Dynamical pricing for one-manufacturer and two-retailers supply chain model

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Abstract—The benefits of dynamic pricing methods have long been known in industries, such as airlines, hotels, and electric utilities, where the capacity is fixed in the short-term and perishable. In recent years, there has been an increasing adoption of dynamic pricing policies in retail and other industries, where the sellers have the ability to store inventory. This paper looks intensively into the 3C (Computer, Communication, Consumer-electronics) products market, which is very dynamic due to technology innovation and short life cycle. Under this circumstance, it becomes crucial for retailers to decide on the correct inventory level to maintain. Meanwhile, the managers also face the problem of selling a given stock of items by the deadline. So we will also investigate the problem of dynamically pricing such items when demand is price sensitive. To tackle these problems, we build a mathematical model for a two-layer supply chain which consists of one manufacturer and two retailers. We identify the optimal pricing strategies for each of the players in the system.

Keywords—Dynamic pricing, Supply chain management, Inventory rationing, Stackelberg game, Perishable product.

I. INTRODUCTION

Companies that sell 3C (Computer, Communication, and Consumer Electronic) products usually face a short product life cycle. With the rapid pace of technological development in recent years, the life cycle of 3C products becomes even shorter. Moreover, business competition puts more pressures on these companies to sell their products quickly. Otherwise, as substitute products enter the market, excess inventory hold almost no value. Meanwhile, insufficient capacity and inventory to fulfill a surge in demand also leads to major losses. According to Elmaghraby and Keskinocak [1], millions of dollars are lost by a significant number of retailers due to lost sales or excess inventory.

Traditionally, a fixed price strategy is preferred and dynamic pricing may be too expensive to be implemented. However, in a market with varying demand, fixed price strategy may not produce the maximum revenue. In modern markets, the internet has helped to implement revenue management more easily, especially for e-Businesses, since menu costs are effectively eliminated or at least reduced. It also helps companies track their customers preferences better, which helps them decide on their pricing strategies.

On the other side, Supply chain management (SCM) is to plan, implement, and control the operations of the supply chain with the purpose of satisfying demand as efficiently as possible. Supply chain management covers all movement and storage of raw materials, work-in-process inventory, and finished products from point-of-origin to point-of-consumption. At the point-of-consumption, product pricing is affected by distribution network configuration, distribution strategy, marketing information, and inventory management. This coupling implies that pricing and inventory strategies can be used together to achieve maximum profit.

For dynamic pricing, Bitran and Caldentey [2] recommended it to companies with high set-up costs, perishable products, short selling horizons and price sensitive demand. Gallego and Ryzin [3] studied the case of maximizing expected revenue through dynamic pricing when retailers must sell a specified number of items by a deadline. Chatwin [4] investigated how the dynamic pricing policy for a perishable product when the selling period reaches its end and when inventory decreases.

Also, dynamic pricing has been widely applied to multi-stage supply chains. Zhang and Lv [5] studied the manufacturers’ wholesale pricing strategy and the retailers’ coordinated inventory-pricing strategy for perishable items. Chen and Simchi-Levi [6] considered a single-product model with continuous time over infinite horizon to decide the pricing and inventory strategies simultaneously. Yu, Huang and Liang [7] explored an information-assymmetric Vendor Managed Inventory supply chain. Their model aims to balance coordinate advertising, dynamic pricing, and inventory strategies. The goal is to achieve coordination between the manufacturer and the retailers. Overall, Elmaghraby and Keskinocak [1] provide a comprehensive review on dynamic pricing and inventory strategies in supply chains.

In this paper, we assume that demand is price sensitive. A linear demand function is used to mimic the sensitivity for different prices. We build a 3C supply chain model with one manufacturer and two retailers and study the pricing strategies and inventory policy. Moreover, we explore the optimal retail price and the optimal inventory level to hold. Our goal is to maximize the revenue.

The remaining parts of this paper are organized as follows. In Section II, we first describe the assumption made for the mathematical model. We then list and explain notations used in this paper. In Section II, we first study the retailer’s problem and
II. THE ASSUMPTIONS AND NOTATIONS

Consider a two-stage supply chain, with one manufacturer and two retailers. The retailers supply the same 3C product to two different markets and there is no direct competition between the retailers. The manufacturer has a certain inventory on hand at the beginning of the time horizon, produces uniformly throughout the cycle, and allows for replenishment orders from the retailers. Each market has a finite time horizon over which sales are permitted and has a predetermined time of peak sales, $T^*_i$. Demand depends on both time and price of the product. Any unsold product at the end of the selling period has zero salvage value. The basic model is shown in Figure 1.

Figure 1. Basic model of one-supplier, two-retailer supply chain.

The problem is modeled as a Stackelberg game with the manufacturer as the leader and the retailer as the follower. That is, the manufacturer announces his wholesale price and discount policy and each retailer devises his replenishment policy accordingly. As the leader, the manufacturer must first predict the best response of the retailer. We assume that the manufacturer has perfect information about the holding cost and pricing policies of each retailer and that the retailer is rational and will always act according to his best response model. Given the retailer’s response, the manufacturer can then formulate his discount policy.

A. Assumptions

In this model, the following assumptions are imposed:

1. The time horizon is finite and divided into $N$ periods.
2. The retailer can only order at the beginning of each period. Lead time is negligible so orders arrive right after the replenishment order is requested.
3. The retail price can only be changed at the beginning of each period and this same constant price is offered for the remainder of the period. For the retailer to be able to dictate market price, we assume that the retailer enjoys monopoly power over the market that it serves.
4. Demand is deterministic and is influenced by both time and retail price of product. Also, the demand of period $j$ reflects only the sales of period $j$. Thus, stock-out quantities are ignored.

B. Notations

The following notations are used in this paper.

| TABLE I |
| Notations |
| $i$ | index for retailers, $i = 1, 2$ |
| $j$ | index for sale periods, $j = 1, 2, \ldots, N$ |
| $T_i$ | finite time horizon for retailer $i$ |
| $T^*_i$ | time of peak sales for retailer $i$ |
| $q_{ij}$ | order quantity of retailer $i$ at the beginning of period $j$, |
| $p_{ij}$ | retail price set by retailer $i$ at the beginning of period $j$ |
| $h_{ij}$ | holding cost per unit of retailer $i$ charged at the end of the period |
| $I_{ij}$ | inventory of retailer $i$ carrying from period $j$ to $j+1$ |
| $d(p_{ij})$ | demand faced by retailer $i$ in period $j$ |
| $D_i$ | total demand quantity faced by retailer $i$ |
| $C(q_{ij}, I_i)$ | unit wholesale price paid by retailer $i$ for order quantity $q_{ij}$ and order period $j$ |

Assume that the demand faced by the retailer is deterministic. Also, without loss of generality, that the demands for consecutive periods are independent and nonnegative. No backlogging is allowed; any demand not satisfied in period $j$ is lost. The product demand is influenced by price and time. Demand increases from the time the product is launched into the market until $T^*$, the time of peak sales. Beyond $T^*$, demand drops to zero. Demand also decreases linearly as price increases. For a particular period, the demand is influenced by the retail price set for period $j$, and is represented by the function:

$$d(p_{ij}) = \begin{cases} a_j - b_j p_{ij}, & \text{for } j = 1, 2, \ldots, T^*_i \\ 0, & \text{for } j > T^*_i \end{cases}$$  \hspace{1cm} (1)$$

where constant $a_j \geq 0$ is simply the demand of period $j$ if $p_{ij}$ is set to zero. Also, $b_j \geq 0$ is the decrease in demand per dollar increase in $p_{ij}$.

On the other side, we assume the manufacturer’s wholesale price function, $C(q_{ij}, I_i)$, to be linear. It is influenced by both the retailer’s order quantity and the order timing. As order quantity increases, the manufacturer provides a larger discount to the retailer and the wholesale price decreases. Also, as time progresses, the manufacturer sets a penalty that acts like a
negative discount to induce buyers to buy earlier. This policy is comparable to real-world scenarios, such as in the airline seat pricing problem, where sellers provide more discounts if buyers order earlier in the season.

The wholesale price function can thus be expressed as:

\[
C(q_y, j) = \begin{cases} 
    C_0 - k_1 q_y + k_2 j, & \text{for } q_y \leq q_0 \\
    C_{\min}, & \text{for } q_y > q_0
\end{cases}
\]  

(2)

where constant \( C_{\min} \geq 0 \) is minimal unit wholesale price which is also the minimal production cost associated to the basic economic scale, \( q_0 \). Also, \( k_1 \) is the discount based on quantity and \( k_2 \) is the penalty cost based on time. The wholesale price function is shown in Figure 2.

![Figure 1. The unit wholesale price and the order quantity.](image)

Equation (2) indicates that the wholesale price is linearly decreasing with respect to order quantity and linearly increasing with time. That is, the retailer can pay a lower price if he either buys more or purchases earlier in the time horizon. In fact, by checking the boundary condition, \( q_y = q_0 \), Equation (2) also implies that

\[
q_0 = \frac{C_0 - C_{\min}}{k_1}.
\]  

(3)

### III. MATHEMATICAL MODELS

#### A. Retailer’s Model

We now consider retailer’s problem. Each retailer starts with zero inventory. The information available to him is the manufacturer’s wholesale price (with discount) policy. The retailer can avail of a discount based on order quantity and incur a low penalty cost based on order timing. This discount has implications on his cost components. He can choose to order everything at the beginning of the first period, enough to satisfy demand for all \( N \) periods. In this way, he can avail of a bigger discount since he orders in bulk and at an earlier timing. However, this will increase his inventory holding cost as supply for later periods is kept for a longer time. Since the manufacturer allows for replenishment, it is possible for the retailer to order a positive quantity in some periods and order nothing in other periods.

Any unsold product at the end of the time horizon has zero salvage value. The market demand is influenced by both time and price, and is assumed to be linear. Given these two factors affecting the demand, the goal of the retailer is to formulate a dynamic pricing policy that will allow him to maximize his profit over the whole time horizon. In summary, the decisions that the retailer has to make at the beginning of the planning horizon are: a) order quantity for each period, and b) dynamic pricing policy. These decisions are made before the demand for the period is fulfilled.

We model retailer’s problem as problem (PR). The retailer’s objective is to maximize profit given a set of boundary constraints.

(PR)

\[
\begin{align*}
\text{Max } & Z(p_y, q_y) = \sum_{j=1}^{N} \left( p_y(a_y - b_y p_y) - h_y I_y - q_y C(q_y, j) \right) \\
\text{s.t. } & I_{i,j+1} = I_{i,j} + q_y - (a_y - b_y p_y) \\
& p_y, q_y, I_y \geq 0
\end{align*}
\]

where \( \sum_{j=1}^{N} p_y(a_y - b_y p_y) \) is the total revenue for all time horizon. Note that \( \sum_{j=1}^{N} h_y I_y \) is the total inventory cost and \( \sum_{j=1}^{N} q_y C(q_y, j) \) is the total wholesale purchase cost. The constraint ties up inventory for multiple periods and no backorder and negative order quantity are allowed.

#### B. Manufacturer’s Model

We now consider manufacturer’s problem. We assume that the manufacturer’s production rate is a given constant. Thus, the manufacturer is only interested in maximizing the revenue received from both retailers. We model retailer’s problem as problem (PM). The manufacturer’s objective is to maximize profit given a set of boundary constraints.

(PM)

\[
\begin{align*}
\text{Max } & S(C_0, k_1, k_2) = \sum_{i=1}^{N} \sum_{j=1}^{N} q_i^*(C_0 - k_1 q_i^* + k_2 j) \\
\text{s.t. } & C_0 - k_1 q_i^* + k_2 j \geq C_{\min} \\
& C_0 \geq C_{\min} \\
& k_1, k_2 \geq 0
\end{align*}
\]

where \( \sum_{j=1}^{N} q_i^*(C_0 - k_1 q_i^* + k_2 j) \) is the total revenue for all time horizon collected from retailer \( i \). Note that the constraint keep the wholesale price to be higher than the minimal production cost.
IV. CONCLUSION AND FUTURE RESEARCH

3C products are known to have short life cycle due to fast changing technology, innovation and competition. It is challenging for manufacturers and retailers to price these products correctly to optimize their profits, and intense competitions among manufacturers and among retailers have escalated this challenge. With dynamic pricing policy, manufacturers and retailers can change their prices according to various factors such as inventory level, time, competition and demand to reap the most profits from selling the 3C products. This paper seek to obtain an optimal pricing policy for the retailers and manufacturer when two competing retailers sharing the same manufacturer in a dynamic pricing environment.

In this paper, we studied the dynamic pricing and inventory control strategy of a two-stage supply chain of one manufacturer serving two retailers in non-competing markets that have different time of sales peak. We have formulated the retailer’s best response problem and solved for the optimal price and order quantity that maximizes profit given the manufacturer’s wholesale price (with discount) policy. The retailer’s optimal retail price is influenced by time, holding cost, and price elasticity of demand. Meanwhile, his optimal order quantity can be influenced either by market demand or by the manufacturer’s wholesale price.

Provided that the retailers always act rationally according to their best response model, the manufacturer can always find a wholesale pricing and discount policy that will maximize profits obtained from the two retailers.

Due to the length limitation, we are not able to show all the procedures and optional solution in this conference paper. Currently, we are organizing many sub-cases of the model for further publication. The optimal will be available upon request.

A number of assumptions were made in this paper, some of which may not be very realistic when applied to a very complex and dynamic supply chain in the real world. We suggest some extension works that can be carried out to obtain a more realistic model.

The first extension is set-up cost for ordering. In the retailer’s model, an order set-up cost for each order request is not considered. In reality, buyers typically incur an order set-up cost that is independent of the order quantity. In our model, a high order set-up cost could possibly induce the retailer to order more in some periods and order nothing in other periods to avoid this set-up cost.

The other extension is the cooperation game between manufacturer and retailers. We assume in our model that the manufacturer and the two retailers all try to optimize locally. However, this could lead to a sub-optimal solution, and better profits may be obtained through collaboration. A study on how to achieve an agreement between the manufacturer and retailers and how to split profits fairly would be of great interest.

REFERENCES


