A Large Deflection Model For The Dynamic Pull-In Analysis Of Electrostatically Actuated Nanobeams In Presence Of Intermolecular Surface Forces

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Abstract: - Nanomechanical switches are fundamental building blocks for the design of NEMS applications, such as nanotweezers and some other nanoscale actuators. However, there is an intrinsic instability, known as the pull-in phenomenon. In practice one must discriminate between the static and the dynamic pull-in aspects. In spite of static pull-in, dynamic pull-in takes into account the inertial and damping effects and the additional effect of an external acceleration. At present, few works have been carried out by neglecting van der waals and Casimir forces in dynamic analysis. At this study a theoretical analysis focusing on the effect of these forces on the pull-in parameters and dynamic behavior of electrostatically actuated nanobeam have been investigated. A large deflection model and first order fringing field correction on electrostatic force are considered. The method of Galerkin decomposition is employed to approximate the system equations by a reduced order model composed of a finite number of discrete modal equations. Also a perturbation method used to predict dynamic behavior of nanobeams that was in a good agreement with results of Runge-Kutta method.

Key-Words: NEMS- Pull-in- Casimir- van der Waals

1 Introduction
An exceptional characteristic of nanoelectromechanical (NEMS) devices is the pull-in phenomenon [1], [2]. In nano-scales the coupling between the electrical and mechanical terms leads to this unstable behavior. The generic device usually consists of a parallel-plate capacitor connected to a clamped beam. Since the electrostatic force is inversely proportional to the square of the deflection and the restoring force of the beam is, in a first approximation, linear with deflection, an unstable system results in the case of a deflection, beyond a critical value. The pull-in voltage ($V_{pi}$), is defined as the voltage that is required to obtain this critical deflection. The main phenomenon is the loss of stability of the equilibrium position. In practice one must distinguish between the static and the dynamic pull-in aspects. Static pull-in is due solely to the electrostatic action. The inertia and damping terms are neglected and the variation of the voltage is considered slow enough, so that equilibrium is at anytime obtained by the static components. Dynamic pull-in takes into account the inertial and damping effects and the additional effect of an external acceleration, which may significantly change the pull-in voltage threshold [3]. Instantaneous application of voltages that are lower than the static pull-in voltages may nevertheless result in a dynamic collapse of the deformable electrode into contact with the fixed electrodes [4]. The reduction of the separation between the components will require NEMS designs to account for intermolecular forces that have been neglected until now. At nano-scales, the decreasing gap between the two electrodes makes surface traction due to molecular interaction. This important force in the structure must be taken into account in the analysis and design of NEMS. When the gap is much less than the plasma wavelength (for a metal) or the absorption wavelength (for a dielectric) of the constituent material of the surfaces (typically below 20 nm), the retardation is not significant and the intermolecular force between two surfaces can be simplified as the van der Waals attraction. In case that the gap is large enough (typically above 20 nm) so that the retardation is pronounced, the intermolecular interaction between two surfaces can be described as the Casimir force (the result of the quantum fluctuations of the vacuum electromagnetic field). In this paper the

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dynamic pull-in behaviors of nano beams in presence of van der Waals and Casimir forces are studied. The Casimir effect on the static pull-in gap and pull-in voltage of NEMS switches was studied in [5]. Lin and Zhao, [6] studied the influence of the Casimir force on the nonlinear behavior of nano-scale electrostatic actuators. In these investigations, a one degree of freedom lumped parameter model was used by the researchers. The effect of the van der Waals force on the static pull-in parameters of nano-switches has been studied elsewhere [7]. Nayfeh et al. [8] studied the pull-in instability in microelectromechanical (MEMS) resonators and find that characteristics of the pull-in phenomenon in the presence of AC loads differ from those under purely DC loads. The analysis presented here focuses on the dynamic aspects of pull-in and find the characteristics of the pull-in phenomenon in the presence of DC loads, and intermolecular forces.

2 Nonlinear equation for beam’s large deflections
We consider a clamped–clamped narrow nano-beam of length l, width b, and thickness h, as shown in Fig.1.

![Fig.1: A schematic of an electrically actuated nano-beam](image)

The nano-beam is located above an infinite ground plane with an initial gap d, both bodies are perfect conductors and are separated by a dielectric medium of permittivity \( \varepsilon_0 \varepsilon_r \), where \( \varepsilon_0 \) is the vacuum permittivity and \( \varepsilon_r \) is the relative permittivity. A positive potential difference V between the two conductors causes the nano-beam to deflect. We incorporate the Von Karman nonlinearity in the expression for the axial strain to account for large deflections. The equation of motion that governs the transverse deflection \( \tilde{w}(\tilde{x}, \tilde{t}) \) is governed by [9].

\[
\tilde{E} I \frac{\partial^4 \tilde{w}}{\partial \tilde{x}^4} + \rho A \frac{\partial^2 \tilde{w}}{\partial \tilde{t}^2} - \left[ \frac{\tilde{E} A}{2l} \int_0^l \left( \frac{\partial \tilde{w}}{\partial \tilde{x}} \right)^2 + \tilde{N} \right] \frac{\partial^2 \tilde{w}}{\partial \tilde{x}^2} = F_e + F_{vdw}
\]

As where \( \tilde{x} \) is the position along the nano-beam, A and I are the area and moment of inertia of the cross section, h the nano-beam thickness b the beam’s width and d is the initial gap between beam and stationary electrode. \( \tilde{t} \) is time, and \( \rho \) is the material density. Effects of transverse shear stresses are neglected since for a typical nano-beam \( l/h > 20 \). A beam is considered narrow, when its width b is less than five times its thickness h. For a narrow beam, \( \tilde{N} = -\tilde{N} \) where \( \tilde{N} = \sigma_0 (1 - \nu) \), \( \sigma_0 \) is the initial uniform biaxial stress in the material, and \( \nu \) is the Poisson’s ratio. \( \tilde{E} \) is the effective modulus. The effective modulus \( \tilde{E} \) simply becomes the Young’s modulus \( E \) for narrow beams (\( b < 5h \)) and becomes the plate modulus \( E/(1 - \nu^2) \) if \( b > 5h \), where \( \nu \) is the Poisson’s ratio.

For convenience, we introduce the non-dimensional variables [8].

\[
\tilde{w} = \frac{w}{d}, \quad \tilde{x} = \frac{x}{l}, \quad \tilde{t} = \frac{t}{\sqrt{\frac{\tilde{E} I}{\rho b h l^4}}}
\]

After substitution (2) and (3) in (1), with non-dimensional variables introduced above, we obtain the
motion equation of the beam assuming van der Waals force as:

\[
\frac{\partial^4 w}{\partial x^4} + \frac{2^2 w}{\partial x^2} \left[ \alpha \int_0^1 \left( \frac{\partial w}{\partial x} \right)^2 + N \right] \frac{\partial^2 w}{\partial x^2} = \frac{\beta v^2}{(1-w)^2} + \frac{\gamma v^2}{(1-w)} + \frac{\delta}{(1-w)^3} \tag{7}
\]

(If we assume Casimir force the term \(\frac{\delta}{(1-w)^3}\) of (7), replaces with \(\frac{\varphi}{(1-w)^4}\).)

The parameters appearing in (7) are

\[
\alpha = 6 \frac{(d^2)}{h^2}, \quad N = \frac{6\varepsilon_0\varepsilon_1 l^4}{E h^3 d^3},
\]

\[
\gamma = \frac{3.9\varepsilon_0\varepsilon_1 l^4}{E h^3 d^2 b}, \quad \beta = \frac{\pi^2 h c l^4}{20E h^3 d^5}, \quad \delta = \frac{2A l^4}{\pi E d^4 h^3}.
\tag{8}
\]

Next, we generate a reduced-order model by discretizing (7) into a finite-degree of freedom system consisting of ordinary-differential equations in time. We use the un-damped linear mode shapes of the straight nano-beam as basic functions in the Galerkin procedure.

### 3 Galerkin formulation

Modal decomposition is performed in this section to facilitate the study of transient behavior of the nano-beam in response to the DC forcing. The method of Galerkin decomposition is employed to approximate the system equations by a reduced order model composed of a finite number of discrete modal equations [12]. The process of Galerkin decomposition starts with separating the dependences of the deflection of the deformed beam, \(w(x, t)\), into temporal and spatial by functions \(u(t)\) and \(q(x)\), respectively, in the form of a series of products

\[
w(x,t) = \sum_{i=1}^{M} u_i(t) q_i(x) \tag{9}
\]

where \(u_i(t)\) is the \(i\)th generalized coordinate and \(q_i(x)\) is the \(i\)th linear un-damped mode shape of the straight nano-beam normalized such that \(\int_0^1 q_i q_i dx = \delta_{ij}\) and governed by [13]

\[
q_i^{iv} = \omega_i^2 q_i \tag{10}
\]

\(q_i = 0\) and \(q_i = 0\) at \(x=0\) and \(x=1\)

\[\tag{11}\]

here \(\omega_i\) is the \(i\)th natural frequency of the nano-beam.

We multiple (7) by \(q_n(x)(1-w)^3\), substitute (8) into the resulting equation, use (10) to eliminate \(q_i^{iv}\), integrate the outcome from \(x=0\) to \(1\), the coupled nonlinear ODEs of the system can be derived as (12) for van der Waals case:

\[
u_i^n + \omega_i^2 u_i + 3 \sum_{i,j,k=1}^{M} u_i u_j u_k \omega_i^2 \int_0^1 q_n q_j q_k q_m dx
\]

\[\tag{12}\]

\[\begin{align*}
&-3 \sum_{i,j=1}^{M} u_i u_j \omega_i^2 \int_0^1 q_n q_j q_k q_m dx \\
&+ \sum_{i,j,k=1}^{M} u_i u_j u_k \omega_i^2 \int_0^1 q_n q_j q_k q_m dx \\
&+3 \sum_{i,j,k=1}^{M} u_i u_j u_k \int_0^1 q_n q_j q_k q_m dx \\
&+ \alpha \sum_{i,j,k=1}^{M} u_i u_j u_k \int_0^1 q_n q_j q_k q_m q_m dx \\
&-3 \sum_{i,j=1}^{M} u_i u_j \int_0^1 q_n q_j q_k q_m dx \\
&- \sum_{i,j,k=1}^{M} u_i u_j u_k \int_0^1 q_n q_j q_k q_m q_m q_m dx \\
&+ N \sum_{i,j=1}^{M} \int_0^1 q_n q_i q_j q_k q_m dx \\
&-3 N \sum_{i,j=1}^{M} \int_0^1 q_n q_i q_j q_k q_m q_m q_m q_m dx \\
&- N \sum_{i,j,k=1}^{M} \int_0^1 q_n q_i q_j q_k q_m q_m q_m q_m q_m dx \\
&\beta v^2 \left[ \int_0^1 q_n dx - \sum_{i,j=1}^{M} u_i \int_0^1 q_n q_j q_k q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m q_m
It must be mentioned that over prime and over dot indicate derivative with respect to non-dimensional time and position. Equation (12) represents a discretized system of \( M \) coupled nonlinear ordinary-differential equations describing the dynamic behavior of an electrically actuated nano-beam assuming van der Waals force. In case of Casimir force equations are similar to (12) with some differences. A single-mode approximation for van der Waals case yields the following equation:

\[
(u^* + \omega_\beta^2 u)
\left[1 - 3u \int_0^1 q^3 \, dx + 3u^2 \int_0^1 q^4 \, dx - u^3 \int_0^1 q^5 \, dx\right]
- N \int_0^1 q \ddot{q} \, dx - \beta V^2 \int_0^1 q^2 \, dx - 2\gamma V^2 \int_0^1 q^3 \, dx u
+ [ -3N \int_0^1 q^2 \ddot{q} \, dx + \gamma V^2 \int_0^1 q^3 \, dx ] u^2
+ \left[ \alpha \int_0^1 \ddot{q}^2 \, dx \cdot \int_0^1 q \ddot{q} \, dx + 3N \int_0^1 q^2 \ddot{q} \, dx \right] u^3
- \left[ 3N \int_0^1 q^2 \ddot{q} \, dx \cdot \int_0^1 q \ddot{q} \, dx + N \int_0^1 q^4 \ddot{q} \, dx \right] u^4
+ [ 3N \int_0^1 q^2 \ddot{q} \, dx \cdot \int_0^1 q^3 \ddot{q} \, dx ] u^5
+ [- \alpha \int_0^1 \ddot{q}^2 \, dx \cdot \int_0^1 q^4 \ddot{q} \, dx ] u^6
+ \beta V^2 \int_0^1 q \, dx + \gamma V^2 \int_0^1 q^3 \, dx + \delta \int_0^1 q \, dx
\] (13)

With one mode approximation, the equation of motion in Casimir force case is in the form of eq. 14.

\[
(u^* + \omega_\beta^2 u). [1 - 4u \int_0^1 q^3 \, dx + 6u^2 \int_0^1 q^4 \, dx
- 4u^3 \int_0^1 q^5 \, dx + u^4 \int_0^1 q^6 \, dx] =
\left[N \int_0^1 q \ddot{q} \, dx - 2\beta V^2 \int_0^1 q^2 \, dx - 3\gamma V^2 \int_0^1 q^3 \, dx \right] u
+ [ -4N \int_0^1 q^2 \ddot{q} \, dx + \beta V^2 \int_0^1 q^3 \, dx +
3\gamma V^2 \int_0^1 q^4 \, dx \right] u^2 + [ \alpha \int_0^1 \ddot{q}^2 \, dx \cdot \int_0^1 q \ddot{q} \, dx
+ \beta V^2 \int_0^1 q^3 \, dx + \gamma V^2 \int_0^1 q^4 \, dx \right] u^3
+ \left[ -4\alpha \int_0^1 \ddot{q}^2 \, dx \cdot \int_0^1 q^4 \ddot{q} \, dx - 4N \int_0^1 q^5 \, dx \right] u^4
+ [ 6\alpha \int_0^1 \ddot{q}^2 \, dx \cdot \int_0^1 q^3 \ddot{q} \, dx + N \int_0^1 q^5 \ddot{q} \, dx \right] u^5
+ \left[ -4\alpha \int_0^1 \ddot{q}^2 \, dx \cdot \int_0^1 q^4 \ddot{q} \, dx \right] u^6
+ \left[ \alpha \int_0^1 \ddot{q}^2 \, dx \cdot \int_0^1 q^5 \ddot{q} \, dx \right] u^7
+ \beta V^2 \int_0^1 q \, dx + \gamma V^2 \int_0^1 q^3 \, dx + \varphi \int_0^1 q \, dx
\] (14)

These equations have been solved by Runge-Kutta method, and results are discussed at next sections. Here we generate another solution using perturbation method (the method of multiple scales). The right terms in (7) can be approximated by Taylor’s series

\[
\frac{\partial^4 w}{\partial x^4} + \frac{\partial^2 w}{\partial t^2} - \left[ \alpha \int_0^1 \left( \frac{\partial w}{\partial x} \right)^2 + N \right] \frac{\partial^2 w}{\partial x^2}
= a + bw + cw^2 + dw^3
\]

where in case of van der Waals,

\[
a = \delta + \gamma V^2 + \beta V^2 \\
b = 3\delta + \gamma V^2 + 2\beta V^2 \\
c = 6\delta + \gamma V^2 + 3\beta V^2 \\
d = 10\delta + \gamma V^2 + 4\beta V^2
\]

and in case of Casimir force,

\[
a = \varphi + \gamma V^2 + \beta V^2 \\
b = 4\varphi + \gamma V^2 + 2\beta V^2 \\
c = 10\varphi + \gamma V^2 + 3\beta V^2 \\
d = 20\varphi + \gamma V^2 + 4\beta V^2
\]

Based on a single degree-of-freedom model of the beams (\( n = 1 \)), (13) can be solved with appropriate accuracy. [9]. Use (9) to eliminate \( q^{iv} \), integrate the outcome from \( x=0 \) to 1, and obtain the nonlinear ODEs of the system can be derived as,

\[
\ddot{u} + Lu + Mu^2 + Nu^3 + O = 0
\]

where,
A general solution of the non-homogeneous (18), on some open interval is a solution of the form

\[ u(t) = u_h(t) + u_p(t) \]  

(20)

where \( u_p(t) \) is a particular solution of (18) and can be determined after finding \( u_h(t) \).

### 4 The method of multiple scales

By introducing new independent variables according to [14]:

\[ T_n = e^{\alpha t} \quad \text{for} \quad n = 0, 1, 2, \ldots \]  

(21)

So the derivatives with respect to \( t \) become:

\[ \frac{d}{dt} = D_0 + \epsilon D_1 + \ldots \] \[ \frac{d^2}{dt^2} = D_0^2 + 2\epsilon D_0 D_1 + \ldots \]  

(22)

By assuming the small factor \( \epsilon = \left( \frac{d}{dt} \right)^2 \), (20) becomes to,

\[ \ddot{u} + Lu = \epsilon(\ddot{u} + N\dot{u}) = 0, N = \epsilon\bar{N}, M = \epsilon\bar{M} \]  

(23)

We assume that the solution of (23) can be represented by an expansion having the form,

\[ u_h(t; \epsilon) = u_0(T_0, T_1, \ldots) + \epsilon u_1(T_0, T_1 \ldots) + \ldots \]  

(24)

Substituting (22) and (24) into (23) and equating the coefficients of \( \epsilon \) to zero, we obtain

\[ D_0^2u_0 + \omega_0^2u_0 = 0 \] \[ D_0^2u_1 + \omega_0^2u_1 = -2D_0D_1u_0 - \bar{M}u_0^2 - \bar{N}u_0^3 \]  

(25) \hspace{1cm} (26)

It is convenient to write the solution of (25) in the form

\[ u_0 = A(T_0) \exp(i\omega_0 T_0) + \bar{A} \exp(-i\omega_0 T_0) \]  

(27)

where \( A \) is an unknown complex function and \( \bar{A} \) is the complex conjugate of \( A \) and \( \omega_0 = L^{0.5} \).

Substituting (27) into (26) leads to:

\[ D_0^2u_1 + \omega_0^2u_1 = -2i\omega_0 \bar{A} \exp(i\omega_0 T_0) - \bar{M}(A^2 \exp(2i\omega_0 T_0) + \bar{A}) \] \[ -\bar{N}(\bar{A}^2 \exp(3i\omega_0 T_0) - 3A^2\bar{A} \exp(i\omega_0 T_0)) + \bar{C} \]  

(28)

where \( \bar{C} \) denotes the complex conjugate of the preceding terms. Any particular solution of (25) has a secular term containing the factor \( \exp(i\omega_0 T_0) \) unless

\[ 2i\omega_0 \bar{A} + 3\bar{N}A^2\bar{A} = 0 \]  

(29)

The solution of (28) is

\[ u_1 = \frac{\bar{M}A^2}{3\omega_0^2} \exp(2i\omega_0 T_0) \] \[ + \frac{\bar{N}A^3}{9\omega_0^2} \exp(3i\omega_0 T_0) - \frac{\bar{M}A\bar{A}}{\omega_0^2} + \bar{C} \]  

(30)

In solving (29), it is convenient to write \( A \) in the polar form

\[ A = \frac{1}{2} \alpha \exp(i\beta) \]  

(31)

where \( \alpha \) and \( \beta \) are real functions of \( T_1 \). Substituting (31) into (29) and separating the results into real and imaginary parts, we obtain

\[ \omega_0 \alpha = 0 \quad \text{and} \quad -\alpha \beta \omega_0 + \frac{3}{8} \bar{N} \alpha^3 = 0 \]  

(32)

It follows that \( \alpha \) is a constant, and hence that

\[ \beta = \frac{3\bar{N}}{8\omega_0^2} \alpha^2 T_1 + \beta_0 \]  

(33)

where \( \beta_0 \) is a constant. Returning to (31), we find that

\[ A = \frac{1}{2} \alpha \exp\left(i\frac{3\bar{N}}{8\omega_0^2} \alpha^2 et + i\beta_0\right) \]  

(34)

where we used the fact that \( T_1 = et \).

Substituting for \( u_0 \) and \( u_1 \) from (27) and (30) into (24) and using (34), we obtain

\[ u_n(t) = \alpha \cos(\omega_0 t + \beta_0) + \epsilon \left( \frac{8\alpha^2}{8\omega_0^2} \cos(2\omega_0 t + 2\beta_0) + \frac{3\bar{N} \alpha^2}{9\omega_0^2} \cos(3\omega_0 t + 3\beta_0) - \frac{\bar{M}A^2}{\omega_0^2} \right) \]  

(35)

where
\[ \omega = \omega_0 \left( 1 + \frac{3N}{8\omega_0} \alpha^2 \epsilon \right) \] (36)

\(\alpha\) and \(\beta_0\) will be found according to initial conditions, then we can find \(u_p(t)\).

Now we can compare perturbation solution, with Runge-Kutta method for equations (13), (14) and (16) for different voltages. As shown in fig.2 and 3, there is a good agreement between these methods, and error in non pull-in response of a nano-beam is less than 5%.

It’s clear that with increasing voltage to pull-in voltage, error increases. We can decrease this error by increasing the number of the Taylor series and number of time-scales. This method can predict the dynamic behavior of nano-beam very well, and can be used for this purpose.

\[ \text{Fig 2. Midpoint deflection time history for different voltages using multiple scales and Runge–Kutta methods, assuming van der Waals force, for the beam properties } E=186.6e9 \text{ Pa, } b=4e-9 \text{ m, } h=3.5e-9 \text{ m, } l=130e-9 \text{ m, non-dimensional initial axial force } N=8.7, \text{ and } d=10e-9 \text{ m) } \]

\[ \text{Fig 3. Midpoint deflection time history for different voltages using multiple scales and Runge–Kutta methods, assuming Casimir force for the beam properties } E=186.6e9 \text{ Pa, } b=4e-9 \text{ m, } h=3.5e-9 \text{ m, } l=130e-9 \text{ m, non-dimensional initial axial force } N=8.7, \text{ and } d=25e-9 \text{ m) } \]

5 Results
The three-mode reduced order model has been used, in this section, to study the behavior of the nano-cantilever under a suddenly applied DC voltage in presence of intermolecular forces. A set of nonlinear ODEs obtained from (12) with \(M = 3\) is numerically solved for zero initial conditions to predict the transient behavior. We use single-mode approximation to simulate dynamic behavior of a beam in Casimir case (equation (14)). Now we show the importance of Vander-Waals and Casimir forces in nano-scale, with one example. As shown in fig.4 pull-in voltage computed with assuming \(F_{\text{vdw}}\) is about 1.82 volt, but if we neglect \(F_{\text{vdw}}\), \(V_{\text{DPI}} = 2.05\) volt (11% error). As shown in fig.5 pull-in voltage was computed with assuming \(F_{\text{Cas}}\). It’s about 19.75 volt, but if we neglect \(F_{\text{Cas}}\), \(V_{\text{DPI}} = 20.8\) volt (7% error). For a particular value of an initial gap and beam properties,
with increasing the excitation voltage, we study the deflection time history at the midpoint of the beam. Periodic motion as a dynamic response of the undamped beam under various suddenly applied voltages (below and above the certain critical value of the voltage known as the dynamic pull-in voltage ($V_{DPI}$)) are shown in figures 6, 7, 8 for van der Waals case and in fig.9 for Casimir case. As shown in fig.6 and fig.7 with increasing the applied voltage, midpoint deflection of the beam increases slowly, since the applied voltages are less than pull-in voltage. With the time period increasing with voltage, a definite softening effect of the electrostatic forces and the inertial nonlinearity is concluded. For a certain voltage above the critical value, the periodic motion gives away to a divergent motion and the beam abruptly collapses onto the electrode. The motions of the beam at $x=l/2$ for a voltages above $V_{DPI}$ are shown in fig.8.
The quantitative estimation of the dynamic pull-in parameters can be made from the corresponding phase plots as shown in fig.10. At voltages lower than some critical value the beam performs periodic motion around an equilibrium position. An increase in the applied voltage leads to an increase in the amplitude of vibrations and a decrease in frequency. At voltages higher than the critical value a divergent motion is developed and the deflection increases abruptly until the beam collapses onto the electrode.

6 Conclusions

The dynamic behavior of a nano-beam, with relatively large gap to beam-length ratios, under electrostatic actuation is studied herein with special emphasis on the nonlinear effects due electric forces, van der Waals and Casimir forces and inertial terms. First the importance of van der Waals and Casimir forces in nano-scales was shown. In these cases there is a large gap between deformable conductor and ground plane, it is essential to consider higher order corrections of electrostatic forces during the formulation of the model, we considered the first order fringing field effect here. The static analysis where the voltages are applied gradually, a linear model can appreciably predict the static behavior for small electrostatic forces as the deflections are small, but here for studying dynamic analysis, for higher strengths of electrostatic forces close to pull-in, coupled effects of
geometric nonlinearity, and nonlinear electrostatic forces with higher order correction terms in presence of van der Waals and Casimir forces cause deviation from the linearized results. We performed the solution of dynamic equations by multiple scales method that was in good agreement with the numerical method (Runge-Kutta) especially in non pull-in behavior. By increasing the voltage to pull-in voltage, in order to decreasing the error, we should use more terms in Taylor series and continue the perturbation method to larger steps. Consideration of nonlinearities gives a better estimation of the stability limits which can be usefully used for design of non-pull-in devices. A reduced order model incorporating the correct number of modes has been evaluated in the present work and has been effectively used to study the transient behavior of nano-beams

In Table 1, the pull-in voltages of a fixed-fixed nano-beams (h=3.5nm, E=166GPa, b=18nm) in the typical intermolecular force type transition regime (d= 20 nm). When the gap is less than 20 nm (for a metal), the intermolecular force between two surfaces is simplified as the van der Waals attraction. In case that the gap is large above 20 nm, the intermolecular interaction between two surfaces can be described as the Casimir force (the result of the quantum fluctuations of the vacuum electromagnetic field). The intermolecular force is modeled as the van der Waals attraction in Case 1 and the Casimir force in Case 2. A comparison shows that the pull-in voltage in Case 1 is always higher than that in Case 2. It is suggested that in such a transition regime, the dynamic pull-in voltage predicted in Case 2 should be used in nano-switch design to guarantee its safe operation. The ratio of dynamic pull-in and static pull-in voltages are always less than 1, and usually is in range of (80% to 95%). So nano-resonators and nano-switches are better to be designed, according to dynamic pull-in voltages or 80% of static pull-in to be safe. It should be noted that in this table in order to compare our results with [15] we neglected large deflection effects on pull-in voltages. It is interesting that in these three cases, \[V_{\text{static}}^{\text{Dynmic}}\] in van der Waals cases are all equal to 91% and \[V_{\text{static}}^{\text{Dynmic}}\] in Casimir cases are all 88%.

<table>
<thead>
<tr>
<th>Beam length (nm)</th>
<th>Dynamic pull-in</th>
<th>Static pull-in [15]</th>
<th>[V_{\text{static}}^{\text{Dynmic}}]</th>
</tr>
</thead>
<tbody>
<tr>
<td>130</td>
<td>13.15</td>
<td>12.581</td>
<td>14.416</td>
</tr>
<tr>
<td></td>
<td>12.801</td>
<td></td>
<td>13.737</td>
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<td></td>
<td>91%</td>
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<td>88%</td>
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<tr>
<td>150</td>
<td>9.849</td>
<td>9.430</td>
<td>10.827</td>
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<tr>
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<td>88%</td>
</tr>
<tr>
<td>180</td>
<td>6.808</td>
<td>6.523</td>
<td>7.5175</td>
</tr>
<tr>
<td></td>
<td>91%</td>
<td></td>
<td>88%</td>
</tr>
</tbody>
</table>

Table 1- Comparison of dynamic and static pull-in voltages with [15]in transient gap (20 nm)

References