Numerical modeling of forest fire initiation and spread

Valeriy A. Perminov
Department of Mathematics and Natural Science
Belovo Branch of Kemerovo State University
Sovetskaya Street, 41
RUSSIA
E-mail: valerperminov@gmail.com

Abstract: In this paper the assignment and theoretical investigations of the problems of forest fire initiation was carried out. It is developed mathematical model for description of heat and mass transfer processes in overterrestrial layer of atmosphere at crown forest fire initiation, taking into account their mutual influence. Mathematical model of forest fire was based on an analysis of experimental data and using concept and methods from reactive media mechanics. The paper suggested in the context of the general mathematical model of forest fires give a new mathematical setting and method of numerical solution of a problem of a forest fire modeling. The boundary-value problem is solved numerically using the control volume methods and method of splitting according to physical processes. In this context, a study - mathematical modeling - of the conditions of forest fire initiation and spreading that would make it possible to obtain a detailed picture of the change in the velocity, temperature and component concentration fields with time, and determine as well as the limiting conditions of forest fire propagation is of interest.

Keywords: Mathematical model, forest fire, combustion, control volume, discrete analogue.

I. INTRODUCTION

One of the objectives of these studies is the improvement of knowledge on the fundamental physical mechanisms that control forest fire initiation and spread. A great deal of work has been done on the theoretical problem of how forest fire initiation. In forest there are two steps for crown forest fire initiation: spread of fire from crown to crown and crown fires are initiated by convective and radiative heat transfer from surface fires. However, convection is the main heat transfer mechanism. Firstly crown forest fire initiation have been studied and modeled by Van Wagner [1]. There are three simple crown properties: crown base height, bulk density and moisture content of forest fuel in this theory. Also crown fire initiation have been studied and modeled in detail (eg: Alexander [2], Van Wagner [3], Xanthopoulos, [4], Rothermel [5,6], Van Wagner, [7], Cruz [8], Albini [9], Scott, J. H. and Reinhardt, E. D. [10]. The discussion of the problem of modeling forest fires is provided by a group of co-workers at Tomsk University (Grishin [11], Grishin and Perminov [12], Perminov [13,14]). The general mathematical model of forest fires was obtained by Grishin [11] based on an analysis of known and original experimental data [11,15], and using concepts and methods from reactive media mechanics. The physical two-phase models used in [16-17] may be considered as a continuation and extension of the formulation proposed by Grishin and Perminov [12-14]. However, the investigation of crown fires has been limited mainly to cases studied of forest fires initiation in two dimensional settings and did not take into account space properties of these phenomena.

II. PHYSICAL AND MATHEMATICAL MODEL

It is assumed that the forest during a forest fire can be modeled as 1) a multi-phase, multistoried, spatially heterogeneous medium; 2) in the fire zone the forest is a porous-dispersed, two-temperature, single-velocity, reactive medium; 3) the forest canopy is supposed to be non-deformed medium (trunks, large branches, small twigs and needles), affects only the magnitude of the force of resistance in the equation of conservation of momentum in the gas phase, i.e., the medium is assumed to be quasi-solid (almost non-deformable during wind gusts); 4) let there be a so-called “ventilated” forest massif, in which the volume of fractions of condensed forest fuel phases, consisting of dry organic matter, water in liquid state, solid pyrolysis products, and ash, can be neglected compared to the volume fraction of gas phase (components of air and gaseous pyrolysis products); 5) the flow has a developed turbulent nature and molecular transfer is neglected; 6) gaseous phase density doesn’t depend on the pressure because of the low velocities of the flow in comparison with the velocity of the sound. Let the coordinate reference point \( x_1, x_2, x_3 = 0 \) be situated at the centre of the domain of surface forest fire source at the height of the roughness level, axis \( 0x_1 \) directed parallel to the Earth’s surface to the right in the direction of the unperturbed wind speed, axis \( 0x_2 \) directed perpendicular to \( 0x_1 \) and axis \( 0x_3 \) directed upward (Fig. 1).

Figure 1
Using the results of [11-14] and known experimental data [15] we have the following sufficiently general equations, which define the state of the medium in the forest fire zone, written using tensor notation:

\[
\frac{\partial \rho}{\partial t} + \sum_{j=1,2,3} \frac{\partial (\rho v_j)}{\partial x_j} = \dot{m}, \quad j=1,2,3; \quad i=1,2,3; \quad (1)
\]

\[
\rho \frac{dv_j}{dt} = -\frac{\partial P}{\partial x_i} + \sum_{\alpha} \rho \frac{\partial v}{\partial x_i} - \rho \sigma c_{\alpha} v_i |\vec{v}| - \rho g_i - \dot{m} v_i; \quad (2)
\]

\[
\alpha \frac{d\rho}{dt} = \frac{\partial}{\partial x_j} \left(-\rho \sigma c_{\alpha} v_j \right) + R_{sa} - \dot{m} c_{\alpha}, \quad \alpha = 1,5; \quad (3)
\]

\[
\frac{\partial}{\partial x_j} \left( \frac{c}{3k} \frac{\partial U}{\partial x_j} \right) - kc U_x + 4k_{s} \sigma T_x + 4k_{g} \sigma T^4 = 0, \quad (5)
\]

\[
\sum_{i=1}^{i} \rho \frac{c_{\alpha}}{\pi} \frac{\partial T_{\alpha}}{\partial t} = q_{3} R_{3} - q_{2} R_{2} +
\]

\[
+ k_{s} \left(c U_{x} - 4 \sigma T_{x}^3 \right) + \alpha_{i} (T - T_{c}); \quad (7)
\]

\[
\sum_{\alpha} c_{\alpha} = 1, \quad p_{\alpha} = \rho R_{T} \sum_{\alpha} \frac{c_{\alpha}}{M_{\alpha}} \vec{v} = (v_{x}, v_{y}, v_{z}), \quad \vec{g} = (0,0,g). \quad (8)
\]

The system of equations (1)-(7) must be solved taking into account the initial and boundary conditions:

\[
x_{i} = -x_{i}, \quad v_{i} = V_{i}, \quad v_{2} = 0, \quad C_{i,0} = C_{i,0}, \quad x_{i} = x_{i}, \quad v_{i} = v_{i} = 0, \quad T = T_{0}, \quad C_{i,0} = C_{i,0}, \quad (8)
\]

\[
x_{i} = x_{i}, \quad v_{i} = 0, \quad v_{2} = 0, \quad C_{i,0} = C_{i,0}, \quad \frac{\partial T}{\partial x_i} = 0, \quad \frac{\partial U_{R}}{\partial x_i} = 0; \quad (9)
\]

\[
x_{i} = x_{i}, \quad v_{i} = 0, \quad v_{2} = 0, \quad v_{3} = 0, \quad C_{i,0} = C_{i,0}, \quad \frac{\partial U_{R}}{\partial x_i} = 0; \quad (10)
\]

\[
x_{i} = x_{i}, \quad v_{i} = 0, \quad v_{2} = 0, \quad v_{3} = 0, \quad C_{i,0} = C_{i,0}, \quad \frac{\partial U_{R}}{\partial x_i} = 0. \quad (11)
\]

Here and above \( \frac{d}{dt} \) is the symbol of the total (substantial) derivative; \( \alpha \) is the coefficient of phase exchange; \( \rho \) - density of gas – dispersed phase, \( t \) is time; \( v_{i} \) - the velocity components; \( T, T_{c} \) - temperatures of gas and solid phases, \( U_{R} \) - density of radiation energy, \( k \) - coefficient of radiation attenuation, \( P \) - pressure; \( c_{\alpha} \) - constant pressure specific heat of the gas phase, \( \sigma_{\alpha}, \rho_{\alpha} \) - specific heat, density and volume of fraction of condensed phase (1 – dry organic substance, 2 – moisture, 3 – condensed pyrolysis products, 4 – mineral part of forest fuel), \( R_{i} \) – the mass rates of chemical reactions, \( q_{1} \) - thermal effects of chemical reactions; \( k_{g}, k_{s} \) - radiation absorption coefficients for gas and condensed phases; \( T_{c} \) – the ambient temperature; \( c_{\alpha} \) – mass concentrations of \( \alpha \)-component of gas - dispersed medium, index \( \alpha = 1,2,3,...,5 \), where 1 corresponds to the density of oxygen, 2 - to carbon monoxide \( CO \), 3 - to carbon dioxide and inert components of air, 4 - to particles of black, 5 - to particles of smoke; \( R \) – universal gas constant; \( M_{\alpha}, M_{g} \) and \( M_{M} \) molecular mass of \( \alpha \)-components of the gas phase, carbon and air mixture; \( g \) is the gravity acceleration; \( c_{d} \) is an empirical coefficient of the resistance of the vegetation, \( s \) is the specific surface of the forest fuel in the given forest stratum. To define source terms which characterize inflow (outflow of mass) in a volume unit of the gas-dispersed phase, the following formulae were used for the rate of formulation of the gas-dispersed mixture \( \dot{m} \), outflow of oxygen \( R_{51} \), changing carbon monoxide \( R_{52} \), generation of black \( R_{54} \) and smoke particles \( R_{55} \).

\[
\dot{m} = (1-\alpha) R_{1} + R_{2} + \frac{M_{R}}{M_{1}} R_{1} + R_{54} + R_{55}, \quad (12)
\]

\[
R_{51} = -R_{3} - \frac{M_{R}}{2M_{2}} R_{52} = v_{g} (1-\alpha) R_{1} - R_{3}, \quad (13)
\]

\[
R_{55} = 0, \quad R_{54} = \alpha_{4} R_{1}, \quad R_{55} = \alpha_{5} v_{3} - R_{2}. \quad (14)
\]

Here \( v_{g} \) – mass fraction of gas combustible products of pyrolysis, \( \alpha_{4} \) and \( \alpha_{5} \) – empirical constants. Reaction rates of these various contributions (pyrolysis, evaporation, combustion of coke and volatile combustible products of pyrolysis) are approximated by Arrhenius laws whose parameters (pre-exponential constant \( k_{i} \) and activation energy \( E_{i} \)) are evaluated using data for mathematical models [11,13].
simplifies the technology of solving problems of predicting the state of the medium in the fire zone numerically. The thermodynamic, thermophysical and structural characteristics correspond to the forest fuels in the canopy of a different (for example pine [11,13]) type of forest. The system of equations (1)–(7) must be solved taking into account the initial and boundary conditions [11-14]. The conditions of symmetry are used because of the patterns of flow and distributions of all scalar functions are symmetrical relatively to the axes \( Ox_2 \).

### III. NUMERICAL METHODS AND RESULTS

The boundary-value problem (1)–(7) we solve numerically using the method of splitting according to physical processes [13]. In the first stage, the hydrodynamic pattern of flow and distribution of scalar functions was calculated. The system of ordinary differential equations of chemical kinetics obtained as a result of splitting [13] was then integrated. A discrete analog was obtained by means of the control volume method using the SIMPLE like algorithm [13,18]. The accuracy of the program was checked by the method of inserted analytical solutions. The time step was selected automatically.

Fields of temperature, velocity, component mass fractions, and volume fractions of phases were obtained numerically. Figures 2 illustrate the time dependence of dimensionless temperatures of gas (1) and condensed phases (2), Figure 3. – mass concentrations of gas components (1- oxygen, 2- gas products of pyrolysis), and Figure 4 - relative volume fractions of solid phases (1), moisture (2) and coke (3) at crown base of the forest (\( V_c=5m/s \)).

\[
R_1 = k_1 \rho \varphi_1 \exp \left( -\frac{E_1}{RT} \right), R_2 = k_1 \rho \varphi_2 T^{-1.2} \exp \left( -\frac{E_2}{RT} \right),
\]

\[
R_3 = k_2 \rho \varphi_3 \exp \left( -\frac{E_3}{RT} \right), R_4 = k_2 M_1 \frac{c_M}{M_3} T^{-2.25} \exp \left( -\frac{E_4}{RT} \right).
\]

The initial values for volume of fractions of condensed phases are determined using the expressions:

\[
\varphi_i = \frac{d(1-v_i)}{\rho_i}, \varphi_{i+} = \frac{W d}{\rho_i}, \varphi_{i+} = \frac{\alpha_i \varphi_{i+} \rho_1}{\rho_i}
\]

where \( d \) - bulk density for surface layer, \( v_i \) – coefficient of ashes of forest fuel, \( W \) – forest fuel moisture content.

It is supposed that the optical properties of a medium are independent of radiation wavelength (the assumption that the medium is “grey”), and the so-called diffusion approximation for radiation flux density were used for a mathematical description of radiation transport during forest fires.

To close the system (1)–(7), the components of the tensor of turbulent stresses, and the turbulent heat and mass fluxes are determined using the local-equilibrium model of turbulence (Grishin, [11]). The system of equations (1)–(7) contains terms associated with turbulent diffusion, thermal conduction, and convection, and needs to be closed. The components of the tensor of turbulent stresses \( \overline{\rho \nu_i \nu_j} \), as well as the turbulent fluxes of heat and mass \( \overline{\rho \nu_i c_i T} \), \( \overline{\rho \nu_i c_i'} \) are written in terms of the gradients of the average flow properties using the formulas

\[
- \overline{\rho \nu_i \nu_j} = \mu_i \left( \frac{\partial V_i}{\partial x_j} + \frac{\partial V_j}{\partial x_i} \right) - \frac{2}{3} K \delta_{ij},
\]

\[
- \overline{\rho \nu_i c_i T} = \lambda_i \frac{\partial T}{\partial x_j}, - \overline{\rho \nu_i c_i'} = \rho D_i \frac{\partial c_i}{\partial x_j},
\]

\[
\lambda_i = \mu_i c_i / Pr_i, \rho D_i = \mu_i / \Sigma c_i, \mu_i = c_i \mu K^2 / \varepsilon,
\]

where \( \mu_i \), \( \lambda_i \), \( D_i \) are the coefficients of turbulent viscosity, thermal conductivity, and diffusion, respectively; \( Pr_i \), \( \Sigma c_i \) are the turbulent Prandtl and Schmidt numbers, which were assumed to be equal to 1. In dimensional form, the coefficient of dynamic turbulent viscosity is determined using local equilibrium model of turbulence [11]. The length of the mixing path is determined using the formula

\[
I = k_1 \times_2 / (1 + 2.5 \times_2 \sqrt{c_d s / h})
\]

taking into account the fact that the coefficient of resistance \( c_d \) in the space between the ground cover and the forest canopy base is equal to zero, while the constants \( k_T = 0.4 \) and \( h= h_3 - h_1 \) (\( h_3 \), \( h_1 \) – height of the tree crowns and the height of the crown base). It should be noted that this system of equations describes processes of transfer within the entire region of the forest massif, which includes the space between the underlying surface and the base of the forest canopy, the forest canopy and the space above it, while the appropriate components of the data base are used to calculate the specific properties of the various forest strata and the near-ground layer of atmosphere. This approach substantially improves the model's ability to simulate the complex interactions within the forest ecosystem.
At the moment of ignition the gas combustible products of pyrolysis burns away, and the concentration of oxygen is rapidly reduced. The temperatures of both phases reach a maximum value at the point of ignition. The ignition processes is of a gas-phase nature, i.e. initially heating of solid and gaseous phases occurs, moisture is evaporated. Then decomposition process into condensed and volatile pyrolysis products starts, the later being ignited in the forest canopy. Note also that the transfer of energy from the fire source takes place due to radiation; the value of radiation heat flux density is small compared to that of the convective heat flux. As a result of heating of forest fuel elements, moisture evaporates, and pyrolysis occurs accompanied by the release of gaseous products, which then ignite.

The vector field of velocities and isotherms of gas phase (Fig. 5-7): 1-6 correspond to the isotherms \( T = 1.1, 1.5, 1.7, 2.0, 3.0 \) and 4 for different instants of time \((t=2, 3, 4 \text{ sec})\). In the vicinity of the source of heat and mass release, heated air masses and products of pyrolysis and combustion float up. The wind field in the forest canopy interacts with the gas-jet obstacle that forms from the surface forest fire source and from the ignited forest canopy base. Recirculating flow forms beyond the zone of heat and mass release, heated air masses and products of pyrolysis and combustion float up. The wind field in the forest canopy interacts with the gas-jet obstacle that forms from the surface forest fire source and from the ignited forest canopy base. Recirculating flow forms beyond the zone of heat and mass release, while on the windward side the movement of the air flowing past the ignition region accelerates. Under the influence of the wind the tilt angle of the flame is increased. As a result this part of the forest canopy, which is shifted in the direction of the wind from the center of the surface forest fire source, is subjected to a more intensive warming up. The isotherms of the gas and condensed phases are deformed in the direction of the wind.

Figures 8-10 presents isolines of oxygen for different instants of time \((t=2, 3, 4 \text{ sec})\): 1-5 correspond to the isolines \( c_1 = 0.9, 0.8, 0.7, 0.5 \) and 0.1.

\[
\begin{align*}
1 - \varphi_1 &= \phi_1 / \phi_{le}, \\
2 - \varphi_2 &= \rho_2 \phi_2 / \rho_c, \\
3 - \varphi_3 &= \rho_3 \phi_3 / \alpha_c \rho_1 \phi_{le}.
\end{align*}
\]
Figures 11-13 presents isolines of gas products of pyrolysis for different instants of time \( t = 2, 3, 4 \) sec.: 1-4 correspond to the isolines \( \bar{c}_2 = 0.02, 0.05, 0.1 \) and 0.5.

The effect of the wind on the forest fire spread is shown in Figures 14(a, b, c) present the horizontal distribution of field of temperature for gas phase in plane \( \delta x_j, x_j \) for different instants of time when a wind velocity \( V_e = 5 \) m/s and moisture of forest combustible materials – 0.6. We can note that the isotherms is moved in the forest canopy and deformed by the action of wind. Also the fields of component concentrations are deformed. It is concluded that the forest fire begins to spread and the fire front is extent.

Figures 15 and 16 (a, b, c) present the distribution of field of concentration of oxygen and volatile combustible products of pyrolysis concentration for the same instants of time when a wind velocity \( V_e = 5 \) m/s and moisture of forest combustible materials – 0.6 (\( \tau \varepsilon = \varepsilon / c_{1e} < \varepsilon 0.23 \)). The lines of equal levels of component concentrations are deformed. It is confirmed that the forest fire begins to spread.
IV. CONCLUSION
This mathematical model and the result of the calculation give an opportunity to evaluate critical condition of the forest fire initiation and spread which allows applying the given model for preventing fires. The model overestimates the rate of the crown forest fires spread. The results obtained agree with the laws of physics and experimental data [11,13,15]).

V. REFERENCES