First Order Model for an $\alpha$-$\beta$-$\sigma$ Radar Tracker

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Abstract: - This paper aims to present a possible solution to improve the radar tracking process. It is well known that most filters currently uses in the process of tracking the kinematic parameters. It is important to remember that the current radar systems are capable of providing a wide range of information about the tracking target. Most of these information are classified as non-kinematic parameters of the target. One of these parameters can be the number of pixels of the target image associated with its PPI display image. Briefly, this parameter may be called "target area" and is available specifically in naval radars. This paper introduces the parameter in the process of tracking and presents a mathematical model of the tracker also called $\alpha$-$\beta$-$\sigma$. Model takes into account the fact that this parameter actually varies depending on distance from the observer.

Key-Words: - Radar tracking;

1 Introduction

It is known that the fundamental problem in tracking is that the origin of the measurement is unknown. In other words, we can not say with certainty whether a particular measurement belongs to a particular tracking target or noise. Moreover, it can happen that a target can not be detected at some point. Usually, to solve the problem, several assumptions regarding the measurements membership are made.

Optimal algorithms must take into account these hypotheses, but is immediately apparent as soon as the necessary computing power requirement is very high. Interesting to note is that the best performance is not necessarily obtained by implementing optimal algorithms.

It must be added that in many applications one of the performance criteria is the correct combination of data and not the error covariance matrix. This problem is known in literature as "Track Purity". It actually consists in the fact that a particular sequence of measurements was correctly attributed to a particular target. When, regions with high density of targets and great noise are considered, it is very easy to mistake association.

After 1990, among the MTT technique specialists began to put the increasingly acute problem of lack of Cramer-Rao bound. Bar Shalom, Chang and Bloom [1], then Y Ho and Agrawal in [2] also come to solve that lack but not solve the problem. One can observe that in [1], a Cramer-Rao bound for covariance matrix of errors is highlighted but its evaluation requires Monte Carlo simulations while the definition introduced in [2] does not require it. Another type of bound is found in [3], where Rihaczek introduces another kind of bound similar to Cramer-Rao.

One of the solutions, that can greatly improve the performance of data association in a tracker, resides in the introducing non-kinematic parameters in the state vector of the tracker. In [4], Daum advances at least five reasons for introducing the non-kinematic parameters in state vector:

1. It is an elegant method to treat a problem of decision using the dynamic estimation techniques formalism [2];
2. The real time decision rules profits by accuracy quantification of the parameters participating in decision process. The accuracy is increased disseminating an error covariance matrix for the non-kinematic variables. This includes the effect of bad associated measurements or unresolved measurements.
3. In some applications the dynamics of the kinematic variables is coupled with the dynamics of the non-kinematic variables. The explicitly modeling of these links improves the decision of association;
4. Standard techniques for mitigating the effect of poor data association used for the
kinematic measurements can be applied in case of non-kinematic measurements. For example: PDA, JPDA, MHT or any other algorithm can be applied to estimate non-kinematic measurements as well as the kinematics;

5. Including the non-kinematic variables in state vector improves the process performance of data association algorithm, whether working with PDA, JPDA, NNPDA, MHT, or alignment algorithms.

This paper introduces as a non-kinematic variable - in tracking process - the number of pixels associated with target (briefly "target area"), in the aim point extractor. This parameter is easy to obtain.

2 Problem Formulation

The mathematical model for an α-β tracker is:

Dynamic equations

\[
\begin{align*}
    x(k+1) &= x(k) + x'(k)T + w_x(k) \\
    x'(k+1) &= x'(k) \\
    y(k+1) &= y(k) + y'(k)T + w_y(k) \\
    y'(k+1) &= y'(k)
\end{align*}
\]

(1) \quad (2) \quad (3) \quad (4)

Measurement equations

\[
\begin{align*}
    z_x(k) &= x(k) + v_x(k) \\
    z_y(k) &= y(k) + v_y(k)
\end{align*}
\]

(5) \quad (6)

where:

\( x, y \) \quad \text{target geometrical position;}

\( x', y' \) \quad \text{target speed projection on x-y axes;}

\( w_x, w_y \) \quad \text{process noise;}

\( v_x, v_y \) \quad \text{measurement noise}

As one can see, the equations numbered (1) to (6) use exclusively kinematic parameters. In terms of mathematical model, ship acts as a geometrical point. On the other hand, the literature highlights that from a target a set of hits are returning. Figure 1 shows such a situation. The number of hits returning from a target is:

\[
P = \frac{\theta \times f \times \frac{1}{\omega}}{360}
\]

(7)

where:

\( P \) \quad \text{number of hits returning from a target;}

\( \theta \) \quad \text{beam width;}

\( f \) \quad \text{impulses frequency}

\( \omega \) \quad \text{angular speed}

Figure 2 presents the starboard image of a target.

Figure 3 presents the same target but on starboard-bow view.

Finally, figure 4 presents the bow image of the same target.

Figures 2, 3 and 4 show how the "image" associated to a target varies depending on the angle that it is "seen." Also, this "image" varies with distance from the observer. This paper aims to use this image in the process of tracking. In other words the system of
equations (1) ÷ (6) may be completed with two more equations, namely:

**Dynamic equations**

\[ \sigma(k + 1) = f[\sigma(k), w_\sigma(k)] \]  
(8)

**Measurement equations**

\[ z_\sigma(k) = \sigma(k) + v_\sigma(k) \]  
(9)

where:

\( \sigma \) target cross-section;  
\( w_\sigma \) process noise;  
\( v_\sigma \) measurement noise.

These two equations complete the model (1) ÷ (6). In formal terms, the aim of this paper is to express equation (8) more precisely. The main problem is to develop the law of variation of “\( \sigma \)” related to distance “\( r \)”. In fact, only this type of variation will be attended to. Remember that the zero order model assumed that this parameter is constant.

### 3 Problem Solution

In the following, one intends to find a relationship between parameter "\( \sigma \)” and the distance “\( r \)” from the observer. Analysis of figures 2-4 allows us to approximate the “target area”, by a rectangle. In these conditions:

\[ \sigma(r) = n_d(r)n_\phi(r) \]  
(10)

where:

\( n_d(r) \) number of increments in the distance to a target located at distance \( r \) from the observer;  
\( n_\phi(r) \) number of increments in azimuth to a target located at a distance ‘\( r \)’ observer.

Accepting for \( n_d(r) \) and \( n_\phi(r) \) a variation inversely proportional to distance:

\[ n_d(r) = \frac{\text{const.}}{r} \]  
(11)

\[ n_\phi(r) = \frac{\text{const.}}{r} \]  
(12)

It is easy to reach:

\[ \sigma(t) = \frac{k}{r(t)^2} \]  
(13)

where:

\( k \) is a normalization constant whose value is fixed in the acquisition process;  
\( r(t) \) target distance from observer.

Differentiating the (13) relationship, one obtains:

\[ \sigma'(t) = -2\frac{k}{r(t)^3} \]  
(14)

or rather:

\[ \sigma'(t) = -2\frac{\sigma(t)}{r(t)} \]  
(15)

Approximating the derivatives:

\[ \frac{\sigma(k + 1) - \sigma(k)}{T} = -2\frac{\sigma(t)}{r(t)} \frac{r(k + 1) - r(k)}{T} \]  
(16)

one reaches to:

\[ \sigma(k + 1) = \sigma(k) - 2\frac{\sigma(k)}{r(k)}[r(k + 1) - r(k)] \]  
(17)

where:

\[ r(k) = \sqrt{x^2(k) + y^2(k)} \]  
(18)

In these conditions, the equations that generate the first order \( \alpha-\beta-\sigma \) filter, for a the ship with linear and uniform movement, are:

**Dynamic equations**

\[ x(k + 1) = x(k) + x'(k)T + w_x(k) \]  
(19)

\[ x'(k + 1) = x'(k) \]  
(20)

\[ y(k + 1) = y(k) + y'(k)T + w_y(k) \]  
(21)

\[ y'(k + 1) = y'(k) \]  
(22)

\[ \sigma(k + 1) = \sigma(k) - 2\frac{\sigma(k)}{\sqrt{x^2(k) + y^2(k)}} \times \]  
\[ \times \left[ \sqrt{x^2(k) + x'(k)T} + \sqrt{y^2(k) + y'(k)T} \right]^2 \]  
\[ - \sqrt{x^2(k) + y^2(k)} \right] + w_x(k) \]  
(23)

**Measurement equations**

\[ z_x(k) = x(k) + v_x(k) \]  
(24)
The significance of the expressions are identical to those used in relationships (1) ÷ (6), (8) and (9).

4 Conclusion

Considering the (22) equation, it must be observed that an important simplification was made. This simplification consist in the absence of $w_x(k)$ and $w_y(k)$ noises. In order to compensate this absence, the term “$w_s(k)$” has been introduced. The term “$w_s(k)$” represents the evolution noise of surface. This technique allows the deduction of the filter equations, considering the noise components uncorrelated between them. This is partially true and may be accepted as an approximation.

The system of equations (19) ÷ (25) is the starting point for implementing the tracking filter. Obviously, this involves known techniques for estimation and prediction. It must be added that the choice to introduce the non-kinematic parameter “$\sigma$” in the $\sigma$-$\beta$ filter, was made in order to emphasize the effects on radar tracking. Remember that $\sigma$-$\beta$ filter was the first filter used in tracking and by consequence is the simplest one.

It is interesting to observe that, subsequent simulations showed that $\alpha$-$\beta$-$\sigma$ filter performance are comparable to those of a Kalman filter that uses only kinematic parameters in the tracking process.

The Kalman filter behavior – with “$\sigma$” parameter in state vector – in tracking process will be presented in a future paper.

4 References