Control of Systems with Time-Varying Delay: A Comparison Study

RADEK MATUŠŮ, ROMAN PROKOP
Department of Automation and Control Engineering
Faculty of Applied Informatics
Tomas Bata University in Zlín
Nad Stráněmi 4511, 76005 Zlín
CZECH REPUBLIC
{rmatusu; prokop}@fai.utb.cz

Abstract: - This contribution is focused on control of single-input single-output (SISO) systems containing harmonically time-varying delay. Three different approaches to continuous-time control are studied and compared. The first technique utilizes a modified PI-PD Smith predictor in a combination with standard forms for minimum of integral squared time error (ISTE). The second methodology exploits a Coefficient Diagram Method (CDM) for another modified Smith predictor structure. And finally, the third approach to synthesis is based on general solutions of Diophantine equations in the ring of proper and Hurwitz-stable rational functions (R_{PS}) for a classical feedback control loop. The comparison of proposed methods is accomplished through a simple simulation example.

Key-Words: - Time-Varying Systems, Time Delay, Algebraic Approaches, Modified Smith Predictor, PI-PD Control, Coefficient Diagram Method

1 Introduction

Systems affected by time delay (TD) have attracted attention of control theory researchers for decades. The ground of this interest can be seen in common presence of TD in real controlled processes and thus in the necessity of quality and easily applicable control algorithms for this class of systems. Unfortunately, TD always deteriorates control conditions and, moreover, the time-varying TD terms induce even much more obstacles.

A possible effective and economical solution for systems with relatively small or limited changes of TD is the usage of robust enough fixed controllers [14]. The worthwhile closed-loop configuration for compensation of dead time has been well known as Smith predictor since 1959. Recently, many new modifications of Smith predictor with improved properties have been introduced – e.g. [2], [5], [9], [12]. Another way how to overcome TD resides in combination of its approximation and subsequent utilization of an algebraic control design method. The advantageous solution represents a fractional approach developed in [7], [19] and applied for robust control of TD systems e.g. in [16]. Moreover, many other stability investigation techniques [3], [15], [20] and robust control methods [4], [6], [8] applicable for time-varying delay systems have been published.

This paper studies three control principles for single-input single-output (SISO) systems with periodically time-varying TD. The results given by continuous-time controller designed in the ring of proper and Hurwitz-stable rational functions (R_{PS}) [16], [17], [18] are compared with those obtained with the use of modified PI-PD Smith predictor [5] and also using the Smith predictor designed by Coefficient Diagram Method (CDM) [2].

The work is organized as follows. In Section 2, basic description of controlled first order time-varying delay system is provided. The Section 3 contains the theoretical backgrounds for the trio of applied methods. Further, the specific controller calculations, simulative comparisons and analyses are presented in Section 4. And finally, Section 5 offers some conclusion remarks.

2 Description of Controlled System

All control design methods are applied on a simple example of first order controlled system with time-varying delay described by differential equation:

$$y'(t) + 0.1y(t) = 0.2u(t - \Theta(t)) \tag{1}$$

with zero initial condition. The TD term harmonically changes from 5 to 15 according to:

$$\Theta(t) = 10 + 5\sin(0.4t) \tag{2}$$

System (1), (2) is considered as a really controlled plant for all simulations. As an alternative notation, it can be used also the non-standard hybrid "transfer function" which depends both on complex variable s and on time t:

$$G(s,t) = \frac{0.2}{s+0.1}e^{-[10+5\sin(0.4t)]s} = \frac{2}{10s+1}e^{-[10+5\sin(0.4t)]s} \tag{3}$$

The nominal system used for the purpose of control design is represented by time-invariant transfer function with average value of TD:

$$G(s)=\frac{0.2}{s+0.1}e^{-10s}=\frac{2}{10s+1}e^{-10s} \tag{4}$$

for all compared techniques.

3 Outline of Applied Control Design Methods

3.1 Modified PI-PD Smith Predictor

The first method is based on the modification of the classical Smith predictor presented in [5]. It exploits the structure with three controllers shown in fig. 1, where G_{c1} is a PI controller, G_{c2} is a PD (or only P where it is appropriate) controller and G_{c3} is the disturbance controller introduced in [11]. Furthermore, w, n, y denote the reference signal, disturbance in the input of the controlled plant, and output signal, respectively.

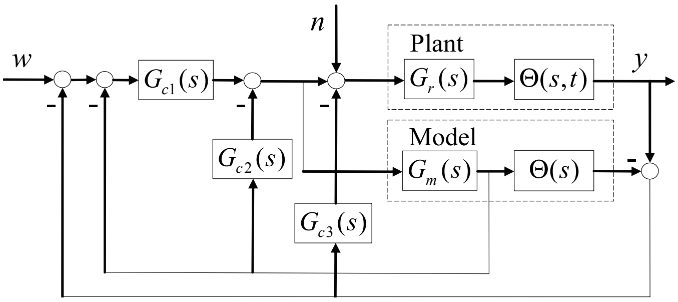

Fig. 1: Modified Smith predictor structure (PI-PD)

Generally, the synthesis is based on usage of standard forms for obtaining the optimal closed-loop transfer function parameters in the meaning of integral squared time error (ISTE) criterion. Control design formulas derived for the case of first order TD plant with example of controller computation can be found in Section 4.

3.2 Modified Smith Predictor Design by Coefficient Diagram Method

The second controller design using the Coefficient Diagram Method (CDM) was proposed in [2]. This method uses the improved Smith predictor structure with the trio of controllers according to fig. 2.

The CDM design is based on the four principles:

- Coefficient diagram: It is a semilogarithmic diagram which allows investigating the stability and response of systems in a single graph. The vertical axis logarithmically shows the coefficients of characteristic polynomial, stability indices, stability limits and equivalent time constant while the horizontal axis represents the order values corresponding to each of coefficients.
- Modification of Kessler standard form: The form developed by Kessler in 1960 has decreased the oscillations and overshoots compared to the original Graham's ITAE form. In this approach, a new form called "Standard Manabe Form" is used. This design should result in quite stable and robust responses with small settling time.
- Lipatov stability analysis: The effect of coefficient variations can not be seen clearly for higher order systems. The conditions for stability or instability of such systems, based on Lipatov's work, are included in CDM design technique.
- Obtaining characteristic polynomial: A method similar to pole placement is applied. However, the main difference is the Manabe form.

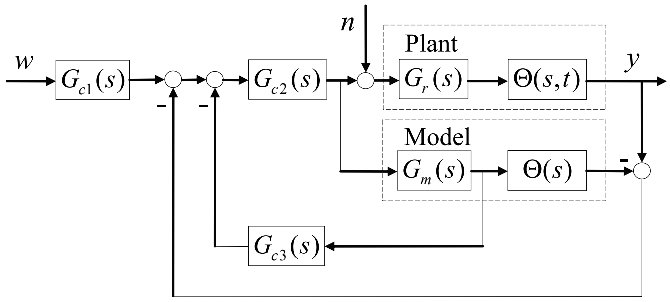

Fig. 2: Modified Smith predictor structure (CDM)

Again, a concise illustration of controller calculation is shown in Section 4. For details about the technique see [1], [2], [10] or other related literature.

3.3 Algebraic Control Design in R_{PS}

The third method adopts an algebraic fractional approach developed in [7], [19] and discussed in [17], [18]. Algebraic tools enable relatively deep insight into control tuning and parametric expression of all suitable controllers.

The first step of algebraic control design in R_{PS} for TD systems is to approximate a TD term by a polynomial approximation in order that the model becomes usable for linear Diophantine equations. A conventional and suitable tool is the Padé approximation.

Then the systems have to be described in R_{PS} as a ratio of two rational fractions:

$$G(s)=\frac{b(s)}{a(s)}=\frac{\dfrac{b(s)}{(s+m)^n}}{\dfrac{a(s)}{(s+m)^n}}=\frac{B(s)}{A(s)} \tag{5}$$

where $n=\max\{\deg(a),\deg(b)\}$ and $m>0$.

The scalar positive parameter m which enters into the synthesis process can be later conveniently used as a "tuning knob" influencing the final control behavior.

This factorization approach can be used for various control structures. For simplicity, the well known classical one-degree-of-freedom (1DOF) configuration was used. The control loop is depicted in fig. 3. In addition to previous two figures, u and v represent control signal and disturbance in the output of the controlled plant, respectively. All signals and functions depicted in this figure should be expressed in R_{PS}.

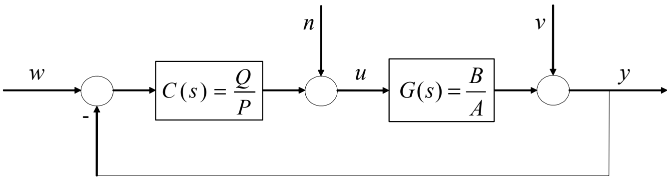

Fig. 3: One-degree-of-freedom closed loop system

The basic task is to ensure internal stability of the system in fig. 3. All stabilizing feedback controllers are given by all solutions of the linear Diophantine equation:

$$AP + BQ = 1 \tag{6}$$

with a general solution $P = P_0 + BT$, $Q = Q_0 - AT$, where T is free in R_{PS} and P_0, Q_0 is a pair of particular solutions.

In other words, ratio:

$$\frac{Q}{P} = \frac{Q_0 - AT}{P_0 + BT}; \quad P_0 + BT \neq 0 \tag{7}$$

represents all possible stabilizing controllers and it is known as Youla – Kučera parameterization. For details and proofs see [7], [19].

Another important property is the convergence of control error e to zero. Under assumption that no disturbances affect the system in fig. 3 $(n = v = 0)$ it follows for this loop:

$$e = \frac{AP}{AP + BQ}\frac{G_w}{F_w} \tag{8}$$

where $\frac{G_w}{F_w}$ is the reference signal w (in R_{PS}). For example, a stepwise reference signal w has the denominator $F_w = \frac{s}{s+m}$.

Substitution of (6) to (8) and subsequent algebraic analysis leads to the outcome that for zero control error:

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0}\left[s \cdot e(s)\right] = 0 \tag{9}$$

the expression F_w must be cancelled from the denominator of (8). Therefore F_w must generally divide product AP (or only P in many practical cases).

One of the main advantages of the mentioned technique is the possibility of tuning of controller parameters by the only scalar parameter m. The optimal choice of m is a nontrivial task. Some recommendations are provided for example in [13]. However, for most simulations and practical events, the primitive "trial and error" method can be successfully applied to find a suitable m.

The details, results and references for the two-degree-of-freedom (2DOF) configuration or for other control problems (disturbance rejection, disturbance attenuation, etc.) can be found e. g. in [16], [17], [18].

An illustration of the controller computation is shown in the following part.

4 Calculations of Controllers and Simulation Results

Remind that a controlled plant with time-varying delay is given by (1), (2) or (3) and mathematical model for control design purpose is supposed in the form (4). The controllers for all PI-PD, CDM and R_{PS} design were experimentally tuned to obtain visually acceptable results without or with only small overshot and short settling time. For better comparability, responses with nearly the same time of reaching the reference value were chosen. Furthermore, the following simulation conditions were used: simulation time $T_S = 600s$, reference value 1 with step to 2 in $1/3$ of T_S, load disturbance injected into the plant input $n = -0.3$ in $2/3$ of T_S, and zero disturbance v in the plant output.

For the first method, modified PI-PD Smith predictor, the controlled system model (without TD) has been supposed in the form:

$$G_m(s) = \frac{\beta_0}{s + \alpha_0} = \frac{0.2}{s + 0.1} \tag{10}$$

The transfer functions of all controllers in fig. 1 are:

$$G_{c1}(s) = K_c\left(1 + \frac{1}{T_i s}\right) = 0.015\left(1 + \frac{1}{1.1s}\right) \tag{11}$$

$$G_{c2}(s) = K_f = -0.1650 \tag{12}$$

$$G_{c3}(s) = K_o = 0.4 \tag{13}$$

Parameters K_c, T_i and K_o have been adjusted by user, while K_f follows from equations:

$$\alpha = \sqrt{\frac{\beta_0 K_c}{T_i}} = 0.0522 \tag{14}$$

$$c_1 = \alpha T_i = 0.05745 \Rightarrow d_1 = 1.3405 \tag{15}$$

$$K_f = \frac{d_1\alpha - \alpha_0 - K_c\beta_0}{\beta_0} \tag{16}$$

The size of d_1 in (15) must be determined on the basis of according to graph from [5]. For the purpose of this paper, the graphical relation has been approximated by the sixth order polynomial:

$$\begin{aligned} d_1 = &-0.0028c_1^6 + 0.0376c_1^5 - 0.1766c_1^4 + \\ &+0.3076c_1^3 + 0.0502c_1^2 + 0.1533c_1 + 1.3315 \end{aligned} \tag{17}$$

Besides, a non-zero value of K_o ensures better disturbance rejection, but there is trade-off between this rejection and oscillation of the control and output signal. The behaviour is "smoother" for $K_o = 0$ (see the corresponding curves from figs. 4 and 5 vs. fig. 6).

Generally, according to [5], the time scale α can be selected by the choice of K_c, c_1 by the choice of T_i and d_1 by the choice of K_f. The same authors subsequently claim that, in practice, K_c will be constrained, possibly to limit the initial value of the control effort, so that the choice of K_c and T_i may involve a compromise between the values chosen for α and c_1.

In CDM, as the second method, the settling time was preset to $t_s = 50s$ and disturbance rejection structure was selected. The resulting controllers are:

$$G_{c1}(s) = 1 \tag{18}$$

$$G_{c2}(s) = \frac{1}{l_1 s} = \frac{1}{43.1141s} \tag{19}$$

$$G_{c3}(s) = k_1 s + 1 = 1.6577s + 1 \tag{20}$$

The coefficients of regulators follow from:

$$l_1 = \frac{K\tau^2}{2.5T} \tag{21}$$

$$k_1 = \tau - \frac{\tau^2}{2.5T} \tag{22}$$

where

$$\tau = t_s / 2.1538 \tag{23}$$

and transfer function of controlled system model (without TD) is assumed in the form:

$$G_m(s) = \frac{K}{Ts+1} = \frac{2}{10s+1} \tag{24}$$

Regarding to the third technique, control design in R_{PS}, the nominal system is obtained using the first order Padé approximation of TD in (4):

$$\begin{aligned} G(s) &= \frac{0.2}{s+0.1}e^{-10s} \approx \frac{0.2(1-5s)}{(s+0.1)(1+5s)} = \\ &= \frac{-0.2s+0.04}{s^2+0.3s+0.02} = \frac{b_1 s + b_0}{s^2 + a_1 s + a_0} \end{aligned} \tag{25}$$

The resulting nominal system has non-minimum phase behaviour (unlike some other approximation methods), but it is not any problem for the R_{PS} design. Furthermore, a higher order Padé approximations than the first one would lead to a more complicated structure of the controller. Thus, the applied way is commonly used for such cases because it simply but also sufficiently approximates behaviour of TD system.

The application of the algebraic approach described in the Subsection 3.3 and the choice $m = 0.12$ give the feedback controller:

$$\begin{aligned} C(s) &= \frac{\tilde{q}_2 s^2 + \tilde{q}_1 s + \tilde{q}_0}{s^2 + \tilde{p}_1 s} = \\ &= \frac{0.2845s^2 + 0.0803s + 0.0052}{s^2 + 0.2369s} \end{aligned} \tag{26}$$

Practically, the parameters of (26) can be calculated from pre-derived equations:

$$\begin{aligned} \tilde{p}_1 &= p_0 + m - p_0 m \frac{b_1}{b_0} \\ \tilde{q}_2 &= q_1 + \frac{p_0 m}{b_0} \\ \tilde{q}_1 &= q_0 + q_1 m + a_1 \frac{p_0 m}{b_0} \\ \tilde{q}_0 &= q_0 m + a_0 \frac{p_0 m}{b_0} \end{aligned} \tag{27}$$

and

$$\begin{aligned} p_1 &= 1 \\ p_0 &= \frac{3m^2 b_0 b_1 - a_0 b_0 b_1 - 3mb_0^2 + a_1 b_0^2 - b_1^2 m^3}{a_1 b_0 b_1 - b_0^2 - a_0 b_1^2} \\ q_1 &= \frac{3m - a_1 - p_0}{b_1} \\ q_0 &= \frac{m^3 - a_0 p_0}{b_0} \end{aligned} \tag{28}$$

The comparison of closed-loop output variables for all methods is shown in fig. 4 while corresponding control signals are plotted in fig. 5. Moreover, the fig. 6

depicts both the output and control signal once more only for PI-PD control design but now without disturbance controller – i.e. $G_{c3}(s) = K_o = 0$. All other parameters and settings remain the same as in (11), (12).

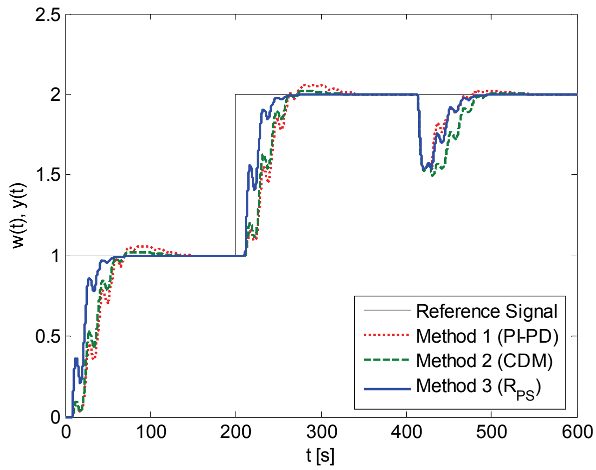

Fig. 4: Comparison of closed-loop control responses

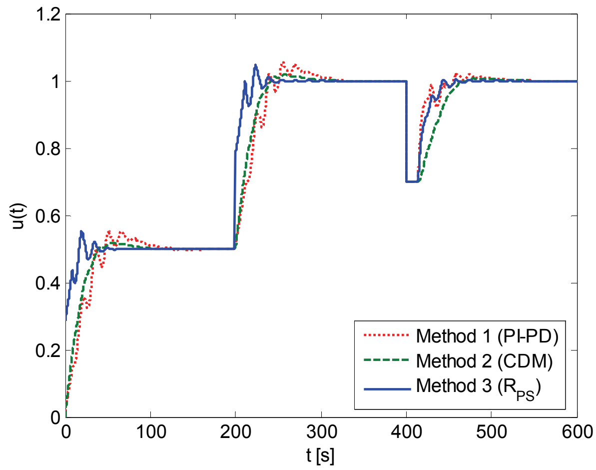

Fig. 5: Comparison of control signals

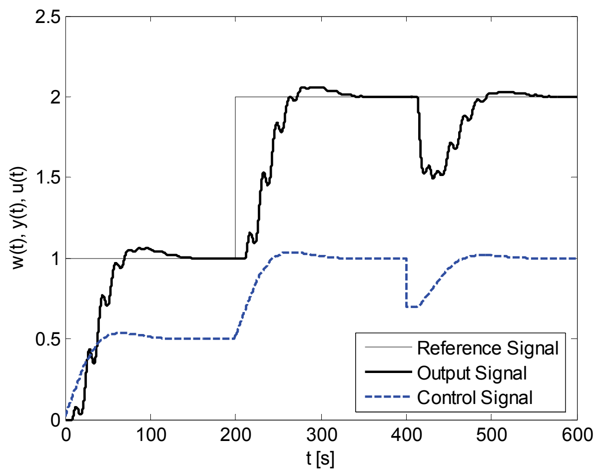

Fig. 6: Output and control signal for Method 1 (PI-PD) and $K_o = 0$

Depicted results of all methods obtained during simulative control of given time-varying delay system should be acceptable for majority of real technological applications. Control design in R_{PS} gives the fastest responses, on the top of that without any overshoots, in comparison with both modified Smith predictors. Moreover, it has relatively good rejection of load disturbance. On the other hand, the cost for it is more aggressive control signal. The modified PI-PD Smith predictor provides probably the best disturbance rejection thanks to the mentioned disturbance controller, but it has the biggest overshoot. The CDM takes the second place from the overshoot point of view and its disturbance rejection is the slowest.

An interesting and objective appreciation of control quality can be obtained by meaning of Integrated Squared Error (ISE) criterion, which is calculated according to:

$$\mathrm{ISE} = \int_0^\infty e(t)^2 \, dt \tag{29}$$

The evaluation of control behaviour (figs. 4, 5 and also 6) from the ISE viewpoint can be found in table 1.

Table 1: Outcomes of ISE calculations

Method	ISE
PI-PD $(K_o = 0.4)$	61.65
CDM	60.22
R_{PS}	38.02
PI-PD $(K_o = 0)$	66.47

Besides, drawbacks of both modifications of Smith predictor are more complicated control loop structure and necessity of TD model in the inner loop. All in all, obtained results indicate that the proposed control design in R_{PS} can be considered as an effective method for studied class of systems.

5 Conclusions

In the paper, control of SISO systems with harmonically time-varying delay has been addressed. Three various continuous-time strategies based on the idea of robustness have been compared. The first two methods use the modified Smith predictor structures in combination with standard forms for minimum of ISTE or design by CDM, respectively. The third method is based on the fractional representation in R_{PS}, general solutions of Diophantine equations and conditions of divisibility. The simulations of control were done in Matlab + Simulink environment.

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