Modeling and Prediction of Laser Generation in UV Copper Bromide Laser via MARS

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Abstract: A case study of the multidimensional dependence of average output laser power on basic input laser parameters in UV Cu+ Ne-CuBr laser is presented. A nonparametric statistical analysis of large amount of experimental data is carry out by multuvariate adaptive regression splines (MARS) technique. The obtained results are in good agreement with practical issues. It is shown that the constructed best MARS models can be applied in estimation and prediction of current and future experiments in order to improve the laser generation.

Key-Words: - Deep ultraviolet copper ion excited neon copper bromide laser, Laser generation, Output laser pour, Multivariate sdaptive regression splines (MARS), Nonparametric regression.

1 Introduction

Development of copper halide lasers continues to be topical [1]. That is due to the fact that in the visible range ($\lambda_1 = 510.6\,\text{nm}$, $\lambda_2 = 578.2\,\text{nm}$), as well as in the ultraviolet range ($\lambda_1 = 248.6\,\text{nm}$, $\lambda_2 = 252.9\,\text{nm}$, $\lambda_3 = 260.0\,\text{nm}$ and $\lambda_4 = 270.3\,\text{nm}$), these lasers operate at highest output power. In particular, because of these and other specific capabilities DUV Cu+ Ne-CuBr laser found wide application in a number of fields [1-3]. The latest theoretical and experimental results on this laser are published in [1, 4-7]. Recently new approach for studying copper halide lasers, based on statistical analysis of the available experimental data was developed. In [8-10] multivariate techniques as factor, regression and cluster analyses for investigating dependences between parameters of copper bromide laser were applied.

In this paper we deal with data of UV Cu+ Ne-CuBr laser. Using the methodology of MARS, the following problems are solved: investigation on the mutual influence of basic input parameters on the output laser power; establishment of the best MARS models of the 0^{th}, 1^{st} and 2^{nd} order for this dependence; comparison between the models constructed and interpretation of the results; cross-validation of the models; implementation of the models for prediction of future experiments.

Experimental results obtained in the Metal Vapour Lasers Laboratory, at the Georgi Nadjakov Institute of Solid State Physics, Bulgarian Academy of Sciences are used for basis of the statistical study. This laboratory is leader in the DUV Cu+ laser development [1-7].

2 Data description

We will consider the available experimental data of UV Cu+ Ne-CuBr laser. The construction of the laser tube is shown in Figure 1.

We will examine data of nine input basic variables which determine the laser operation. They are: *D* (mm) – inside diameter of the laser tube, *L* (cm) – electrode separation (length of the active area), *PIN* (W) – input electrical power, *PRF* (KHz) – pulse repetition frequency, *PNE* (Torr) - neon gas pressure, *PH*2 (Torr) – hydrogen gas pressure, *PL* (W/cm) - specific electrical power per unit length, *TR* (K) – temperature of the reservoir, *pd* (Torr.cm). The dependent variable is *Pout* (mW) – multiline average output laser power.

The data is of historical type. It consists of 237 experiments. Here we have to mention the complexity, long duration and high cost of each conducted experiment.

The data of consideration are not normally distributed, which was checked by applying a nonparametric Kolmogorov-Smirnov test. This is why the well known parametric regression methods as multiple linear regression, do not give satisfactory results for our data.

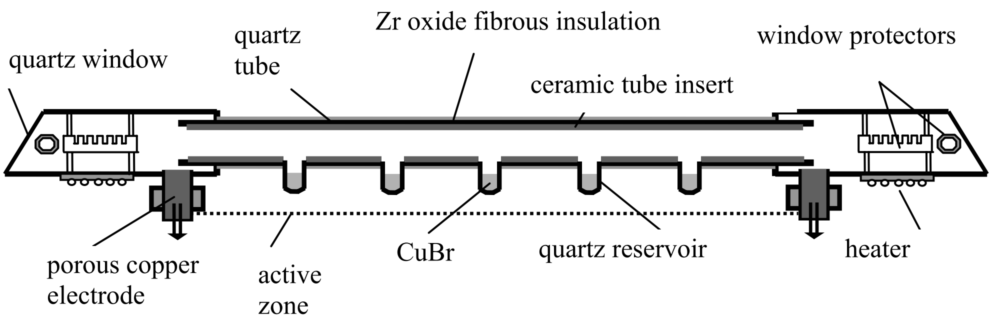

Fig. 1. Construction of the UV Cu+ Ne-CuBr laser tube.

3 Construction of the best MARS models

3.1 Brief description of MARS method

The mathematical basis for MARS was developed by Friedman in 1991 [11]. The created algorithms and their first working program versions have been integrated in the currently existing MARS software product. The product has gained popularity and has been applied with increasing success in the last few years [12].

In essence MARS generates flexible adaptive models through partially linear regressions, i.e. data nonlinearities are approximated using separate sloped intercepts in different subintervals of the set defined for each predictor variable. A broken line is used, instead of looking for one common regression curve approximating the data. The slope of the regression line varies from one interval to another at the so called nodes. The node shows where the behavior of the function changes. In the classic spline, nodes are given in advance, while with MARS, the most suitable place for them is determined using a fast algorithm when certain suitable optimization conditions are met (for example a SSE minimum – the sum of the squares of the errors).

The other basic element of MARS is the basis function (*BF*) for transformation of predictors. The basis function is called a “hockey stick” and has the form:

$$\max(0, X - c) \text{ or } \max(0, c - X)$$

where *c* is a constant (value of the node). The regression spline is constructed as a linear or nonlinear combination of basic functions. An example of univariate MARS model of the dependence between *PIN* and *Pout* with three basis functions is shown in Figure 2.

Within this study only the best MARS models are presented. They are calculated by using the upper nine independent laser variables *D*, *L*, *PIN*, *PRF*, *PNE*, *PH*2, *TR*, *pd* and *PL* as predictors and *Pout* as a response variable. The best models are selected so as to allow no overfitting of the model, as well as by using the algorithm for applying the least squares method with a GCV (generalized cross validation measure) criterion [11-13].

In this study we set a limit of no more than 20 basic functions. Models of 0^{th}, 1^{st} and 2^{nd} order of interactions between predictors were obtained. Below are a part of the derived models.

The main statistics of the constructed models are summarized in Table 1.

3.2 Zero order MARS model

Firstly we present the MARS model of the 0^{th} order without interaction between predictors. It includes the following fifteen piecewise linear basic functions:

$$\begin{aligned}
BF1 &= \max(0, PIN - 1440);\\
BF2 &= \max(0, 1440 - PIN);\\
BF3 &= \max(0, PNE - 19.34);\\
BF4 &= \max(0, 19.34 - PNE);\\
BF5 &= \max(0, PH2 - 0.04);\\
BF6 &= \max(0, 0.04 - PH2);\\
BF7 &= \max(0, D - 8);\\
BF9 &= \max(0, PNE - 21.88);\\
BF11 &= \max(0, PNE - 18.5);\\
BF13 &= \max(0, PNE - 20);\\
BF15 &= \max(0, PIN - 1400);\\
BF17 &= \max(0, PD - 10.75);\\
BF18 &= \max(0, 10.75 - PD);\\
BF19 &= \max(0, L - 86);\\
BF20 &= \max(0, 86 - L);
\end{aligned} \qquad (1)$$

The basis functions include six predictors. Sorted by their decreasing importance in the model, they are: *PIN*, *PNE*, *PH*2, *pd, D* and *L*. Their graphs are similar of the example in Figure 2.

The model is

$$\widehat{Pout} = 941.5008 + 4.7347\,BF1 - 1.1363\,BF2 - 751.6722\,BF3 - 52.3815\,BF4 - 13445.6182\,BF5 - 8704.9668\,BF6 + 40.5457\,BF7 + 78.3509\,BF9 + 300.3305\,BF11 + 373.0804\,BF13 - 3.7940\,BF15 - 46.5438\,BF17 - 100.3301\,BF18 + 267.5647\,BF19 + 18.7600\,BF20 \quad (2)$$

With the help of MARS model (1)-(2) it is easy to calculate the estimate of *Pout* when predictor values are known. For example, a maximum laser output power *Pout* = 1300 mW has been measured at *D*=5.7 mm, *PIN*=1900 W, *PNE*=19.3 Torr, *PH*2=0 Torr, *TR*=560 K, *PL*=5.52 W/cm, *L*=86 cm, *pd*=11.04 and *PRF*=25 KHz [6]. After substituting the latter in (1)-(2) we find the approximate estimate $\widehat{Pout} = 1113$ mW.

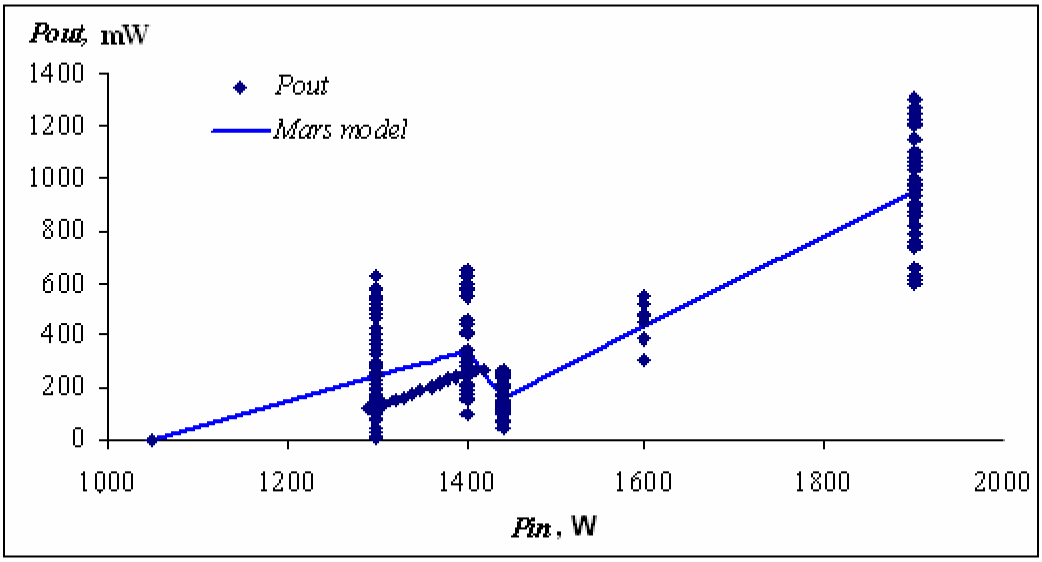

Fig. 2. Data of *Pout* vs *PIN* with the best MARS model, using 3 basic functions with two nodes: 1400 and 1440.

3.3 Second order MARS model

The 2nd order best MARS model uses the following basics functions

$$\begin{aligned}
BF1 &= \max(0, PIN - 1440);\\
BF2 &= \max(0, 1440 - PIN);\\
BF3 &= \max(0, PNE - 19.34);\\
BF4 &= \max(0, 19.34 - PNE);\\
BF5 &= \max(0, PNE - 19.3)\,BF1;\\
BF6 &= \max(0, 19.3 - PNE)\,BF1;\\
BF7 &= \max(0, PH2 - 0.04)\,BF4;\\
BF8 &= \max(0, 0.04 - PH2)\,BF4;\\
BF9 &= \max(0, D - 8)\,BF4;\\
BF12 &= \max(0, 10.55 - pd)\,BF2;\\
BF13 &= \max(0, PIN - 1600)\,BF3;\\
BF15 &= \max(0, PNE - 21.57)\,BF2;\\
BF17 &= \max(0, D - 8)\,BF2;\\
BF18 &= \max(0, 8 - D)\,BF2;\\
BF19 &= \max(0, PL - 3.78)\,BF17;\\
BF20 &= \max(0, 3.7 - PL)\,BF17;
\end{aligned} \quad (3)$$

In (3) the predictors are: *PIN*, *PNE*, *PH*2, *D*, *PL* and *pd*. The equation to calculate the estimates of *Pout* is:

$$\widehat{Pout} = 264.4585 + 2.1350\,BF1 - 0.86218\,BF2 - 181.8927\,BF3 + 150.007\,BF4 + 0.6079\,BF5 - 1.4871\,BF6 - 5020.7812\,BF7 - 3753.4465\,BF8 - 3.4747\,BF9 - 1.4942\,BF12 - 3.0753\,BF13 + 1.1135\,BF15 + 1.1723\,BF18 + 0.6320\,BF19 + 0.2412\,BF20; \quad (4)$$

In particular, the predicted value for *Pout*=1300 mW by the model (3)-(4) is approximately $\widehat{Pout} = 1246$ mW.

3.4 Notes on the constructed models

We will briefly discuss the obtained results in this section. From Table 1 it can be seen that the 2nd order models have the best statistical indexes. However, the models of higher order of interaction between predictors do not show further improvement of the basic statistics.

As an example, the contribution of the interaction between *PNE* and *PH*2 to *Pout* is shown in Figure 3. A maximum for *PNE* in the interval [1.9, 2] is observed. This is also seen in the basic functions *BF*4, *BF*5, *BF*7 and *BF*8 in (3).

It can be noted that six of the nine predictors show influence over laser generation. Variables *PRF*, *L* and *TR* are not included in the models. The statistical tests of all three models are very good - in particular, the coefficient of determination $R^2 \approx 0.966$ for the 2nd order model. This indicates that model (3)-(4) explains 96.6% of the data sample. In general, equation (4) presents nonlinearities in the studied dependence by piecewise polynomials up to third degree in different subregions.

It can be concluded that the models fit our data well.

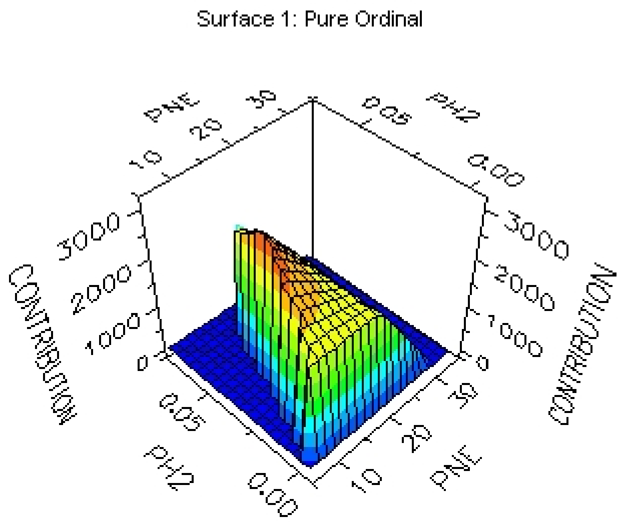

Fig 3. The contribution of the predictors *PNE* and *PH*2 on *Pout* in model (3)-(4).

4 Cross-validation of the models and prediction of future experiment

4.1 Cross-validation

To check validity of the MARS models for our data the usual split sampling technique was carried out (see [14-15]). In this method the observed data are divided randomly into two sub-samples, for instance in proportion 70-30%. The 70% subset is called “training” sub-sample and is used to estimate the model. Subsequently the obtained model is applied to predict the dependent variable for the remaining data or “validation” sub-sample.

Here we present the data from applying this technique to the model (3)-(4). The obtained results for our training and validation samples are given in the right four columns of Table 1. The results for the test sub-sample of 70% are derived using the same model setup with up to 20 basic functions with 2nd order interactions. The validation of the 30% sub-sample describes 95.8% of the corresponding sample of experimental data, which is a very good result. We can conclude that the model (3)-(4) is valid and possesses good prediction power.

Table 1. Basic statistics of constructed MARS models, 2nd order MARS model from a random training 70% sub-sample and its 30% validation sub-sample.

MARS model	0th order	1st order	2nd order	0th order 70% training sub-sample	1st order 70% training sub-sample	2nd order 70% training sub-sample	30% validation sub-sample
R^2	0.9322	0.9635	0.9657	0.9311	0.9660	0.9702	0.9576
R^2 adjusted	0.9276	0.9609	0.9581	0.9254	0.9622	0.9669	0.9570
GCV	0.9077	0.9464	0.9438	0.9009	0.9370	0.9448	-
Direct predictors	6	6	6	6	6	6	-
Terms in model	15	16	15	13	17	17	-

4.2 Prediction of future experiment

The obtained MARS models can be used for prediction of new experiments. To this end we apply model (3)-(4) at following fixed laser parameters: *D*=5.7 mm, *PH2*=0.04 Torr. Dependance of average output power *Pout* on Ne buffer-gas pressure at different values of input power *PIN* is shown in Figure 4: the solid line is the experimental data from [6], the triangles are the values estimated by the MARS model (3)-(4) at *PIN*=1900 W; the two other lines are predictions of future experiments by means of (3)-(4) at *PIN*=2000 W and 2100 W, respectively.

We can add here that in principle MARS models give averaged local behavior of the dependent variable, as it was demonstrated in Figure 2.

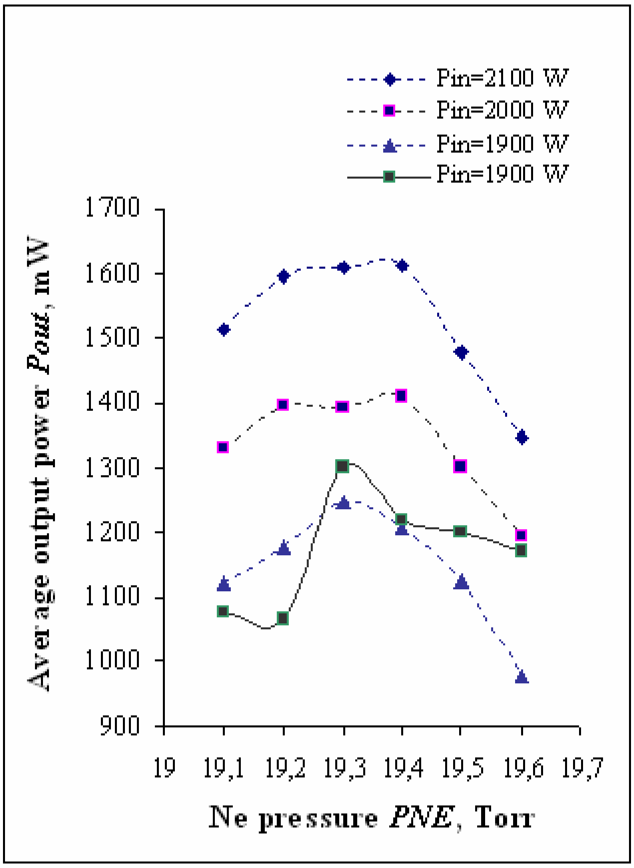

Fig. 4. Comparison of experimental data of laser power *Pout* (solid line) against estimates (triangles) at *PIN*=1900; the two other lines are predictions via MARS model (3)-(4) at *PIN*=2000 W and *PIN*=2100 W, respectively.

5 Conclusion

In this article we presented the main results of statistical modeling of multiline average output power in a UV C+ Ne-CuBr laser by means of the flexible nonparametric MARS method. The models demonstrate good prediction ability in estimation and prediction existing and future experiments. This methodology could be successfully applied in other areas of laser technology.

6 Acknowledgements

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