EVALUATION OF EFFECTIVE FORCES ON PERFORATED-BALL VELOCITY METER (PVM) USING INVERSE PROBLEM OF MORISON EQUATION IN LARGE SCALE

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ABSTRACT

Due to random nature of ocean waves, determination of hydrodynamic forces on steel platforms generated by waves is quite complicated. The role of mathematical-empirical models in evaluation of wave forces on cylindrical elements of platforms is crucial. One such model is the Morrison’s equation. A Perforated-ball Velocity Meter (PVM) device measures the wave induced forces on a cantilevered ball-tube assembly and the wave kinematics is computed from these forces by solution of an inverse problem given by the Morrison’s equation. The drag and inertia coefficient of ball is used in the inverse Morison problem. The behavior and convergence of the numerical scheme for this inverse problem has been studied in detail. In the present paper, experimental result from large scale models at wave flume Delta of Delft hydraulic Laboratory in Netherlands and forces on Perforated-ball Velocity Meter (PVM) has been used to analyze the determination of water particle kinematics. The measured kinematics have compared with calculated kinematics obtained from the free surface elevation measurements using the inertia wave theory and the Wheeler stretching theory and exhibit with graphs. The comparisons have been satisfactory. It is also shown that the kinematics measured by the PVM can yield the free surface elevation with good accuracy.

KEY WORDS: Perforated-ball Velocity Meter - Morison’s Equation - Wave Kinematics - Inverse Problem Method.

INTRODUCTION

Analysis of Measured wave forces on model structures in the laboratory requires information about particle kinematics. among velocity measuring instruments, the Perforated-ball Velocity Meter (PVM) has certain advantages over others when used in large hydrodynamic test facilities. It can measure all the three components of velocities and the measurements can be made at different water depths simultaneously, by fixing the sensors at these depths. The PVM has been found to have advantages of simplicity and ruggedness and is free from the effect of stall, electrical noise and cross-talk. The PVM has been used in full-scale measurements in Christchurch Bay Tower [1] and further developed in the work of Chaplin and Subbiah [2].

A PVM device measures the wave-induced forces on a cantilevered ball-tube assembly and the wave kinematics is computed from these forces by solution of an inverse problem given by the Morison’s equation. The numerical scheme for the inverse problem, namely, obtaining kinematics from force measurements, which is developed in this paper, is more general than that used by Chaplin and Subbiah [2]. This, therefore, can handle a wider variation of configurations, which can be adopted in designing the PVM assembly[3].

In the present paper, the forces on Perforated-ball Velocity Meter (PVM) collected from large scale laboratory model in wave flume "Delta" (Delft hydraulic Laboratory, Netherlands) has been used to analyze the determination of water particle kinematics.

During September and October 1993 a series of experiments were undertaken to examine the wave loading on two large scale circular cylinders in the Delft Hydraulic Laboratory’s Delta wave flume in the Netherlands (DHL). This
flume is 250 m long, 5 m wide, 7 m deep and during the tests, was filled with water to a depth of about 5 m. The waves were generated by a programmable, hydraulically driven, piston type wave-maker and their energy was dissipated at the other end of the flume through the use of a 1:6 sloping concrete beach. This facility is capable of generating regular and random waves with a range of periods of about 3 to 10 s and wave heights up to about 2 m over most of the range of periods. For the random wave experiments the JONSWAP spectrum was used and the results presented in this paper are for experiments in long crest random waves with a significant wave height of 1.5 m and a peak period of 5.9 s. Two perforated-ball velocity meter were used during the course of the Experiments in elevation 1.5 m and 2.5 m [4].

In this paper used perforated-ball velocity meter located at 2.5 m and figure 2 to 4 shown Experimental data used this paper.

**PRINCIPLE OF THE PVM**

The PVM is essentially a drag device, in which a perforated hollow ball is mounted on a strain-gauged cantilever. Fig. 1 shows the schematic set-up and loading on the balls of a single stage velocity meter. The strain-gauged arm senses two mutually perpendicular components of force exerted by wave on the ball. The water particle velocity components are obtained from the measured forces. By fixing strain gauged arms in pairs at right angles to each other, and by keeping the plane of the cantilevers either vertical or horizontal, it is possible to measure all the three components of force. The angle between the wave direction and one of the cantilever arms is denoted α (angle with the other arm being (90-α)). Usually α = 45° is adopted in wave flume experiments and any value of α can be used in wave basin experiments. The purpose of the perforations on the ball is to render the drag coefficient of the ball insensitive to Reynold’s number and incident turbulence.

The forces measured by the PVM are \( f_j \) (j=1 to 4). Using the geometry of the arrangement, these forces can be resolved into x, y and z directions as

\[
\begin{align*}
    f_x &= \frac{f_1 + f_2}{\sqrt{2}} \\
    f_y &= \frac{f_2 - f_1}{2} \\
    f_z &= \frac{f_1 - f_2}{\sqrt{2}}
\end{align*}
\]

(1)

**HYDRODYNAMIC COEFFICIENTS OF THE BALL**

The components of the wave force on the ball is given by the Morison’s equation:

\[
\begin{pmatrix}
    f_x \\
    f_y \\
    f_z
\end{pmatrix} = \frac{1}{2} C_d \rho A \begin{pmatrix}
    u \\
    v \\
    w
\end{pmatrix} \begin{pmatrix}
    u \\
    v \\
    w
\end{pmatrix} + C_m \rho V \begin{pmatrix}
    u \\
    v \\
    w
\end{pmatrix}
\]

(2)

where \( u, v \) and \( w \) are the velocity components in x, y and z directions respectively, \( C_d \) and \( C_m \) are the drag and inertia coefficients of the perforated ball, respectively, \( A \) is the frontal area of the ball, \( V \) its enclosed volume, \( \rho \) the fluid density and

\[
q = \sqrt{u^2 + v^2 + w^2}
\]

(3)

A series of regular wave tests were carried out to evaluate the hydrodynamic coefficients of the perforated ball. The forces sensed by the perforated ball alone were obtained by deducting the forces resisted by the tube from their total values. The Keulegan–Carpenter (KC) number of the ball is given by

\[
KC = \frac{\sqrt{u_m^2 + w_m^2} T}{D}
\]

(4)

where \( u_m \) and \( w_m \) are the maximum normal velocity components in the horizontal and vertical directions, respectively, at the ball location, \( T \) the wave period and \( D \) the ball diameter. For the PVM developed, the data for the ball are
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\[ D = 0.099 \text{ m}, \quad A = 0.0077 \text{ m}^2 \] and \[ V = 5.08 \times 10^{-4} \text{ m}^3 \]. Then plotting the hydrodynamic coefficients against KB, the following regression formulae were obtained[5]:

\[
\begin{align*}
C_d &= 0.0004KC^2 - 0.0263KC + 1.5421 \\
C_m &= 0.0037KC^2 - 0.1816KC + 4.8839 \quad \text{for } KC \geq 50 \\
C_d &= 0.0011KC^2 + 0.0247KC^2 - 0.3263KC + 3.0864 \\
C_m &= 0.0003KC^2 - 0.0083KC^2 + 0.1271KC + 0.4438 \quad \text{for } KC < 50
\end{align*}
\] (5)

**INVERSE MORISON PROBLEM**

The measured forces \( f_j \) \((j=1 \text{ to } 4)\), can be resolved into component forces \( f_j(t) \); \( f_x(t) \) and \( f_y(t) \) as per Eq. (1). The task is to compute the corresponding component velocities \( u(t) \), \( v(t) \) and \( w(t) \) which are consistent with these force components. It should be recognized here that \( f_x \), \( f_y \) and \( f_z \) are the components of the total measured force, which is due to the ball. Hence, the total force was taken to be that due to the ball alone. Therefore, once the tube force is included in the Morison’s equation and hydrodynamic coefficients of the tube are known from experiments, as has been done in the present work.

The procedure of obtaining the velocity components is iterative and a starting point is provided by the liberalized Morison’s equation of the form:

\[
\begin{align*}
\{f_j\} &= \{\overline{\kappa}_j\} \{u\} + \{\overline{\kappa}_m\} \{\dot{u}\} \\
\overline{\kappa}_d &= \frac{1}{2} \rho C_d A \dot{q} \\
\overline{\kappa}_m &= \rho C_m V
\end{align*}
\] (7)

In the above, an over-bar denotes an estimate of the overall representative value. Starting value of these quantities, namely, \( \overline{\kappa}_d \), \( \overline{\kappa}_m \) and \( \dot{q} \); must be assumed to start the iteration process.

Let the discrete Fourier Transform (FT) of \( f_j(k) \) be \( F_j(k) \) \((j=x, y \text{ or } z)\) be and FT of \( u(t), v(t) \) and \( w(t) \) be \( \overline{U}_j(k) \), \( \overline{U}_j(k) \) and \( \overline{U}_z(k) \); respectively, where \( k \) is an integer denoting the frequency index and related to the frequency as \( f_j = \frac{2k\pi}{N\Delta t} \) where \( f_j \) is frequency in Hz of \( k \) th component of FT and \( N \) is the number of intervals of the force record sampled at \( \Delta t \) intervals of time. \( N \) and \( \Delta t \) are assumed equal for all \((j=x, y \text{ or } z)\) and same values are adopted for all other FT quantities. It may be noted that index \( k \) takes negative to positive values. In actual calculations a discrete FT is computed by FFT procedure.

Assuming the harmonic variation of \( \{u\} \) and \( \{\dot{u}\} \) as \( e^{-i\omega t} \); where \( \omega \) is the circular frequency, the FT of Eq. (7) gives the first approximation of \( U_j(k) \) as

\[
U_j^{(1)}(k) = \frac{F_j(k)}{k^{(0)}_d + i\overline{\kappa}^{(0)}_m \pi f(k)}; \quad j = x,y \text{ or } z
\] (9)

where the bracketed superscript denotes the iteration number and \( \overline{K}^{(0)}_d \) and \( \overline{K}^{(0)}_m \) denote starting values chosen. The inverse FT (IFT) of \( U_j(k) \) gives the first approximation of the velocity components \( u^{(1)}(t) \); \( v^{(1)}(t) \) and \( w^{(1)}(t) \); which, when differentiated with respect to time \( t \), gives the first approximation of the acceleration components \( \dot{u}^{(1)}(t) \); \( \dot{v}^{(1)}(t) \) and \( \dot{w}^{(1)}(t) \); Also, the first approximation of \( q^{(1)}(t) \) can be obtained from \( u^{(1)}(t) \); \( v^{(1)}(t) \) and \( w^{(1)}(t) \) using Eq.(3). The average of \( q^{(1)}(t) \) over the time series segment being used yields \( \overline{q}^{(1)}(t) \).

Now, one can construct envelop functions of \( u(t), v(t) \) and \( w(t) \) as \( \hat{u}(t) \); \( \hat{v}(t) \) and \( \hat{w}(t) \); respectively, and envelop functions of \( \dot{u}(t) \); \( \dot{v}(t) \) and \( \dot{w}(t) \) as \( \hat{\dot{u}}(t) \); \( \hat{\dot{v}}(t) \) and \( \hat{\dot{w}}(t) \); respectively. Towards this, one needs to compute Hilbert transform [3] of the time series. Consider the time series of \( u(t) \) only, whose FT is \( \overline{U}_s(k) \); Then, construct the function

\[
H_s(k) = (1 + \text{sgn} k) \overline{U}_s(k) = \left\{ \begin{array}{ll}
2U_s(k) & \text{for } k > 0 \\
0 & \text{for } k \leq 0
\end{array} \right.
\] (10)

and its IFT as \( \hat{h}_s(t) \); It may be verified that the function \( \hat{h}_s(t) \) is complex with real part \( u(t) \) and its imaginary part is denoted as \( \hat{\dot{u}}(t) \); which is called the Hilbert transform of \( u(t) \). In other words,

\[
\hat{h}_s(t) = u(t) + \hat{\dot{u}}(t)
\] (11)

The envelop function of \( u(t) \), denoted \( \hat{u}(t) \); is then

\[
\hat{u}(t) = \sqrt{u^2(t) + \hat{\dot{u}}^2(t)}
\] (12)

The above procedure can be adopted to obtain the envelope functions of \( u, v, w, \dot{u}, \dot{v} \) and \( \dot{w} \).

If the dominant wave motion is in the x direction, a slowly varying time series of frequency \( \omega_s \), denoted \( \hat{\omega}(t) \); may be obtained from

\[
\hat{\omega}^{(1)}(t) = \frac{\hat{\dot{u}}^{(1)}(t)}{u^{(1)}(t)}
\] (13)

which is the first approximation. Then a time dependent KC number, for the ball, can be computed for each component direction as first approximation:

\[
\{KC^{(1)}(t)\} = \frac{2\pi}{\hat{\omega}^{(1)}(t)D} \{ u^{(1)}(t) \}
\] (14)

Where
The KC numbers for the ball and the tube are then given by
\[ KC_{t} = 1 \left( KC_{b} / KC_{t} \right)^{1/2} \]
for some values of these coefficients in Eq. (11):
\[ k_{b}^{(1)}(t) = \frac{1}{2} \rho C_{b}^{(1)}(t) \dot{a} q^{(1)}(t) \]
\[ k_{m}^{(1)}(t) = \rho C_{m}^{(1)}(t) \dot{y} \]
(17)

The mean values of \( k_{b}^{(1)}(t) \) and \( k_{m}^{(1)}(t) \) over the time series segment being used yield their average values, denoted \( \bar{k}_{b}^{(1)}(t) \) and \( \bar{k}_{m}^{(1)}(t) \); respectively. These and \( \bar{\dot{y}}^{(1)} \) are then used in starting the second iteration using Eq. (9) by obtaining \( U_{j}^{(2)}(t) \) and the steps that follow as described above.

The iteration is continued until convergence of the velocity vector \( \{ u(t) \} \) with a specified tolerance (say \( 1 \times 10^{-8} \)) on its norm vector. It was found desirable to filter the velocities and accelerations above 5 Hz at each step for better accuracy and faster convergence. The procedure was applied in turn to segments of 10200 data points and it converged in each case within a few cycles.

**SCHEME FOR NUMERICAL COMPARISON**

The wave kinematics computed from the computer code implementing the Inverse Morison Problem algorithm needs to be compared with an alternative source in order to validate the performance of the PVM. The alternative could either be a well-proven velocity meter such as LDA or an analytical method. Since no alternative velocity meter was available, an analytical method is adopted in this work to validate the PVM measurements. This is based upon the fact that if the wave elevation is recorded at the same longitudinal location as the PVM assembly, then a Fourier analysis of this record can yield the wave kinematics by the use of any appropriate wave theory. The wave theories used for this purpose are the linear wave theory, wave elevation of a long crested sea, propagating in x direction, can be expressed as a sum of its Fourier components,
\[ \eta(x,t) = \sum_{j=1}^{n} a_{j} \cos \theta_{j} \]
(18)
where \( a_{j} \); \( K_{j} \); \( \omega_{j} \) and \( \beta_{j} \) are the amplitude, wave number, wave frequency and phase, respectively, of the \( j \)th harmonic:
The total number of harmonics \( N \) is related to the cut-off frequency. For the \( j \)th wave component, the velocity potential is [5]:
\[ \phi_{j} = \frac{a_{j} \omega_{j}}{k_{j}} \frac{\cosh k_{j}(z + d)}{\sinh k_{d}} \sin \theta_{j} \quad (-d \leq z \leq 0) \]
(19)
where \( d \) is the water depth. The wave number \( k_{j} \) and wave frequency \( \omega_{j} \) satisfy the linear dispersion relation,
\[ \omega_{j}^{2} = k_{j} g \tanh k_{j} d \]
(20)

The Wheeler stretching method maps the linear wave velocity field for \( -d \leq z_{w} \leq 0 \) the instantaneous wave surface through a co-ordinate transfer [6]:
\[ z_{w} + d = \frac{z + d}{1 + \eta \frac{d}{d}} \quad (-d \leq z \leq \eta) \]
(23)
The stretched wave velocities are obtained by replacing \( z \) with \( z_{w} \) in Eq. (22) and then using Eq. (23):
\[ \left\{ \begin{array}{c} u \\ w \end{array} \right\} = \sum_{j=1}^{N} a_{j} \omega_{j} \left\{ \begin{array}{c} k_{j} \frac{\cosh k_{j}(z + d)}{\sinh k_{d}} \cos \theta_{j} \\ k_{j} \frac{\sinh k_{j}(z + d)}{\sin k_{d}} \sin \theta_{j} \end{array} \right\} \quad (-d \leq z \leq \eta) \]
(24)

This method is applicable up to the instantaneous free surface \( \eta \). Hence, from measured \( \eta(t) \); one can compute \( u(t) \) and \( w(t) \) from Eq. (24), which can be compared with the experimental results using the PVM.

The results of one typical measurement obtained by the PVM developed have been compare using above method and are shown in figs. 2 to 4. The quality of comparisons is similar to those presented by Chaplin and Subbiah.
NUMERICAL FEATURES OF THE INVERSE MORISON PROBLEM

The convergence properties of the iteration scheme of the inverse Morison problem has been studied extensively and found to be quite robust. For the sample as shown in Fig.5, three sets of initial values of \( C_d, C_m \), and \( q \) were chosen. The values in the first set could be called ‘reasonable’ whereas all the values in the second set are much higher than actual and those in the third set much lower than actual. In fact, the chosen values of the coefficients of the second and third sets are beyond the range of the coefficients given in Eqs. (5) and (6). This can be ascertained by calculating the coefficients at limiting values of KC numbers of the ball and the tube. The convergence of; as the iteration progresses, is shown in Fig. 5. The quality of \( \|u(t)\| \) convergence of \( \|w(t)\| \) is similar and hence not shown. The updated average values of the coefficients of the linearised Morison’s equation (Eq. (7)) as well as \( \bar{q} \) as obtained from the steps 9 and 4 of Table 1 are shown in Table 2 for all the three cases. The tolerance on vector norm of \( \|u(t)\| \) in all the cases was \( 1 \times 10^{-8} \). It may be seen from this table that within about six iterations, the error norm of about \( 1 \times 10^{-4} \) was achieved and it took about 12–13 iterations to achieve a norm of \( 1 \times 10^{-8} \). Also, \( \bar{k_d}, \bar{k_m} \), and \( \bar{q} \); despite widely off from their actual values in the first iteration, settles to values close to actual in about four iterations. Many other combinations of starting values, some lower and others higher than actual were chosen and iteration scheme has been found to be equally robust in all of them.
DISCUSSION

The PVM was studied in detail in the large scale. In this paper, a few aspects of the PVM have been improved or studied in greater detail. Firstly, the iterative inverse Morison problem proposed for the ball–tube transducer assembly in this work is sufficiently general to admit a variety of size combinations for its design. This is particularly important because the adoption of a relatively smaller cross section of the tube, which will make its effect negligible, has attendant problems of vibration and deflection of the cantilever transducer assembly. Secondly, the iterative process has been numerically studied in detail and found to be quite insensitive to the initial estimates of parameters and its convergence is seen to be at least quadratic. Thirdly, it is demonstrated that it is possible to effectively compare the measured kinematics by Fourier analysis of the measured wave elevation records instead of using an alternative costly velocity meter such as LDA. Fourthly, it is shown that the measured kinematics could be used to predict free surface elevations also with good accuracy.

A typical result of measurement of $U$ in a regular wave in the wave basin using 4 stages is shown in Fig.6, where the measured kinematics have been compared with those obtained from Wheeler stretching theory using time series of 7. The comparison is reasonably good for all stages. The quality of comparison for $w$ is as good and hence not shown.

Validation tests of the three-component perforated-ball velocity meter (PVM) in uni-directional and multi-directional waves suggest that it is a viable alternative to an electromagnetic flow meter. Computing velocity records from force measurements involves the solution of what is referred to here

\[ SET2 : C_d = 1, \quad C_m = 0.6, \quad q = 0.15 \]

<table>
<thead>
<tr>
<th>Iteration No.</th>
<th>$u$</th>
<th>$w$</th>
<th>$k_d$</th>
<th>$k_m$</th>
<th>$\bar{q}$</th>
</tr>
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</tr>
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</tr>
<tr>
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<td>0.388221</td>
<td>0.002262</td>
</tr>
<tr>
<td>4</td>
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<tr>
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<tr>
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<tr>
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<td>0.061089</td>
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\[ SET3 : C_d = 0.1, \quad C_m = 0.1, \quad q = 0.01 \]

<table>
<thead>
<tr>
<th>Iteration No.</th>
<th>$u$</th>
<th>$w$</th>
<th>$k_d$</th>
<th>$k_m$</th>
<th>$\bar{q}$</th>
</tr>
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</tr>
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</table>

\[ SET2 : C_d = 3, \quad C_m = 3, \quad q = 3 \]
as the inverse Morison problem. Achieving converged results for the case where drag and inertia forces are both important, and where both coefficients are functions of Keulegan Carpenter number (and the inertia coefficient also depends on ellipticity) presented an unexpectedly difficult problem that seems not to have been addressed before. The success of the technique described in this paper (in which the Keulegan Carpenter number and the frequency are treated as slowly varying parameters) suggest that historical element in Morison loading has been adequately represented in a quasi-steady model. It should be noted however that the perforated-ball has well-defined Morison coefficients that (in the range of inertia here) have been addressed before. The success of the technique described in this paper (in which the Keulegan Carpenter number and the frequency are treated as slowly varying parameters) suggest that historical element in Morison loading has been adequately represented in a quasi-steady model. It should be noted however that the perforated-ball has well-defined Morison coefficients that (in the range of inertia here) are independent of Stokes parameter, and it is not subject to the effects of vortex shedding.

A possible improvement to the PVM presented to align the cantilever arms of the balls in the horizontal and vertical direction rather than 45° the configuration used here. Although supporting the instrument would then be slightly more difficult, the effect of small geometrical errors would be reduced. As an instrument for measuring particle velocities, the PVM has advantages of physical simplicity and ruggedness, and is cheap produce. Its shortcoming arise from its non-linear nature; accuracy is reduce at low velocities, and the complexity of the numerical computation that is required seem to rule out real-time signal processing. The techniques of system identification that have been explored in the context of Morison equation with mixed success might lead to improvements in this respect.

REFERENCES