Memristors: A New Approach in Nonlinear Circuits Design

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Abstract: - The conception of memristor as the fourth fundamental component in circuit theory, creates a new approach in nonlinear circuit design. In this paper the complex dynamics of Chua's canonical circuit implemented by using a memristor instead of the nonlinear resistor, was studied. The proposed memristor is a flux-controlled one, described by the function $W(\varphi) = dq(\varphi)/d\varphi$, where $q(\varphi)$ is a cubic function. Computer simulation of the dynamic behaviour of a Chua circuit incorporating a memristor, confirmed very important phenomena concerning Chaos Theory, such us, the great sensitivity of circuit behavior on initial conditions, the route to chaos through the mechanism of period doubling, as well as antimonotonicity.

Key-Words: - Memristor, nonlinear circuits, period doubling, antimonotonicity.

1 Introduction

Until 1971, electronic circuit theory had been spinning around the three, well known, fundamental components: resistors, capacitors and the inductors. It was that year that Leon Chua from the University of California at Berkeley, reasoned from symmetry arguments, that there should be a fourth fundamental element, which he named memristor (short for memory resistor) [1].

As it is known, circuit elements reflect relationships between pairs of the four electromagnetic quantities of charge, current, voltage and magnetic flux. But a link between charge and flux was missing (Fig. 1). Chua dubbed this missing link by introducing memristor and created a crude example to demonstrate its key property i.e. that it becomes more or less resistive (less or more conductive) depending on the amount of charge that has flowed through it.

In 2008, Hewlett-Packard scientists, working at its Laboratories in Palo Alto-California, reported the realization of a new nanometer-scale electric switch, which "remembers" whether it is "on" or "off" after its power is turned off [2]. The memristor created in HP labs, is based on a film of titanium dioxide, part of which is doped to be missing some oxygen atoms. Researchers believe, that memristor might become a useful tool either for constructing nonvolatile computer memory, which is not lost even after the power goes off or for keeping the computer industry on pace to satisfy Moore's law, i.e. the exponential growth in processing power every 18 months.

Recently, Itoh and Chua proposed several nonlinear oscillators based on Chua's circuits. In these implementations Chua's diode was replaced by memristors [3]. Also, in Refs [4] and [5] cubic memristors were used in well known nonlinear

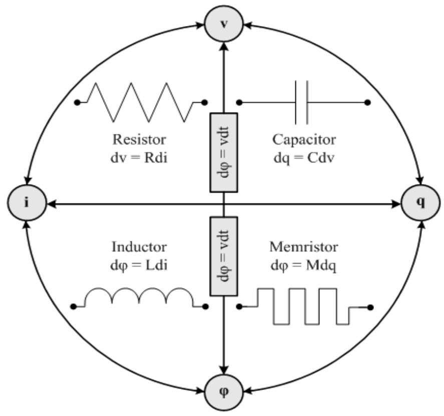


Fig 1. The four basic circuit-element relationship.

circuits. Consequently, it is clear that a new scientific area inside nonlinear circuit theory has started to shape, in a way that nonlinear elements are being replaced by memristors.

In this paper, the study of the simulated dynamic behavior of a canonical Chua circuit, with a cubic memristor, is presented. In Section 2, the proposed system is studied. In Section 3, numerical simulations demonstrate very important phenomena, such us, the great sensitivity of the circuit on initial conditions, the route to chaos through the mechanism of period doubling, and the antimonotonicity. Finally, conclusion remarks are included in Section 4.

2 The Canonical Chua's Circuit with Cubic Memristor

Chua's canonical circuit [6-9] is a nonlinear autonomous 3rd-order electric circuit (Fig. 2). In this circuit G_n is a linear negative conductance, while the nonlinear resistor is replaced by a memristor. The proposed memristor M is a flux-controlled memristor described by the function $W(\varphi(t))$, which is called *memductance*, and is defined as follows:

$$W(\varphi) = \frac{dq(\varphi)}{d\varphi} \quad (1)$$

where $q(\varphi)$ is a smooth continuous cubic function of the form:

$$q(\varphi) = -\alpha \cdot \varphi + b \cdot \varphi^3 \quad (2)$$

with a, b > 0. As a result, in this case the *memductance* $W(\varphi)$ is provided by the following expression:

$$W(\varphi) = \frac{dq(\varphi)}{d\varphi} = -\alpha + 3 \cdot b \cdot \varphi^2 \quad (3)$$

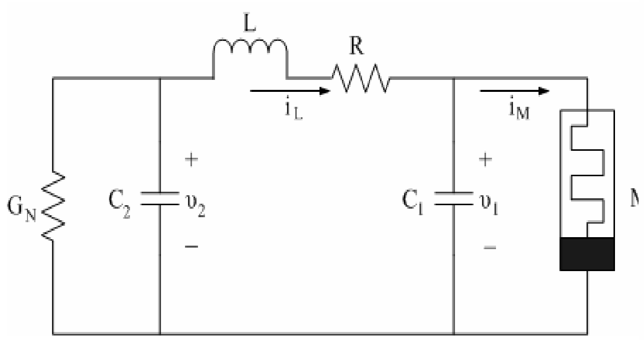


Fig 2. Memristor-based Chua canonical circuit.

The current i_M through the memristor is:

$$i_M = W(\varphi) \cdot \upsilon \quad (4)$$

And the state equations of the circuit are the following:

$$\begin{cases} \frac{d\varphi}{dt} = \upsilon_1 \\ \frac{d\upsilon_1}{dt} = \frac{1}{C_1}\left(i_L - W(\varphi)\upsilon_1\right) \\ \frac{d\upsilon_2}{dt} = -\frac{1}{C_2}\left(i_L + G_N\upsilon_2\right) \\ \frac{di_L}{dt} = \frac{1}{L}\left(-\upsilon_1 + \upsilon_2 - i_L R\right) \end{cases} \quad (5)$$

3 Dynamics of the Circuit

Numerical simulation of the system state equations (5), by employing a fourth order Runge-Kutta algorithm, is presented in this section. The circuit parameters were set to the following values:

R=300Ω, L=100mH, G_n=–0.40mS,
$\alpha = 0.5 \cdot 10^4\, C/Wb$ and $b = 4 \cdot 10^4\, C/Wb^3$

and the initial conditions were set: $(\varphi)_0$=0Wb, $(\upsilon_1)_0$=0.006V, $(\upsilon_2)_0$=0.02V and $(i_L)_0$ =0.001A.

Bifurcation diagrams υ_1 versus C_2 were plotted, for a variety of constant values for capacitance C_1. The comparative study of these bifurcation diagrams provides with a sense of the qualitative changes of the dynamics of the memristor, as C_1 is set to different discrete values. Bifurcation diagram, υ_1 versus C_2, for C_1 = 50nF is shown in Fig. 3. The system remains in a period-1 stable state, as C_2 decreases. In Fig. 4 the period-1 limit cycle can be observed for C_1 = 50nF.

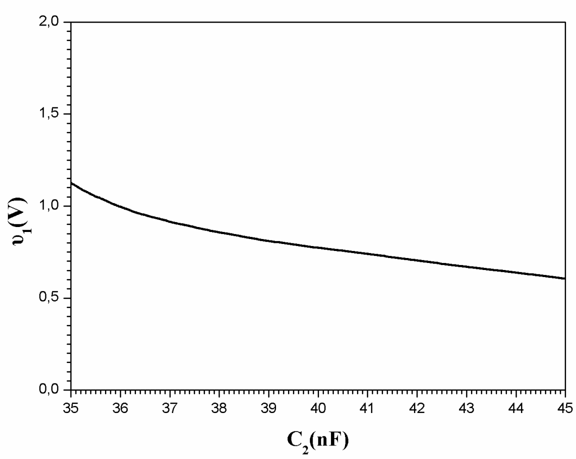


Fig 3. Bifurcation diagram, υ_1 vs C_2, for C_1=50nF.

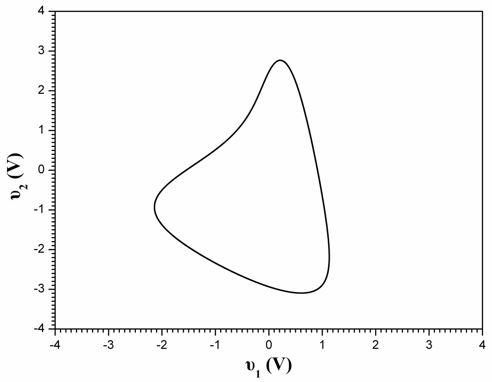


Fig 4. Phase portrait υ_2 vs υ_1, for C_1=50nF.

For C_1=38nF, the bifurcation diagram υ_1 versus C_2, follows the scheme: period−1 → period−2 → period−1, as shown Fig. 5. As C_2 is decreased, the system always remains in a periodic state but two different periodic states are emerging. This scheme is called "primary bubble" [10].

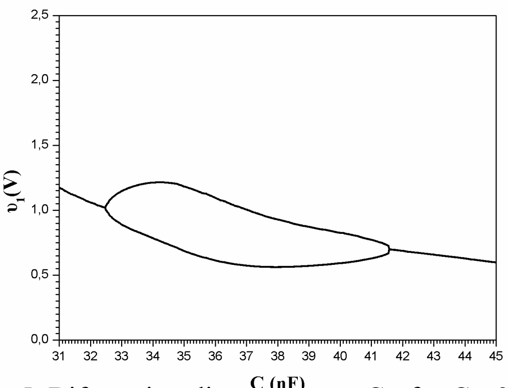


Fig 5. Bifurcation diagram, υ_1 vs C_2, for C_1=38nF.

Bifurcation diagram, υ_1 versus C_2, in the case of C_1=36nF is shown in Fig. 6. The system remains again in a periodic state, but a period−4 state is now formed. In this case the system follows the scheme: period−1 → period−2 → period−4 → period−2 → period−1.

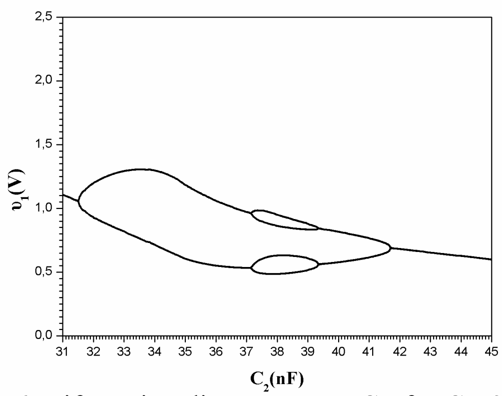


Fig 6. Bifurcation diagram, υ_1 vs C_2, for C_1=36nF.

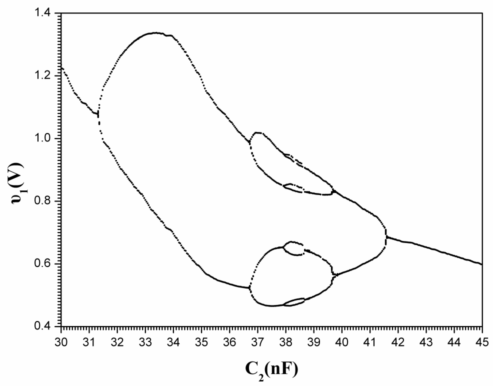


Fig 7. Bifurcation diagram, υ_1 vs C_2, for C_1=35.1nF.

In bifurcation diagram, υ_1 versus C_2, for C_1 = 35.1nF a period−8 is formed, correspondingly, as shown in Fig. 7.

As C_1 is decreased, chaotic regions appear. This could be observed in Fig. 8, where the bifurcation diagram, υ_1 versus C_2, for C_1 = 35nF is presented.

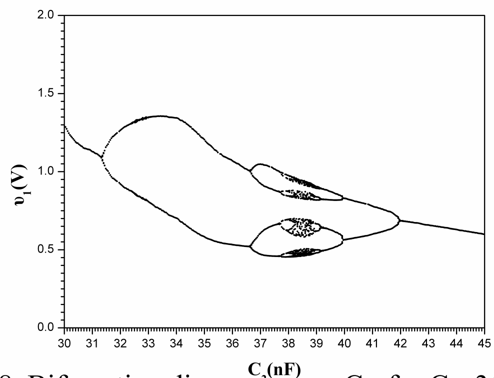


Fig 8. Bifurcation diagram, υ_1 vs C_2, for C_1=35nF.

Apparently, each bubble is now clearly chaotic. These chaotic regions, inside the bubbles, become larger as C_2 is decreased (Fig. 9). For the chaotic bubbles in Figs 8 and 9, the initial and the final dynamic state are in period-1 state and as a result, they are characterized as "*period-1 chaotic bubbles*".

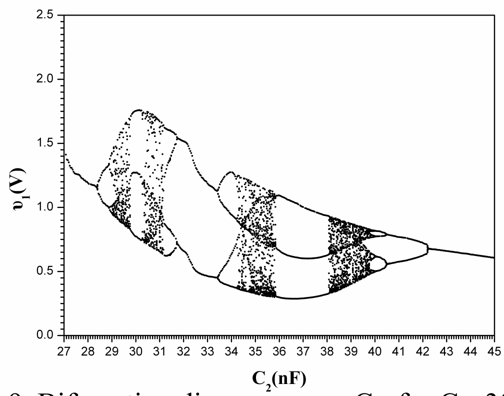


Fig 9. Bifurcation diagram, υ_1 vs C_2, for C_1=30nF.

In general, in many nonlinear dynamical systems, the forward period-doubling bifurcation sequences are followed by reverse period-doubling sequences, as a parameter is varied in a monotone way. This phenomenon is called *antimonotonicity*.

Bier and Bountis, demonstrated that reverse period-doubling sequences are expected to occur, when a minimum number of conditions is fulfilled [10]. The main point was, that a reverse period-doubling sequence is likely to occur in any nonlinear system, provided that there is a symmetry transformation, under which state equations remain invariant.

Indeed, in the case under question, state equations (5) remain invariant under the following transformation:

$$\varphi \rightarrow -\varphi,\ \upsilon_1 \rightarrow -\upsilon_1,\ \upsilon_2 \rightarrow -\upsilon_2,\ i_L \rightarrow -i_L \qquad (6)$$

It has also been demonstrated in the literature, that reverse period-doubling commonly arises in nonlinear dynamical systems that involve the variation of two parameters [10, 11]. In the studied circuit, these parameters appear to be the two capacitances C_1 and C_2. Moreover, what is important is the fact that the period-doubling "trees" should develop symmetrically towards each other, along some line in parameter space.

Reverse period doubling is destroyed for C_2<29.72nF, when C_1=25nF, as demonstrated in Figure 10. It is apparent in this case that the chaotic regions are enlarged. The bifurcation diagram in Fig 10, demonstrates the fact that the memristor follows the period-doubling route to chaos, a very common mechanism in chaotic systems.

In Figure 11, the phase portraits υ_2 versus υ_1 for a route to chaos via period doubling, in the case of C_1 = 25nF, are presented. In Figure 12 the chaotic spiral attractors for C_1 = 25nF and C_2 = 37nF is shown.

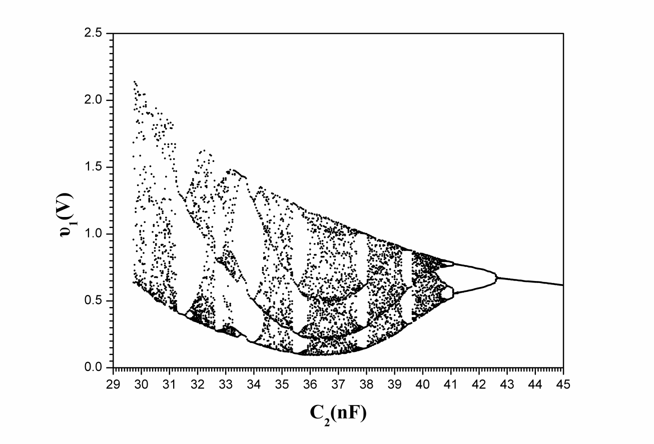


Fig 10. Bifurcation diagram, υ_1 vs C_2, for C_1=25nF.

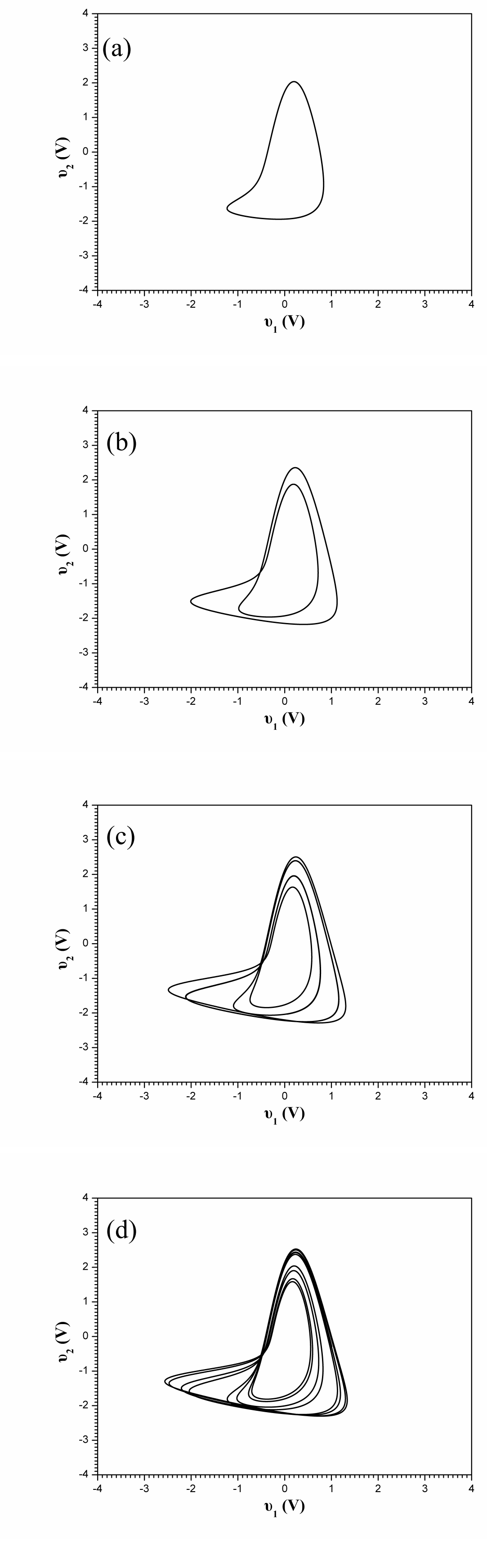


Fig 11. Phase portraits υ_2 vs υ_1, for C_1=25nF and (a) C_2=44nF (period-1), (b) C_2=42nF (period-2), (c) C_2=41nF (period-4), (d) C_2=40.8nF (period-8).

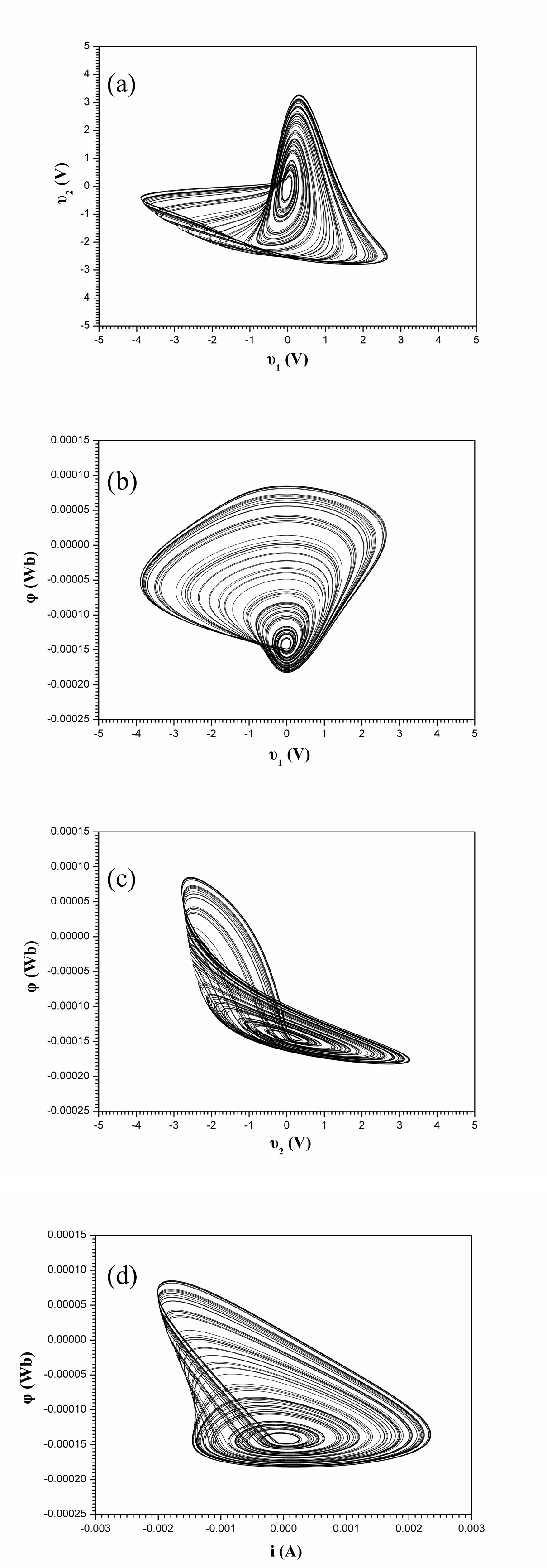
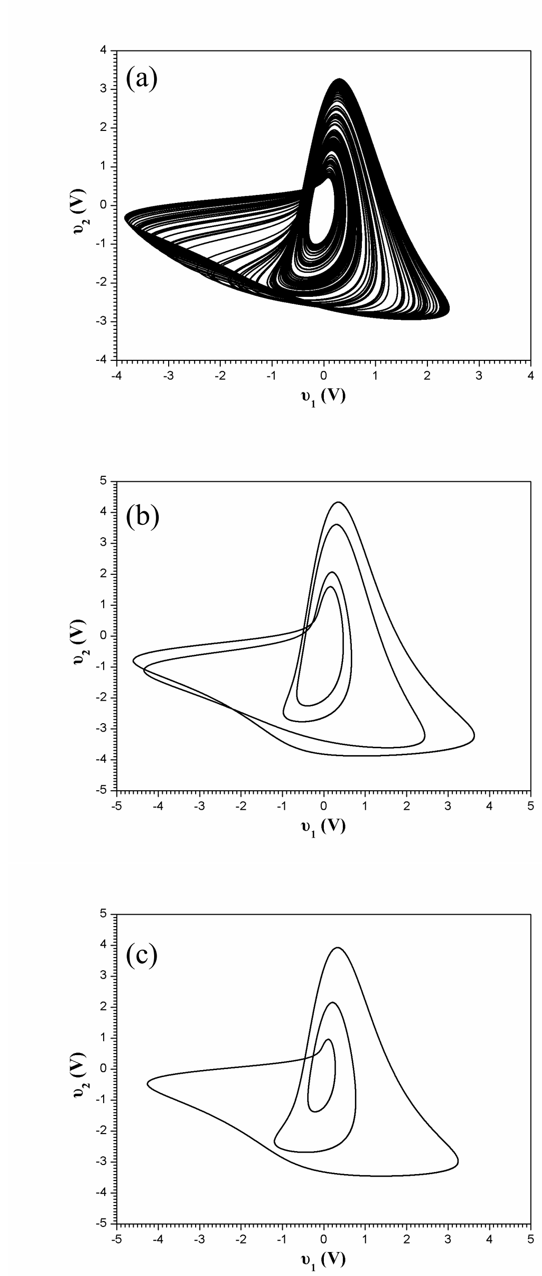


Fig 12. Chaotic spiral attractors for C_1 = 25nF and C_2 = 37nF in (a) $\upsilon_1 - \upsilon_2$ plane, (b) $\upsilon_1 - \varphi$ plane, (c) $\upsilon_2 - \varphi$ plane and (d) $i - \varphi$ plane.

The creation of bubbles is also very sensitive to initial conditions. The chaotic spiral attractors coexist with periodic limit cycles, resulting to a state where the circuit can be driven to quite different states, depending on the initial conditions. This phenomenon is clearly demonstrated in Figure 13, where different coexisting attractors are produced by slightly changing the initial condition of the first system parameter φ, (in the case of C_1=28nF and C_2=37nF). As a result, chaotic spiral attractor (Fig. 13a) coexists with five different limit cycles of period-(1, 2, 3, 4 and 6) according to the value of the initial condition of parameter φ.



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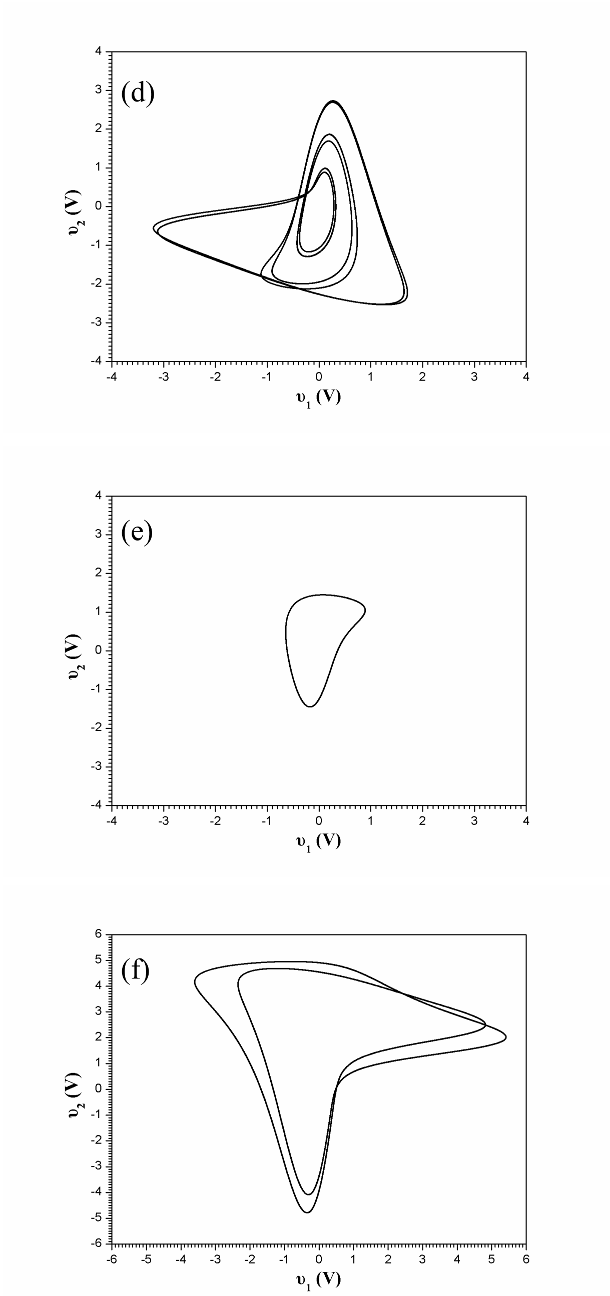

Fig 13. Phase portraits υ_2 vs υ_1, for C_1=28nF and C_2=37nF, with $(\upsilon_1)_0$=0.006V, $(\upsilon_2)_0$=0.02V, $(i_L)_0$=0.001A (period–1):
(a) $(\varphi)_0$=0Wb (chaotic state),
(b) $(\varphi)_0 = 0.5\cdot10^{-3}$ Wb (period–4),
(c) $(\varphi)_0 = 1\cdot10^{-3}$ Wb (period–3 state),
(d) $(\varphi)_0 = 1.2\cdot10^{-3}$ Wb (period–6 state),
(e) $(\varphi)_0 = 1.5\cdot10^{-3}$ Wb (period–1 state),
(f) $(\varphi)_0 = 1.8\cdot10^{-3}$ Wb (period–2 state).

4 Conclusion

In this paper the dynamic behavior of a Chua's canonical circuit, in which the nonlinear resistor has been replaced by a cubic memristor was studied. Using tools of nonlinear analysis, such as bifurcation diagrams and phase portraits, various phenomena concerning Chaos Theory were observed. In general, this work is the first attempt to approach nonlinear circuits implemented by the use of memristors. Future work should be concetrated on the use of memristors in other nonlinear circuits, as well as on the study of synchronization phenomena between coupled nonlinear circuits with memristors.

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