A Fuzzy Multi Objective Model for Supplier Selection

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Abstract: - Supplier selection and determination of order quantities with selected suppliers is one of the most important activities in supply chain management. Hence, supplier selection problem should be performed by scientific methods and systematic approaches. In this study, a fuzzy multi objective model with an approach for supplier selection is presented. The fuzzy multi objective model with minimum and maximum order quantity constraint, delivery delay time constraint, defect number constraint and shortage number constraint is developed, then supplier selection problem solved by the extended model with fuzzy criteria weights that calculates by logarithmic least squares. In order to better description of the proposed approach and presented model, an example presented.

Key-Words: - Supplier selection, Fuzzy multi objective model, Logarithmic least squares approach.

1 Introduction
Supplier selection decisions are an important component of production and logistics management for many firms. Such decisions entail the selection of individual suppliers to employ, and the determination of order quantities to be placed with the selected suppliers. Selecting right suppliers significantly reduces the component purchasing cost and improves corporate competitiveness, which is why many experts believe that the supplier selection is the most important activity of a purchasing department. Basically there are two kinds of supplier selection problem: (1) Single sourcing. Constraints are not considered in the supplier selection process. In other words, all suppliers can satisfy the buyer’s requirements of demand, quality, delivery, etc. The buyer only needs to make one decision, which supplier is the best. (2) Multiple sourcing. Some limitations such as supplier’s capacity, quality and delivery are considered in the supplier selection process. In other words, no one supplier can satisfy the buyer’s total requirements and the buyer needs to purchase some part of demand from one supplier and the other part from another supplier to compensate for the shortage of capacity or low quality of the first supplier. In these circumstances buyers need to make two decisions: which suppliers are the best, and how much should be purchased from each selected supplier [1]. Supplier selection is a multiple criteria decision-making (MCDM) problem which is affected by several conflicting factors. Consequently, a purchasing manager must analyze the trade off among the several criteria. MCDM techniques support the decision-makers (DMs) in evaluating a set of alternatives. Depending upon the purchasing situations, criteria have varying importance and there is a need to weight criteria. The vast majority of the decision models applied to the supplier selection are linear weighting models and mathematical programming models. In linear weighting models weights are given to the criteria, the biggest weight indicating the highest importance. Ratings on the criteria are multiplied by their weights and summed in order to obtain a single figure for each supplier. The supplier with the highest overall rating can then be selected. Given an appropriate decision setting, mathematical programming allows the decision maker to formulate the decision problem in terms of a mathematical objective function that subsequently needs to be maximized or minimized by varying the values of the variables in the objective function. According to the number of objective functions, the supplier selection problem can be divided into two groups [1]: (1) Single objective (2) Multiple objectives. In a real situation for a supplier selection problem, many input information are not known precisely. At the time of making decisions, the value of many criteria and constraints are expressed in vague terms such as “very high in quality” or “low in price”. Deterministic models cannot easily take this vagueness into account. In these cases the theory of fuzzy sets is one of the best tools for handling uncertainty. For example, Bellman and Zadeh [2] suggested a fuzzy programming model for decision-making in fuzzy environments. Over the years, several techniques have been developed to solve the problem efficiently such as Analytic hierarchy process (AHP), analytic network process (ANP), linear programming
(LP), mathematical programming, multi-objective programming, data envelopment analysis (DEA), neural networks (NN), case based reasoning (CBR) and fuzzy set theory (FST) methods. For example, Onut & et al. [3] present a combined method on the basis of fuzzy ANP and fuzzy TOPSIS for supplier selection that FANP is applied to calculate criteria weights and fuzzy TOPSIS is used to select a proper supplier. Dickson [4] firstly identified and analyzed the importance of 23 criteria for supplier selection based on a survey of purchasing managers. He showed that quality is the most important criterion followed by delivery and performance history. Weber et al. [5] reviewed 74 articles discussing supplier selection criteria, and showed that net price is the most important criterion for supplier selection. They also concluded that supplier selection is a multi criteria problem and the priority of criteria depends on each purchasing situation. A comprehensive review of criteria for supplier selection is presented in Ghodsypour and O’Brien [6]. They concluded that the number and the weights of criteria depend on purchasing strategies. Gaballa [7] is the first author who applied mathematical programming to supplier selection in a real case. He used mixed integer programming to minimize the total discounted price of allocated items to the suppliers. Weber and Current [8] used a multi objective approach to systematically analyze the trade-offs between conflicting criteria in supplier selection problems. Ghodsypour and O’Brien [9] developed an integrated AHP and linear programming model to consider both qualitative and quantitative factors in purchasing activity. Karpak et al. [10] used a goal programming model to minimize costs and maximize delivery reliability and quality in supplier selection when assigning the order quantities to each supplier. Degraeve and Roodhooft [11] developed a total cost approach with mathematical programming to treat supplier selection using activity-based cost information. Ghodsypour and O’Brien [12] developed a mixed-integer non-linear programming approach to minimize total cost of logistics, including net price, storage, ordering costs and transportation in supplier selection. Xia and Wu [13] proposed an AHP approach and multi objective mixed integer programming for determined the number of suppliers. Wang et al. [14] used the fuzzy programming model for vendor selection problem in supply chain. Ustun and aktar [15] proposed an integrated multi objective mixed integer linear programming and ANP method for supplier selection. Amid et al. [16] proposed a fuzzy multi objective linear model with capacity constraint. In this study, the fuzzy multi objective model developed with minimum and maximum order quantity constraint, delivery delay time constraint, defect number constraint and shortage number constraint, then supplier selection problem solved by the extended model with fuzzy criteria weights that calculates by logarithmic least squares[17]. The rest of paper is organized as follow: Section 2 describes the proposed multi objective model and section 3 presents integrated approach for supplier selection problem solution. Section 4 describes numerical example and section 5 presents conclusions.

2 Problem Formulations

A general multi objective model for the supplier selection problem can be stated as follows [8, 12]:

\[
\min Z_1, Z_2, \ldots, Z_K \\
\max Z_{K+1}, Z_{K+2}, \ldots, Z_P \\
s.t. \\
x \in X_d
\]

Where \(Z_1, Z_2, \ldots, Z_K\) are the negative objectives or criteria-like cost, late delivery, etc, and \(Z_{K+1}, Z_{K+2}, \ldots, Z_P\) are the positive objectives or criteria such as quality, on time delivery, after sale service and so on. \(X_d\) is the set of feasible solutions which satisfy the constraint such as buyer demand, supplier capacity, etc. A typical linear model for supplier selection problems is: \(\min Z1; \max Z2, Z3\) with

\[
Z_1 = \sum_{i=1}^{n} P_i x_i \\
Z_2 = \sum_{i=1}^{n} F_i x_i \\
Z_3 = \sum_{i=1}^{n} S_i x_i \\
s.t: \\
\sum_{i=1}^{n} x_i \geq D \\
x_i \leq C_i \quad i=1,2,\ldots,n \\
x_i > 0 \quad i=1,2,\ldots,n
\]

Where \(D\) is demand over period, \(x_i\) is the number of units purchased from the \(i^{th}\) supplier, \(P_i\) is per unit net purchase cost from supplier \(i\), \(C_i\) is capacity of \(i^{th}\) supplier, \(F_i\) is percentage of quality level of \(i^{th}\) supplier, \(S_i\) is percentage of service level of \(i^{th}\) supplier, \(n\) is number of suppliers. Amid et al. [16] developed a fuzzy multi objective linear model for supplier selection
problem and presented appropriate operators for model solution. Their model find a vector \( x \) which minimizes objective function \( Z_k \) and maximizes objective function \( Z_l \) with

\[
\begin{align*}
\text{Min} Z_k &= \sum_{i=1}^{\infty} c_{ki} x_i & k = 1,2,\ldots, p \quad (10) \\
\text{Max} Z_l &= \sum_{i=1}^{\infty} c_{li} x_i & l = p+1, p+2,\ldots,q \quad (11)
\end{align*}
\]

where, \( c_{ki}, c_{li}, a_{ri} \) and \( b_r \) can be crisp or fuzzy values.

### 2.1 The proposed model

The proposed model is based on the model that presented by Amid et al. [16]. The proposed model solves the supplier selection problem with fuzzy criteria weights in objective functions that is calculated by logarithmic least squares. Also, minimum and maximum order quantity, delivery delay time, defect number and shortage number considered as constraints.  

**Decision variables:** The decision variables, which are represented as \( x_i \) are the number of parts to be purchase from supplier i.  

**Objective function:** The objective functions of the fuzzy multi objective model are in the order of maximization of positive criteria and minimization of negative criteria. The Criteria weights of the two objective functions have been calculated based on logarithmic least squares method.  

**Constraints:** The constraints of the decision problem are buyer’s demand and minimum and maximum order quantity constraint, delivery delay time constraint, defect number constraint and shortage number constraint.  

The mathematical models are constructed by using the following notation:

**Indices:**

\( i \) 1, 2, \ldots, n index of suppliers  
\( j \) 1, 2, \ldots, m index of criteria  
\( J \in \{ J, J' \} \) negative criteria  
\( J' \) positive criteria  

**Decision variables:**

\( x_i \) Number of parts to be purchased from supplier i  

**Parameters:**

\( c_{ij} \) Score of supplier i from criteria j  

\( w_j \) Criteria weights obtained from logarithmic least squares method  
\( D \) Demand of the part  
\( s_{li} \) Minimum order quantity from supplier i  
\( s_{ui} \) Maximum order quantity from supplier i  
\( t_i \) Delivery delay time of supplier i  
\( T \) buyer’s maximum acceptable delivery delay time  
\( p_i \) Defect number of supplier i  
\( P \) buyer’s maximum acceptable defect number  
\( b_i \) Shortage number of supplier i  
\( B \) Buyer’s maximum acceptable shortage number

The objective functions and the constraints of this model are as follows:

\[
\begin{align*}
\text{Min} \sum_{i=1}^{n} \sum_{j=1}^{m} w_j c_{ij} x_i & \quad (j \in J, \text{for negative criteria}) \\
\text{Max} \sum_{i=1}^{n} \sum_{j=1}^{m} w_j c_{ij} x_i & \quad (j \in J', \text{for positive criteria}) \\
\sum_{i=1}^{n} x_i & \geq D \quad (16) \\
\sum_{i=1}^{n} x_i t_i & \leq T . D \quad (17) \\
\sum_{i=1}^{n} x_i p_i & \leq P . D \quad (18) \\
\sum_{i=1}^{n} x_i b_i & \leq B . D \quad (19) \\
\end{align*}
\]

3 Problem Solution

3.1 The model solution

One of the best approaches for solving multi objective programming problem with fuzzy objective function \( (c^p, c^m, c^o) \) is using following models: Consider the following linear programming model with fuzzy objective function:
max \( Z = CX \)
\[ s.t. \]
\( AX \leq B \)
\( X \geq 0 \)

Center value \((c^m X)\) have to be maximized, left value \(((c^m - c^p) X)\) minimized and right value \(((c^o - c^m) X)\) maximized. Then each objective can be formulated as follow:

\[ \begin{align*}
\min & \quad Z_1 = (c^m - c^p) X \\
\max & \quad Z_2 = c^m X \\
\max & \quad Z_3 = (c^o - c^m) X \\
\end{align*} \]

\[ s.t. : \]
\( AX \leq b \)
\( X \geq 0 \)

One method to solve this multi objective optimization problem is to optimize the objective by computing membership function. In this method, first, problem is solved in best and worst condition for each objective function independently. Then on the basis of them, linear membership function can be obtained:

\[ Z_i^l = \min(c^m - c^p) X \]
\[ s.t. : \]
\( AX \leq b \)
\( X \geq 0 \)

\[ Z_i^u = \max(c^m - c^p) X \]
\[ s.t. : \]
\( AX \leq b \)
\( X \geq 0 \)

\[ \mu_{Z_i}(Z_1) = \begin{cases} 
1 & Z_1 \leq Z_i^l \\
\frac{Z_i^u - Z_i^l}{Z_i^u - Z_i^l} & Z_i^l \leq Z_1 \leq Z_i^u \\
0 & Z_1 \geq Z_i^u 
\end{cases} \]

Membership functions for \( Z_2, Z_3 \) can be obtained as above relation and finally the problem converts to a crisp linear programming problem as follow:

\[ \max \lambda \]
\[ s.t. : \]
\( \lambda(Z_i^u - Z_i^l) \leq Z_i^u - (c^m - c^p) X \)
\( \lambda(Z_2^u - Z_2^l) \leq c^m X - Z_2^l \)
\( \lambda(Z_3^u - Z_3^l) \leq (c^o - c^m) X - Z_3^l \)
\( AX \leq b \)
\( X \geq 0 \)
\( \lambda \geq 0 \)

It can obtain optimal values with solving the above problem.

### 3.2 Logarithmic least squares method

Logarithmic least squares method presented by Mehdizadeh and Ayobi [21] for supplier evaluation. Triangular fuzzy numbers is used in all pairwise comparison matrices. Hence, criteria weights were calculated as the triangular fuzzy numbers and then these fuzzy criteria weights are inserted to the fuzzy TOPSIS methodology to rank the alternatives.

In the pairwise comparison of attributes, DM can use triangular fuzzy numbers to state their preferences. A scale of 1 - 9 can be defined for triangular fuzzy numbers. When comparing attribute i with attribute j, \( \tilde{1} \), \( \tilde{3} \), \( \tilde{5} \), \( \tilde{7} \) and \( \tilde{9} \) indicate equal importance among the compare attribute, weak importance of i over j, strong importance of i over j, very strong importance of i over j and extreme importance of i over j, where \( i=1,2,\ldots,m, \) and \( j=1,2,\ldots,n \). comparison scale is shown in table 1. To evaluate the DM preferences, pairwise comparison matrices are structured by using triangular fuzzy numbers \((l, m, u)\). The \( m \times n \) triangular fuzzy matrix can be given as follow [19]:

<table>
<thead>
<tr>
<th>Comparison scale</th>
<th>Linguistic scale for importance</th>
<th>Fuzzy number</th>
<th>Triangular fuzzy scale</th>
</tr>
</thead>
<tbody>
<tr>
<td>Equal importance</td>
<td>( \tilde{1} )</td>
<td>(1, 1, 3)</td>
<td></td>
</tr>
<tr>
<td>weak importance</td>
<td>( \tilde{3} )</td>
<td>(1, 3, 5)</td>
<td></td>
</tr>
<tr>
<td>Strong importance</td>
<td>( \tilde{5} )</td>
<td>(3, 5, 7)</td>
<td></td>
</tr>
<tr>
<td>very strong importance</td>
<td>( \tilde{7} )</td>
<td>(5, 7, 9)</td>
<td></td>
</tr>
<tr>
<td>extreme importance</td>
<td>( \tilde{9} )</td>
<td>(7, 9, 9)</td>
<td></td>
</tr>
</tbody>
</table>
reasonable and effective, and it is used in this study. 

### 3.3 The fuzzy integrated approach

The proposed fuzzy integrated approach algorithm for supplier selection problem is given in Fig.1. The initialization phase includes two stages (Identify the groups of suppliers in the decision environment and determine supplier information), phase I includes two stages (Form the pairwise comparison matrix and calculate the criteria weights by logarithmic least squares method) and phase II includes three stages (Form the multi objective programming model and determine the number of parts to be purchased from supplier i).

#### 4 Numerical results

The supplier selection model is implemented in an auto-parts supplier company in Iran [21]. The determined criteria are given as follows:

- $C_1$: Standardization
- $C_2$: Product and process audit
- $C_3$: Customer PPM
- $C_4$: Competitive price
- $C_5$: Price reduction rate
- $C_6$: Order realization
- $C_7$: Just in time delivery

$C_1$ is negative criteria and the others are positive. $C_1$, $C_2$ and $C_3$ are related to quality criteria, $C_4$ and $C_5$ are related to cost criteria and $C_6$ and $C_7$ are related to delivery criteria. For calculating fuzzy weights of criteria, firstly the experts completed the designed forms and then the pairwise matrix can be filled. Secondly, the fuzzy weights of criteria by logarithmic least squares method calculated. For supplying an auto-part to a market, two suppliers managed in the company. There are three main criteria and seven sub-criteria. Quality, cost and delivery are main criteria and standardization, product and process audit, customer PPM, competitive price, price reduction rate, order realization and just in time delivery are sub-criteria.

The value of sub-criteria and constraints of suppliers are presented in Table 2. The weight of criteria calculated by logarithmic least squares method and the results of the solution process are given in Table 3 (Suppose $B=T=P=2$). The multi objective model of sub-criteria is presented as Max $Z_1$ (standardization and Product and process audit, Competitive price, Order realization and Just in time delivery) and Min $Z_2$ (customer PPM):

For solving the mentioned example and obtaining the optimal solution, the presented method in section 3 can be used.
Finally, the fuzzy multi objective obtained as follow:

\[
\begin{align*}
\text{max } Z_1 &= (0.094,0.177,0.28)(40x_1 + 40x_2) \\
&+ (0.052,0.093,0.156)(18x_1 + 10.8x_2) \\
&+ (0.031,0.056,0.092)(71.84x_1 + 80x_2) \\
&+ (0.119,0.174,0.344)(47x_1 + 43x_2) \\
&+ (0.174,0.282,0.479)(53x_1 + 53x_2) \\
\text{min } Z_2 &= (0.091,0.157,0.24)(120x_1 + 160x_2) \\
\end{align*}
\]

\[
\begin{align*}
s.t.:
& x_1 + x_2 \geq 274000 \\
& 2x_1 + x_2 \leq 548000 \\
& 2x_1 + 2x_2 \leq 548000 \\
& x_1 + 2x_2 \leq 548000 \\
& 100000 \leq x_1 \leq 250000 \\
& 100000 \leq x_2 \leq 250000 \\
\end{align*}
\]

Finally, the fuzzy multi objective obtained as follow:

\[
\begin{align*}
\text{max } w &= w \times 23014 \leq 3849562 - 14.163 \times x_1 - 13.852 \times x_2 \\
&+ w \times 67266 \leq 35.901 \times x_1 + 34.992 \times x_2 - 9678708 \\
&+ w \times 62160 \leq 26.271 \times x_1 + 25.431 \times x_2 - 7052094 \\
&+ w \times 195360 \leq 7.92 \times x_1 + 10.56 \times x_2 - 2434080 \\
&+ w \times 464720 \leq 6254880 - 18.84 \times x_1 - 25.12 \times x_2 \\
&+ w \times 245680 \leq 3306720 - 9.96 \times x_1 - 13.28 \times x_2 \\
\end{align*}
\]

\[
\begin{align*}
x_1 + x_2 &\geq 274000 \\
2x_1 + x_2 &\leq 548000 \\
x_1 + 2x_2 &\leq 548000 \\
x_1 &\geq 100000 \\
x_1 &\leq 250000 \\
x_2 &\geq 100000 \\
x_2 &\leq 250000 \\
w &\geq 0 \\
\end{align*}
\]
5 Conclusion
Supplier evaluation is one of the most important activities in supply chain management. Most of the companies can improve their competitive advantage by means of good supplier management. Hence, supplier evaluation should be performed by a systematic approach. In this study, an integrated logarithmic least squares method and a fuzzy multi objective model was proposed for supplier selection. In this study a fuzzy multi objective model with minimum and maximum order quantity constraint, delivery delay time constraint, defect number constraint and shortage number constraint developed, then supplier selection problem solved by the extended model with fuzzy criteria weights that calculates by logarithmic least squares. Logarithmic least squares method is reasonable and effective for calculating triangular fuzzy weights of criteria. As an example, The integrated approach implemented for a company with two suppliers.

Three major criteria such as quality, price and delivery and sub criteria from each three major criteria have considered in proposed approach and that is main advantage of the suggested approach. For further research, suppliers inventory investment, purchaser and supplier warehouses space can be considered as problem constraints. Also an approach for problems with both fuzzy objective functions and fuzzy constraints can be presented. Other multi objective decision making approaches such as AHP, fuzzy AHP, ANP and fuzzy ANP for criteria weights in proposed fuzzy approach can be used.

References:

### Table 3.

<table>
<thead>
<tr>
<th>Sub criteria</th>
<th>Fuzzy weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standardization</td>
<td>(0.094,0.177,0.28)</td>
</tr>
<tr>
<td>Product and process audit</td>
<td>(0.052,0.093,0.156)</td>
</tr>
<tr>
<td>Customer PPM</td>
<td>(0.091,0.157,0.24)</td>
</tr>
<tr>
<td>Competitive price</td>
<td>(0.031,0.056,0.092)</td>
</tr>
<tr>
<td>Price reduction rate</td>
<td>(0.04,0.06,0.1)</td>
</tr>
<tr>
<td>Order realization</td>
<td>(0.119,0.174,0.344)</td>
</tr>
<tr>
<td>Just in time delivery</td>
<td>(0.174,0.282,0.479)</td>
</tr>
</tbody>
</table>


