Modeling of Three-phase Controlled Rectifier using a DQ Method


Abstract—Power converter models are normally time-varying because of their switching behaviors. This paper presents the DQ modeling method to eliminate the switching action to achieve time-invariant model. The power system studied is the AC distribution system. The small-signal model of the power system is obtained by using a linearization technique. The small-signal simulations are used to validate the DQ linearized model. The results show that an excellent agreement between the mathematical model and the three-phase benchmark model is achieved.

Keywords—Constant power loads, DQ method, simulation, modeling, controlled rectifier.

I. INTRODUCTION

It is well known that the power converter model is time-varying because of the switching behavior. Several approaches are commonly used for eliminating the switching actions to achieve time-invariant model. Then, the classical linear control theory can be easily applied to the model for a system analysis and design. The first method is the generalized state-space averaging (SSA) modeling method. This method has been used to analyze many power converters in DC distribution systems [1]-[3], as well as uncontrolled and controlled rectifiers in single-phase AC distribution systems [4], and 6- and 12- pulse diode rectifiers in three phase systems [5]. The second is an average-value modeling method, which has been used for 6- and 12- pulse diode rectifiers in many publications [6]-[8], as well as generators with line-commutated rectifiers [9]-[13]. These rectifiers can be modeled with good accuracy as a constant DC voltage source. However this method is not easily applicable to the analysis of the stability of the general AC power system with multi-converter power electronic systems. Another technique widely used for AC system analysis is that of DQ-transformation theory [14]-[16], in which power converters can be treated as transformers. The DQ modeling method can also be easily applied for modeling a power system comprising vector-controlled converters where the SSA model and the average-value model are not easily applicable. Moreover, the resulting converter models can be easily combined with models of other power elements expressed in terms of synchronously rotating frames such as generators, front-end converters, and vector-controlled drives. The DQ models of three-phase AC-DC power systems have been reported in the previous works [14]-[16]. But these do not include a constant power load (CPL). Applying the DQ modeling approach for stability studies of the power system including a CPL has been addressed in [17]-[19]. The DQ method for modeling the three-phase uncontrolled rectifier has been reported in [17]. Therefore, this paper extends the work in [17] to create the DQ model of the three-phase controlled rectifier. The DQ model will be verified by using the intensive time domain simulation.

The paper is structured as follows. In Section II, the power system definition and assumptions are explained. The DQ dynamic model of the system is fully addressed in Section III. The steady-state value calculation for the small-signal model derived from the proposed method is presented in Section VI. In Section V, the model validation using a small-signal simulation is shown. Finally, Section V concludes and discusses the advantages of the DQ method to model the power converter.

II. POWER SYSTEM DEFINITION AND ASSUMPTIONS

The power system studied in this paper is depicted in Fig. 1. It consists of a three-phase voltage source, transmission line, 6-pulse controlled rectifier, DC-link filters, and an ideal CPL connected to the DC bus. The ideal CPL is used to represent actuator drive systems by assuming an infinitely fast controller action of the drive system. Hence, the ideal CPL can be considered as a voltage-dependent current source given by:

$$I_{\text{CPL}} = \frac{P_{\text{CPL}}}{V_{\text{out}}}$$

where $V_{\text{out}}$ is the voltage across the CPL and $P_{\text{CPL}}$ is the power level of CPL.

It is assumed that the three-phase voltage source is balanced. The equivalent parameters of a transmission line are represented by $R_{eq}$, $L_{eq}$ and $C_{eq}$. The DC-link filters are shown by elements $r_L$, $L_P$, and $C_P$. $E_{dc}$ and $V_{out}$ are the output terminal voltage of a controlled rectifier and the voltage across the DC-link capacitor $C_F$, respectively. A phase shift between the source bus and the AC bus is $\lambda$ as shown in Fig. 1.
The effect of $L_{eq}$ on the AC side causes an overlap angle $\mu$ in the output waveforms that causes as a commutation voltage drop. This drop can be represented as a variable resistance $r_{\mu}$ that is located on the DC side [17],[20] as shown in Fig.2. The $r_{\mu}$ can be calculated by:

$$r_{\mu} = \frac{3\alpha L_{eq}}{\pi}$$

where $\omega$ is the source frequency.

![Three-phase controlled rectifier with overlap angle resistance](image)

Fig.2: Three-phase controlled rectifier with overlap angle resistance

It can be seen from Fig.2 that $E_{abc}$ represents the output voltage from the switching signal without an overlap angle effect, while $E_{dc}$ represents the voltage at the rectifier output terminal taking into account the voltage drop effect.

Since the commutation effect has been moved on to the DC side, the switching signals for 3-phase controlled rectifier can be applied without considering the effect of overlap angle. This is show in Fig. 3 in which $\alpha$ is the firing angle of thyristors.

![Controller rectifier switching signal](image)

Fig.3: The controller rectifier switching signal

The switching function of $S_{a}$ in Fig.3 can be expressed by a Fourier series. In this paper, neglecting the harmonics of the power system, the switching functions can be written for three phases as:

$$S_{abc} = \frac{2\sqrt{3}}{\pi} \left[ \sin(\alpha + \phi - \alpha) \sin(\alpha - \frac{2\pi}{3} + \phi - \alpha) \sin(\alpha + \frac{2\pi}{3} + \phi - \alpha) \right]$$

where $\phi$ is a phase angle of the AC bus voltage and $\alpha$ is the firing angle.

The relationship between input and output terminal of controlled rectifier is given by:

$$I_{in,abc} = S_{abc} I_{dc}$$

$$E_{dc,1} = S_{abc} V_{bus,abc}$$

It can be seen from (4) that the fundamental input current is in phase with the switching signals. In addition, for a controlled rectifier, the fundamental input current lags the fundamental input voltage by $\alpha$ [20].

Equations (3)-(5) will be used to derive the model of controlled rectifier by using DQ modeling method in Section III. The model assumptions in this paper are as follows:

- The rectifier is operated under a continuous conduction mode (CCM).
- The output DC current of the rectifier is constant.
- The amplitude of the three-phase source is constant and balanced.
- Only one commutation occurs at a time.
- All harmonics in the system are neglected (concerning only fundamental component).

III. DQ Dynamic Model of the System

In this section, the DQ modeling method is applied to derive a mathematical model of the system as depicted in Fig.1.

Firstly, the controlled rectifier is transformed into a two axis frame (DQ frame) rotating at the system frequency $\omega$ by means of:

$$T[\theta(t)] = \frac{\sqrt{3}}{3} \begin{bmatrix} \cos(\theta(t)) & \cos(\theta(t) - \frac{2\pi}{3}) & \cos(\theta(t) + \frac{2\pi}{3}) \\ -\sin(\theta(t)) & -\sin(\theta(t) - \frac{2\pi}{3}) & -\sin(\theta(t) + \frac{2\pi}{3}) \end{bmatrix}$$
where $\theta(t) = \omega t - \frac{\pi}{2} + \phi_1$

Combining equations (4)-(6) results in:

$$I_{in.dq} = S_{dq} I_{dc} \quad \text{(7)}$$

$$E_{dc/1} = S_{dq} V_{bus.dq} \quad \text{(8)}$$

Secondly, the switching functions in (3) can be transformed into a DQ frame by means of (6) to give:

$$S_{dq} = \frac{3}{2} \cdot \frac{2^{2/3}}{\pi} \begin{bmatrix} \cos(\phi_1 - \phi + \alpha) & -\sin(\phi_1 - \phi + \alpha) \end{bmatrix}^T \quad \text{(9)}$$

The vector diagram for the DQ transformation is as shown in Fig. 4 where $V_i$ is the peak amplitude phase voltage, $I_i$ is the peak amplitude current, $V_{bus}$ is the peak amplitude AC bus voltage, and $S$ is peak amplitude of the switching signal, here equal to $2\sqrt{3}/\pi$ as shown in (3).

From (7)-(9), the controlled rectifier can be easily represented as a transformer having d and q-axis transformer ratio $S_d$, $S_q$ that depend on the phase of the DQ frame ($\phi$), the phase of $V_{bus}$ ($\phi_1$), and the firing angle of thyristors ($\alpha$). As a result, the equivalent circuit of the controlled rectifier in the DQ frame derived by using DQ modeling method is shown in Fig.5.

![Fig.4: The vector diagram for DQ transformation](image1)

Finally, using (6), the cable section can be transformed into DQ frame [21]. The DQ representation of the cable is then combined with the controlled rectifier as shown in Fig.5. As a result, the equivalent circuit of the power system in Fig.1 can be represented in the DQ frame as depicted in Fig.6.

The equivalent circuit in Fig.6 can be simplified by fixing the rotating frame on the phase of the switching function ($\phi_1 = \phi - \alpha$). This results in the circuit as shown in Fig.7.

![Fig.5: The controlled rectifier equivalent circuit in the DQ frame](image2)

![Fig.6: The equivalent circuit of the system in Fig.1 on DQ frame](image3)
Applying the Kirchhoff’s voltage law (KVL) and the Kirchhoff’s current law (KCL) to the circuit in Fig.7 obtains the set of nonlinear differential equations. We define:

State variables: $x = [I_{ds}, I_{qs}, V_{bus,d}, V_{bus,q}, I_{dc}, V_{out}]^T$

Input: $u = [V_m, P_{CPL}]^T$

Output: $y = [V_{out}]$

The set of nonlinear differential equations is given as follows:

\begin{align*}
I_{ds} &= -\frac{R_{eq}}{L_{eq}} I_{ds} + \omega I_{qs} - \frac{1}{L_{eq}} I_{bus,d} + \frac{1}{L_{eq}} V_{sd} \\
I_{qs} &= -\omega I_{ds} - \frac{R_{eq}}{L_{eq}} I_{qs} - \frac{1}{L_{eq}} I_{bus,q} + \frac{1}{L_{eq}} V_{sq} \\
V_{bus,d} &= \frac{1}{C_{eq}} I_{ds} + \omega V_{bus,q} - \frac{\sqrt{3}}{2} \frac{2\sqrt{3}}{\pi \omega_{eq}} I_{dc} \\
V_{bus,q} &= -\omega V_{bus,d} + \frac{1}{C_{eq}} I_{qs} \\
I_{dc} &= \sqrt{\frac{3}{2}} \frac{\sqrt{3}}{\pi \omega_{eq}} V_{bus,d} - \left( \frac{r_F + r_m}{L_F} \right) I_{dc} - \frac{1}{L_F} V_{out} \\
V_{out} &= \frac{1}{C_F} I_{dc} - \frac{1}{C_F} P_{CPL} V_{out}
\end{align*}

The equation (10) is a nonlinear equation. It is well known that the linearized model can be used for a controller system design via a linear control theory. In addition, the linearized model can be also used to analyze the small-signal stability of the power system including a CPL [17]-[19]. Therefore, (10) is linearized using the first order terms of the Taylor expansion so as to achieve a set of linear differential equations around an equilibrium point. The DQ linearized model of (10) is then of the following form:

\begin{align*}
\delta \dot{x} &= A(x_o, u_o) \delta x + B(x_o, u_o) \delta u \\
\delta \dot{y} &= C(x_o, u_o) \delta x + D(x_o, u_o) \delta u
\end{align*}

where

\begin{align*}
\delta x &= \begin{bmatrix} \delta I_{dc} \\ \delta I_{qs} \\ \delta V_{bus,d} \\ \delta V_{bus,q} \\ \delta I_{dc} \\ \delta V_{out} \end{bmatrix} \\
\delta u &= \begin{bmatrix} \delta V_{in} \\ \delta P_{CPL} \end{bmatrix} \\
\delta \dot{y} &= \begin{bmatrix} \delta V_{out} \end{bmatrix}
\end{align*}

\begin{align*}
A(x_o, u_o) &= \begin{bmatrix}
-\frac{R_{eq}}{L_{eq}} & \omega & -\frac{1}{L_{eq}} & 0 & 0 & 0 \\
-\omega & -\frac{R_{eq}}{L_{eq}} & 0 & -\frac{1}{L_{eq}} & 0 & 0 \\
\frac{1}{C_{eq}} & 0 & 0 & \omega \frac{\sqrt{3}}{2} \frac{2\sqrt{3}}{\pi \omega_{eq}} & 0 & 0 \\
0 & \frac{1}{C_{eq}} & -\omega & 0 & 0 & 0 \\
0 & 0 & \frac{\sqrt{3}}{2} \frac{2\sqrt{3}}{\pi \omega_{eq}} & 0 & -\left( \frac{r_F + r_m}{L_F} \right) & -\frac{1}{L_F} \\
0 & 0 & 0 & \frac{1}{C_F} & \frac{P_{CPL}}{C_F V_{out,dc}^2} V_{out,dc} & 0
\end{bmatrix} \\
B(x_o, u_o) &= \begin{bmatrix}
\frac{3}{2} \cos(\lambda_c + \alpha) \\
\frac{3}{2} \sin(\lambda_c + \alpha) \\
0 \\
0 \\
0 \\
-\frac{1}{C_F} V_{out,dc}^2
\end{bmatrix}
\end{align*}
The DQ linearized model in (10) needs to define \( V_{\text{out},o} \) and \( \lambda_o \). The power flow equation can be applied to determine the steady state value at the AC side of the power system in Fig.1. This leads to a system of nonlinear equations:

\[
\begin{align*}
\frac{V_{\text{bus}}}{Z} \cos(\gamma - \lambda) - \frac{V_{\text{bus}}^2}{Z} \cos(\gamma) &= P_{\text{bus}} \\
\frac{V_{\text{bus}}}{Z} \sin(\gamma - \lambda) - \frac{V_{\text{bus}}^2}{Z} \sin(\gamma) &= Q_{\text{bus}}
\end{align*}
\]

(12)

where the following steady-state values are: \( V_{\text{bus},o} \) – voltage at AC bus (rms), \( \lambda_o \) - phase shift between \( V_s \) and \( V_{\text{bus}} \) as mentioned above. Note that \( Z \angle \gamma \) is the transmission line impedance, while the active and reactive power (per phase) at the AC bus is given by:

\[
\begin{align*}
P_{\text{bus}} &= V_{\text{bus}} I_{\text{bus}} \cos \alpha = \left( P_{\text{CPL}} \right) / 3 \\
Q_{\text{bus}} &= V_{\text{bus}} I_{\text{bus}} \sin \alpha
\end{align*}
\]

(13)

It can be seen that the \( P_{\text{bus}} \) and \( Q_{\text{bus}} \) depend on the firing angle of thyristors (\( \alpha \)). Equation (12) can be solved by using a numerical method such as Newton Raphson to achieve \( V_{\text{out},o} \) and \( \lambda_o \) at the steady-state conditions. Consequently, \( V_{\text{out},o} \) for DQ linearized model in (11) can then be calculated by:

\[
V_{\text{out},o} = \frac{3\sqrt{3}}{\pi} (\sqrt{2} V_{\text{bus},o}) \cos(\alpha) - \frac{3 I_{\text{eq}}}{\pi} I_{\text{dc},o} - r_F I_{\text{dc},o}
\]

(14)

where

\[
I_{\text{dc},o} = \frac{\sqrt{3} e^{j0} - V_{\text{bus},o} e^{-j\lambda_o}}{Z e^{j\gamma}} \cdot \frac{3}{\sqrt{2} \pi} \text{ and } Z = \sqrt{R_{\text{eq}}^2 + (oL_{\text{eq}})^2}
\]

\[
\gamma = \tan^{-1} \left( \frac{oL_{\text{eq}}}{R_{\text{eq}}} \right)
\]

V. SMALL-SIGNAL SIMULATION

The DQ linearized model in (11) is simulated for small-signal transients against a corresponding three-phase benchmark circuit model in SimPowerSystems of SIMULINK. The set of parameters for the example system according to Fig.1 is given as follows: \( V_s=230 \) V_{\text{rms}}/phase, \( f=50 \) Hz, \( R_{\text{eq}}=0.15 \) \Omega, \( L_{\text{eq}}=30 \) mH, \( C_{\text{eq}}=2 \) mF, \( C_F=4000 \) \mu F, \( r_F=0.03 \) \Omega, and \( L_F=8 \) mH. Fig.8 shows the \( V_{\text{out}} \) response of the system in Fig.1 to a step change of \( P_{\text{CPL}} \) from 7 to 9kW that occurs at \( t=0.3s \). (\( \alpha = 10 \) degrees).

Similarly, Fig.9 is the response to a step change of \( P_{\text{CPL}} \) from 7 to 9kW for \( \alpha \) equal to 20 degrees.

VI. CONCLUSION

In this paper, the DQ modeling method is presented for modeling a three-phase AC distribution system with a three-phase controlled rectifier, DC-link filters, and an ideal CPL connected to the DC bus. The proposed approach is very useful for modeling the AC distribution system and also concerning a phase shift between source bus and AC bus. Moreover, the resulting converter models can be easily
combined with models of other power elements expressed in terms of synchronously rotating frames such as generators, front-end converters, and vector-controlled drives. For the future work, the DQ linearized model in this paper will be used to analyze the system stability due to the CPL.

REFERENCES


