / \bar{d}) / \bar{d}$$

$$F_d(d) = 1 - \exp(-d / \bar{d})$$

(5)

(6)

The probability that a “ballistic” photon traveling along the optical axis reaches the receiver without being scattered is

$$P(d > L_{12}) = 1 - F_d(L_{12}) = \exp(-\tau)$$

(7)

Thus the attenuation due to fog measured with a receiver with narrow field of view will be

$$\alpha = \tau 10 \log e = 4.34 \tau \text{[dB]}$$

(8)

which is, in fact, the well-known Beer-Lambert law.

When a photon interacts with a droplet, it is absorbed with the probability

$$P_{abs} = 1 - a$$

(9)

where $a$ is the single particle albedo. Otherwise it is scattered at a random angle $\theta$ with arbitrary azimuthal rotation $\varphi$, Fig. 2, [7].

#### Fig. 2 Scattering event.

The azimuth angle is uniformly distributed in $[0, 2\pi]$. The PDF of scattering angle $\theta$ can be obtained from the Mie phase function $P(\theta)$ as

$$f_\theta(\theta) = P(\theta) \sin(\theta) / 2$$

(10)

The phase function depends on the wavelength and spectrum of droplet diameters. It can be obtained together with the albedo, using, for example, the MiePlot software [10].

### 2.3 Single scattering model

Let us assume the most probable mechanism is a single scattering, i.e. a collision with at most one droplet along the path. For simplicity, the scattering volume will be constrained conically by the beam divergence half-angle $\theta_{div}$ and the receiver field-of-
view half-angle $\theta_{RX}$, Fig. 3.

![Fig. 3 Worst-case scenario.](image1)

The greatest difference between the path lengths of ballistic and scattered photons is depicted in Fig. 3. The worst-case difference of arrival times is

$$t_{d, \text{max}} = \frac{d_1 + d_2 - L_{12}}{c} = \frac{L_{12}}{c} \left( \sin \theta_{TX} + \sin \theta_{RX} \right),$$

(11)

where $c$ is the speed of light. Fig. 4 shows the results for $L_{12} = 1\text{km}$ and typical values of the half-angles.

![Fig. 4 Worst-case limit for pulse broadening. (single scattering approximation, Fig.3)](image2)

Fig. 5 shows a model for the computation of statistics of arrival times. For a chosen time difference $t_d$ between arrivals of ballistic and scattered photons the paths $d_1$ and $d_2$ must satisfy the condition

$$d_1 + d_2 - L_{12} = \Delta_L = c t_d.$$  

(12)

For a constant $t_d$, the distance $d_1$ to the first scattering must lie within the range $[d_{1\text{min}}, d_{1\text{max}}]$, which corresponds to the boundaries of the scattering volume, see Fig. 5.

![Fig. 5 Geometry of scattering volume.](image3)

$$d_{1\text{min}} = \frac{L_{12} + \Delta_L / 2}{1 + \frac{L_{12}}{\Delta_L} (1 - \cos \theta_{TX})}$$  

(13a)

$$d_{1\text{max}} = L_{12} + \Delta_L - \frac{L_{12} + \Delta_L / 2}{1 + \frac{L_{12}}{\Delta_L} (1 - \cos \theta_{RX})}$$  

(13b)

For $d_1 \in [d_{1\text{min}}, d_{1\text{max}}]$ the angles $\theta_1$ and $\theta_2$ in Fig. 5 are

$$\cos \theta_1 = \frac{d_1^2 + L_{12}^2 - d_2^2}{2d_1 L_{12}}, \quad \cos \theta_2 = \frac{d_2^2 + L_{12}^2 - d_1^2}{2d_2 L_{12}},$$

(14)

where $d_3 = L_{12} + \Delta_L - d_1$.

Due to the geometrical rotational symmetry, the probability that the photon delay lies in $[t_d, t_d+dt]$ will be

$$f_{\tau}(t_d)dt_d = \int_{t_{d\text{min}}}^{t_{d\text{max}}} f_{TX}(\theta_1) \frac{d\theta_1}{dt_d} f_d(d_1) P_{RX}(d_1)dt_1,$$

(15)

where $f_{\tau}$ is the PDF of photon delay. Product $f_{TX}d\theta_1$ is the probability of photon occurrence in the annulus $[\theta_1, \theta_1+d\theta_1]$. For the ideal Gaussian beam we obtain

$$f_{TX} = 4 \theta_{TX}^{-2} e^{-2\left(\frac{\theta_{TX}}{\theta_{TX}}\right)^2}.$$  

(16)

For a constant $d_1$ the angle $\theta_1$ is a function of chosen delay $t_d$, i.e. a function of $\Delta_L$. Differentiating (14) with respect to $t_d$ we obtain

$$\frac{d\theta_1}{dt_d} = \frac{c d_2}{d_1 L_{12} \sin \theta_1}.$$  

(17)

The probability of receiving the scattered photon $P_{RX}$ depends on the product of scattered irradiance and the perpendicular projection of the receiving aperture. For a circular aperture with diameter $D_{RX}$ we obtain

$$P_{RX} = f_\theta(\theta_1 + \theta_2) \frac{D_{RX}^2 \cos \theta_2}{8 d_1^2 \sin(\theta_1 + \theta_2)}.$$  

(18)

As all photons are “transmitted” at the same time the PDF of $t_d$ is, in fact, the impulse response of the channel. Several approximations of the pulse have been proposed [8]. The simplest one is the gamma function

$$h(t) = k_1 t \exp(-k_2 t) \quad \text{for} \ t \geq 0$$  

(19)

with coefficients $k_1$ and $k_2$ obtained by curve fitting. The time origin of (19) corresponds to the propagation delay along the line of sight, i.e. $L_{12}/c$.

The normalized frequency response of the channel is then
\[ H_n(\omega) = \frac{1}{(1 + j\omega/k_2)^n} , \]  

(20)

which has a double pole \( p_{1,2} = -k_2 \) and the 3dB bandwidth

\[ f_{-3dB} = k_2 \sqrt{2} - 1/2\pi . \]

(21)

3 Numerical simulation

For subsequent simulations we will use the parameters of the dense continental fog from Table 1. Fig. 6 shows the corresponding Mie phase function for \( \lambda = 850nm \). The path length was \( L_{12} = 1\text{km} \) and attenuation for unscattered photons \( \alpha_1 = 20\text{ dB/km} \).

![Fig. 6 Mie phase function for continental fog.](image)

Table II shows the parameter \( k_2 \) of the gamma function obtained by fitting the theoretical response (19) to the simulated data and the equivalent bandwidth calculated using (22).

<table>
<thead>
<tr>
<th>( \theta_{TX} )</th>
<th>( \theta_{RX} )</th>
<th>( k_2 )</th>
<th>( f_{-3dB} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 mrad</td>
<td>10 mrad</td>
<td>( 2.5 \times 10^{11} )</td>
<td>25.6 GHz</td>
</tr>
<tr>
<td>2 mrad</td>
<td>20 mrad</td>
<td>( 1.3 \times 10^{11} )</td>
<td>13.3 GHz</td>
</tr>
<tr>
<td>5 mrad</td>
<td>20 mrad</td>
<td>( 5 \times 10^{10} )</td>
<td>5.12 GHz</td>
</tr>
<tr>
<td>10 mrad</td>
<td>20 mrad</td>
<td>( 2.55 \times 10^{10} )</td>
<td>2.61 GHz</td>
</tr>
</tbody>
</table>

4 Conclusions

The paper presented a simple methodology for the simulation of multipath propagation in atmospheric optical channel under the single-scattering scenario. The numerical results show that multipath propagation may be an important phenomenon for the next-generation FSO systems operating at 10 Gbs, where the pulse duration is 0.1 ns. The advantage of the semi-analytical solution is low computational cost in comparison with the Monte-Carlo statistical methods.

Acknowledgements

This work was supported by the Czech Science Foundation under projects No. 102/08/0851, No. 102/09/0550, the Czech Ministry of Education under research project No. MSM0021630513. The research leading to these results has received funding from the European Community’s Seventh Framework Programme (FP7/2007-2013) under grant agreement No. 230126.

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