Abstract: Due to insufficient stiffness of gear transmission and presence of backlashes, oscillations and reduced positioning accuracy may occur in automatic control systems. This paper gives an analysis of the impact mechanism of gear backlashes, as well as the ways of reducing or eliminating their influence on the system performance. Introduction of divided gears can eliminate backlash introduced effects but can also decrease the resonance maximum frequency and move it near the cut-off frequency or even in the pass-band of the system making it unstable. Examples and calculations are presented which show how to eliminate these problems.

Key-Words: Automatic control system, Geared servo system, Gear backlash

1 Introduction
Many servo systems, especially those that manipulate large masses like tank turret, use a gear train structure to generate high torque and reduce angular speed. The selection of servo drive (electro-mechanical or hydraulic type) can be affect the disturbance transfer from its source to the system. Some drive types can inherently dump the disturbance (even if the control loop is open) while others just accept the whole disturbance and leave the control loop to deal with it. Moreover, the quality of the selected servo drive has also a significant influence on the stability of control (accuracy of target tracking in tanks for example), which is analyzed in literature [1], [2] and is not a subject of this paper. This paper analyzes the gear as a part of the servo system.

Reducing gears are devices whose basic role is to perform mechanical adaptation of the load moment and the motor driving moment. Gear transmission is one of the basic forms of mechanical equipment which has the broadest application in accompanying systems, systems of gyro stabilization, regulators and other automatic control systems. Different types of gears are applied and the most widespread are reducing gears with spur wheels and bevel wheels, while other types are applied more rarely and only in special cases.

There are few nonlinear anomalies in geared servo systems (for example hindsight axis drive in tanks - turret by the azimuth and gun’s tube by elevation): inertial variations, static and coulomb friction and backlash are the most important.

These anomalies cause a lot of problems in stabilization, especially during the target tracking when it is necessary to switch the direction of the rear side axes. In such situations tracking velocities are generally very low, and the effects of friction and backlash are very significant [3], [4], [5]. Besides, reducing gears present a considerable source of possible oscillations (instability) in automatic control systems due to the presence of backlashes and static and coulomb friction and also because reducing gears (chassis and all mobile parts) represent an elastic element.

2 Impact of Gearing Transmission Nonlinearities on Automatic Control Systems
The presence of backlashes introduces, when the boosting coefficient in the system is increased, the stability reduction and increased tranquilizing time and even the undamped oscillations [6], [7], [8], [9]. Nonlinearities of the backlash type increase amplitude and phase deformations which lead to degradation of quality indicators. Backlashes occur due to lateral floats in cogged couples and due to the phenomenon of shaft and gear deformation. These elastic deformations are more pronounced when changing direction of rotation.
Moreover, backlashes are a highly nonlinear effect that can disturb, even jeopardize operation of linear control system, especially in the low speed region where the torque, as a rule, changes the sign frequently to track the low speed reference (effects of friction are dominant).

This is particularly dangerous in systems in which the controlled object is enclosed by feedback loop (this is the case in gyroscopic stabilization systems). Every time the target tracking direction changes, rate gyroscope detects that the stabilization object is not moving, due to backlashes and static friction. The regulator as a response increases the control signal. When the system overcomes static friction, angular velocity of rotation becomes significantly higher than necessary, so the regulator decreases the control signal abruptly. Angular velocity is also abruptly decreased to the necessary level or even changes sign. This leads to the appearance of stepwise minimal velocities (Fig. 1).

This problem is even more complicated in stabilization systems with electrohydraulic activating devices. Namely, besides static friction of the reducing gear, these systems require taking into the account static friction of the hydro-pump rotating control block, pump dead zone and leaking within the hydro-pump - hydro-motor system (hydrostatic gearing).

For very small angles of pump block deflection, there is a dead zone of several miliradians. Nonlinearity is very pronounced in the vicinity of zero in that case. Besides that, hydro-pump block deflection is small for low angular velocities, so the useful flow rates are of the same order of magnitude as the leakage due to the manufacturing imperfections. For small pump deflection angles the working pressure is very low, so the driving moment is practically zero. This is the additional cause for the minimum electromotor velocities to be irregular (stepwise), as can be seen in Fig. 1.

The first channel represents the command signal. Hydro-motor angular velocity depends on the block deflection angle of the pump with variable working volume (second channel). Tachometer located on the hydro-motor axis measures angular velocity of the incoming reducing gear shaft. Angular velocity of the stabilization object is measured by rate gyroscope.

Each time the torque sign is changed, backlashes cause the sudden increase of the measured velocity and the linear controller responds with instantaneous decreasing of the commanded torque. This disturbance spoils the quality of control and can even result in oscillations. That is why controllers should be designed to keep the numbers of torque sign-changes as low as possible.

![Figure 1. Influence of static friction on minimum velocities](image)

The size of lateral floats depends on the selected quality and accuracy of axle distance of coupled gears and it is directly proportional to the size of the backlash.

The impact of reducing gear stiffness and quality of connection between reducing gears and load in the mechanical part of the system will be analyzed by means of load motion equation in elastics systems, in the motor-reducing gear-load system:

\[
J_0 \frac{d^2 \theta_0}{dt^2} = M_0(t) + C_r \left( \frac{\theta_m}{N} - \theta_0 \right) - F_0 \frac{d \theta_0}{dt} \tag{1}
\]

Equivalent equation of motion may be written and reduced to motor shaft as follows:

\[
\left( J_m + \frac{J_0}{N^2} \right) \frac{d^2 \theta_m}{dt^2} = M_m(t) - \frac{C_r}{N^2} \left( \frac{\theta_m}{N} - \theta_0 \right) - F_m \frac{d \theta_m}{dt} \tag{2}
\]

where:

\[
C_r \left( \frac{\theta_m}{N} - \theta_0 \right) \quad [Nm] - \text{stiffness moment};
\]

\[
C_r \quad [Nm/rad] - \text{reducing gear stiffness constant};
\]

\[
F_0 \quad [Nm/(rad/s)] - \text{speed friction load constant};
\]

\[
F_m \quad [Nm/(rad/s)] - \text{speed friction motor constant};
\]

\[
\theta_m \quad [rad] - \text{motor rotating angle};
\]

\[
\theta_0 \quad [rad] - \text{load rotating angle}.
\]

Laplace Transforms of the Equations (1) and (2) are:
\[ (T_0 s^2 + 2\xi_0 T_0 s + 1)\theta_0(s) = \frac{M_0(s)}{C_r} + \frac{\theta_m(s)}{N}, \]  
\[ (T_m s^2 + 2\xi_m T_m s + 1)\theta_m(s) = \frac{M_m(s)N^2}{C_r} + N\theta_0(s), \]  
where:
\[ T_0 = \sqrt{\frac{J_0}{C_0}}; \quad T_m = \sqrt{\frac{J_m N^2}{C_r}}; \quad \xi_0 = \frac{F_0}{2\sqrt{J_0 C_r}}; \]
\[ \xi_m = \frac{F_m N}{2\sqrt{J_m C_r}}; \quad \nu = 1 + \frac{J_0}{N^2 J_m}. \]

Now a structural schematic diagram of the motor starting system by means of “elastic” reducing gear can be drawn (Fig. 2). Presence of the positive feedback contour is clearly observed on the schematic diagram. When constructing frequency characteristics of this contour on the basis of Equations (3) and (4), it is clear that resonance maximums will appear at frequencies \( \omega_0 = \frac{1}{T_0} \) and \( \omega_m = \frac{1}{T_m} \). They will be more noticeable if \( C_r \) is larger since the damping coefficients \( \xi_0 \) and \( \xi_m \) will be significantly reduced.

\[ J_m N^2 s^2 \theta_0 + F_m N^2 s \theta_0 + C_r (\theta_0 - \theta_1) = N M_m \]

By solving this system for \( \theta_1 \) we get the transfer function:
\[ G_s(s) = \frac{\theta_1(s)}{M_m(s)} = \frac{1}{F_m N \left( 1 + \frac{J_0 + J_m N^2}{C_r} + \frac{J_0 s^2 + J_m N^2 s^3}{C_r F_m} \right)}, \]
\[ G_s(s) = \frac{1}{F_m N s (1 + T_m s)(1 + 2T_0 s + T_0^2 s^2)}, \]

where:
\[ T_a = T_0 - \frac{J_m N^2}{F_m N}; \quad T_u = \frac{J_0 N}{C_r (J_0 + J_m N^2)} \]
\[ T_a = \frac{aT_0}{C_r}; \quad T_u = \frac{a J_m N^2}{C_r} \]

When servo systems with feedback are designed (Fig. 4) this maximum has to be considered since it can cause instability of the system, particularly in the area of small gears stiffness \( C_r \). If the system is implemented with pure integrator: \( 1/s \) (dotted line), then there is no pole at the frequency \( 1/T_m \) (this pole is the consequence of the term: \( K/(T_m s + 1) \)).
When the stiffness of the gears is small, for example when a spring is inserted to eliminate the backlash of the shared gears, the frequency $\omega_0$ approaches the pass-band of the system and there will be no redundant amplification stability (Fig. 5). Also, increase of the load moment of inertia will further decrease the redundant amplification.

**Figure 4. Bode diagram of the stabilization servo system**

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**Figure 5. Shared gears reduce system stiffness and stability redundancy**

**3 Ways of Eliminating or Reducing the Impact of Backlashes and Stiffness on System Performance**

**Increasing the stiffness coefficient.** The impact of the gear elasticity reduction may be completely eliminated, or at least reduced, by increasing the stiffness coefficient $C_r$. From Equations (3) and (4) it is obvious that the resonance maximum will be increased, but it will in turn move to the area of higher frequencies. Increase of the stiffness coefficient $C_r$ is achieved by increasing the mass, and therefore increasing the size or corresponding constructive solution in the sense of reducing the gear casing which is often limited by the fitting-in conditions and permitted weight.

This primarily refers to servo systems which are expected to provide high-quality monitoring of mobile objects. By adjusting the mentioned moment, this servo system disturbance (being, as a rule, a stochastic value), becomes considerably more definite.

A solution, based on the principle of a copy lathe applied in practice is given in Fig. 6. Limited degree of freedom of movement in “X” direction, in parallel with the conjugate axis action, is introduced to the whole reducing gear (that is, its casing). The running small cylinder on the internal side of the following gear (rolling path), enables a very accurate radial clearance in cogging (tooting) and prevents serrations, and the radial force which is always present in coupling, prevents over cogging. Self-aligning coupling enables very high-quality coupling by means of which, amongst other things, additional deformations are eliminated in case of bad coupling of insufficiently stiff reducing gear.

**Figure 6. Self-aligning gear with an object**

**Introduction of divided gears and reducing gears.** Impact of lateral floats on backlashes is removed by introduction of divided gears and reducing gears. Namely, this coupled gear has one divided (two-part) gear, whose halves are braced in coupling with the other gear, by means of one (torsion) or more extensive (compressive) springs. Often, only the outlet reducing gear is derived as divided, for in its coupling, the impact of lateral floats on the quality of servo system monitoring is the greatest.

This method decreases the resonance maximum frequency and moves it near the cut-off frequency or even in the pass-band of the system making it unstable. Table 1 gives the measured values of the transfer function $G(s)$ for one gyro stabilization system.

<table>
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<th>f[Hz]</th>
<th>L[db]</th>
<th>$\phi$[°]</th>
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<tr>
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**Table 1**
Table 1. “Elastic” gear transfer function module and phase

The module $L$ is in decibels and the phase $\phi$ is in degrees. Transfer function is defined in Equation (4) as the ratio of the main feedback (from gyroscope) and the local feedback (from tachometer):

$$G(s) = \frac{U_{gyro}(s)}{U_{sp}(s)} = \frac{1}{\left(\frac{1}{\omega_m}s + 1\right)}$$

In the experiment, the motor was loaded with the real load (guns tube). Tachometer was connected directly to the motor shaft, before the gear. Gyroscope was connected directly to the guns tube. Fig. 7 gives the transfer function $G(s)$ as a diagram of the module $L$ and phase $\phi$, in a real system of gyro stabilization, recorded as mentioned signal ratio as a function of frequency $f$ in Hz. The module diagram is interpolated using cubic spline interpolation and the phase diagram is fitted using sixth order polynomial “least square” approximation.

Using standard procedure in MATLAB one can approximate this transfer function with the third order transfer function, Equation (5). Corresponding diagrams are given in Fig. 8.

The values of coefficients $K$ and $\xi$, and the second order pole frequency $\omega_0$ are set directly from the diagram in Fig. 7, but to set the first-order pole frequency $\omega_m$, another experiment was necessary. The experiment is described in the following paragraph.

$$G(s) = \frac{U_{gyro}(s)}{U_{tacho}(s)} = \frac{K}{\left(\frac{1}{\omega_m}s + 1\right)\left(\frac{1}{\omega_0}s^2 + \frac{2\xi}{\omega_0}s + 1\right)}$$

$$G(s) = \frac{0.26}{2.152\times10^{-5}s^3 + 6.605\times10^{-4}s^2 + 0.04512s + 1}$$

where: $K = 0.26$, $\xi = 0.076$, $\omega_m = 24\text{ rad/s}$, $\omega_0 = 44\text{ rad/s}$.  

![Figure 7. Transfer function module and phase diagram](image-url)
The first order pole is set to $\omega_m = 24$ rad/s by means of an experimental measurement of the transfer function of real, loaded hydro-motor. The transfer function of the motor is defined as a ratio $U_{\text{tacho}}/U_{\text{hp}}$ where $U_{\text{tacho}}$ is the voltage from tachometer connected to the motor shaft, and $U_{\text{hp}}$ is the voltage from the gauge which measures the pump-block angle (proportional to fluid flow).

Fig. 7 (measured values) and Fig. 8 (calculated transfer function) show a high resonant peak in the transfer function around the frequency $\omega_0$. In servo system design where the load is covered by the main feedback loop, this peak must be taken into account since it may cause system instability, especially in the area of decreased stiffness of reducing gear $C_r$. In that case the frequency $\omega_0$ approaches the passband of the system and there is no redundant stability. It may also be observed that the increase of the load inertia moment will cause even greater reduction of excessive boosting (strengthening).

In order to avoid possible consequences (reduced operation accuracy and breakdown of the reducing gear), frequency $\omega_0$ should be at least 5 times larger than the system cut-off frequency.

**Hardening technologies.** The impact of elastic deformations of the gears is significantly reduced by teeth and shaft surfaces hardening technologies, such as: cementing, nitriding and similar.

### 4 Conclusion

Static friction and backlash cause a lot of problems in stabilization, especially during the target tracking with very low velocities when it is necessary to switch the direction of the rear side axes. In automatic control systems, which are required to provide high-quality monitoring, regulation or stabilization, application of reducing gears with divided gears or self-aligning coupling considerably reduces the negative effects occurring due to insufficient reducing gear stiffness, as well as presence of lateral floats in the cogged transmission.

**Acknowledgment:** This research is supported by Ministry of Science, Republic of Serbia, Grant 144007.

**References:**


