Nonlinear Model & Controller Design for Magnetic Levitation System

ISHTIAQ AHMAD[1], MUHAMMAD AKRAM JAVAID[2]
[1]Department of Civil Engineering
[2]Department of Basic Sciences
University of Engineering & Technology, Taxila,
PAKISTAN
ishtiaq@uettaxila.edu.pk, www.uettaxila.edu.pk

Abstract: - This paper aims at development of non linear dynamic model for Magnetic Levitation System and proposed linear and nonlinear state space controllers. The linear controller was designed by linearizing the model around equilibrium point, while nonlinear controller was based on feedback linearization where a nonlinear state-space transformation is used to linearize the system exactly. Relative degree of the system was determined and conditions were found that ensure relative degree be well defined. Magnetic Levitation system considered in this study is taken as a ferromagnetic ball suspended in a voltage controlled magnetic field. Dynamic behaviour of the system was modeled by the study of electromagnetic and mechanical subsystems. State space model was derived from the system equations. Linear full state feedback controller along with linear observer was designed and was compared with nonlinear full state feedback with nonlinear observer. Both linear and nonlinear controllers were simulated using matlab and results are presented.

Key-Words: - Magnetic Levitation System; Nonlinear Model; Exact Linearization; Electromagnet; unstable

1 Introduction

A lot of research effort in control system field has been focused on the control of a Magnetic Levitation System (MLS). They are widely used in various fields such as frictionless bearings, high-speed Maglev passenger trains, levitation of wind tunnel models etc. MLS are generally highly nonlinear and open loop unstable systems. This unstable aspect of MLS and its inherent nonlinearities make the modeling and control problems very challenging. Several dynamic models of magnetic force have been proposed over the past years and with these models various control strategies have been used comparing their performance. Both the linear and non-linear techniques have been used. Linear system model only works well over a small region of operating point [1].

Wong obtained an approximate linear model, with an open-loop pole in the right-half plane. A phase-lead (linear) compensator was used to stabilize the system for step responses of 1.5 mm around the operating point [2].

Guess and Alciatore examined the differences between the conventional magnetic levitation system model and actual system. Effects of un-modeled dynamics on the stability of a simulated system were also studied. PID controller proved to be effective for set point regulation and for tracking a changing input [3].

Valer and Lia build a nonlinear model for magnetic levitation system and proposes systems linearization principle (the expansion in Fourier series and the preservation of the first order terms) in order to linearize the acquired nonlinear model [4].

Ying-Shing Shiao, (2001) employed system linearization and phase-lead compensation with virtual pole cancellation to design the controller of unstable nonlinear system to maintain better stability in a levitated ball. Such magnetic levitation systems (MLS) with small operating ranges have been proposed by the various researchers [5].

[6] presented a nonlinear model for the magnetic force of magnetic levitation device and model was then used to propose a control technique for position control of a magnetically levitated permanent magnet. A Lyapunov based stability analysis was performed to prove the stability of the control technique. It was reported that the proposed controller performed a precise positioning operation over an operation range of 30 mm, which is an improvement over available control strategies in the literature for large gap magnetic levitation systems.

In [7] the author carried out a comparative study of linear and nonlinear controllers for Maglev system and stated that, feed-back linearization controller has provided significantly better trajectory tracking.

2 Magnetic Levitation System

Magnetic levitation system considered in the current analysis is consisting of a ferromagnetic ball suspended in a voltage-controlled magnetic field. Fig. 1 shows the schematic diagram of magnetic levitation system.

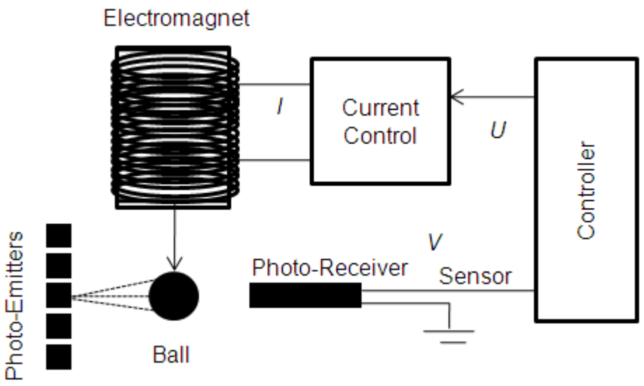

Figure 1: Schematic Diagram of Magnetic Levitation System

Coil acts as electromagnetic actuator, while an opto-electronic sensor determines the position of the ferromagnetic ball. By regulating the electric current in the circuit through a controller, the electromagnetic force can be adjusted to be equal to the weight of the steel ball, thus the ball will levitate in an equilibrium state. But it is a nonlinear, open loop, unstable system that demands a good dynamic model and a stabilized controller.

3 System Dynamics and Modeling

Dynamic behaviuor of magnetic levitation system can be modeled by the study of electromagnetic and mechanical sub systems.

3.1 Electromagnetic Dynamics Modeling

Electromagnetic force produced by current is given by the kirchoff's voltage law;

$$u(t) = V_R + V_L = iR + \frac{dL(x)i}{dt}$$

Where,
u : applied voltage,
i : current in the coil of electromagnet,
R : coil's resistance and
L : coil's inductance.

3.2 Mechanical Modeling

Free body diagram of ferromagnetic ball suspended by balancing the electromagnetic force $f_{em}(x,i)$ and gravitational force f_g is shown in Fig. 2.

Net force f_{net} acting on the ball is given by Newton's 3rd law of motion while neglecting friction, drag force of the air etc. [2]

$$f_{net} = f_g - f_{em}$$

$$m\ddot{x} = mg - C(\frac{i}{x})^2$$

Where
m: mass of ball,
x: position of the ball,
g: gravitational constant and
C: magnetic force constant.

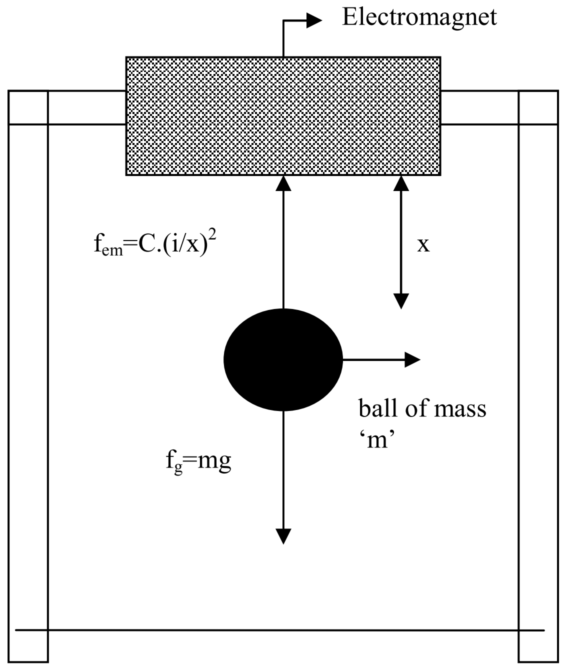

Figure 2. Free Body diagram of Magnetic levitation system.

3.3 Non Linear Model

On the basis of electro-mechanical modeling nonlinear model of magnetic levitation system can be described in terms of following set of differential equations;

$$v = \frac{dx}{dt} \text{------------}(1)$$

$$u = Ri + \frac{dL(x)i}{dt} \text{-------}(2)$$

$$m\ddot{x} = mg - C(\frac{i}{x})^2 \text{------}(3)$$

Equation (2) indicates that L(x) is a nonlinear function of balls position x [2], [8]. Various approximations have been used for determination of inductance for a magnetic levitation system. If we take the approximation that inductance varies with the inverse of ball's position, that is

$$L(x) = L + \frac{L_0 x_0}{x} \text{-------}(4)$$

Where L is the constant inductance of the coil in the absence of ball, L_o is the additional inductance contributed by the presence of the ball, x_0 is the equilibrium position. Substituting (4) into (2) results in

$$u(t) = iR + \frac{d}{dt}(L_c + \frac{L_0 x_0}{x})i$$

$$u(t) = iR + L\frac{di}{dt} - (\frac{L_0 x_0 i}{x^2})\frac{dx}{dt}$$

Substituting $L_0 x_0 = 2C$ [9], we get

$$u(t) = iR + L\frac{di}{dt} - C(\frac{i}{x^2})\frac{dx}{dt} ----(5)$$

3.4 Vector format

Taking x=x1, v=x2, i=x3, Equations (1), (3) and (5) can be expressed in vector format where position of ball is taken as output as under;

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} x2 \\ g - \frac{C}{m}(\frac{x_3}{x_1})^2 \\ -\frac{R}{L}x_3 + \frac{2C}{L}(\frac{x_2 x_3}{x_1^2}) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix} u$$

$$y = \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

$$\dot{x} = f(x) + g(x)u$$

3.5 Linear Model

The system was linearized around a point $x_1=x_{01}$, which results in state vector as;

$$X_0 = \begin{bmatrix} x0_1 & x0_2 & x0_3 \end{bmatrix}^T$$

At equilibrium, time rate derivative of x must be equal to zero i.e. $x0_2=0$. Also equilibrium current can be evaluated from Equation (3) and it must satisfy the following condition;

$$x0_3 = x0_1\sqrt{\frac{gm}{C}}$$

Thus we can write the linearized model in state space form as under;

$$A = \begin{bmatrix} 0 & 1 & 0 \\ \frac{Cx_{03}^2}{mx_{01}^3} & 0 & -2\frac{Cx_{03}}{mx_{01}^2} \\ 0 & 2\frac{Cx_{03}}{Lx_{01}^2} & -\frac{R}{L} \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ \frac{1}{L} \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

3.6 Exact Feedback Linearizing Controller

Relative degree of the system

In order to determine relative degree of the system the following definition was used [10];

$$y^r = L^r fh(x) + LgLf^{r-1}h(x)u$$

'r' is relative degree of the system when $LgLf^i h(x) = 0 \quad where \quad 0 \le i < r-1$

$$LgLf^{r-1}h(x) \ne 0$$

Using the definition

For i=0

$$L_g h(x) = 0$$

i=1

$$L_g L_f h(x) = 0$$

$$i = 2$$

$$L_g L^2{}_f h(x) = -\frac{-2C}{mL}\left(\frac{x_3}{x_1^2}\right) ------(6)$$

At equilibrium point the term $L_g L^2{}_f h(x) \ne 0$ means that degree of the system is 3. In order to assure that the relative degree of the system is well defined, x1 and x3 should be greater than zero. This is quiet acceptable as x1≤0 means that the levitated object touches the magnetic coil or exists inside the coil while x3<0 would result in negative current.

Diffeomorphism and Feed back Transformation

Considering the nonlinear change in coordinates

$$z = \begin{bmatrix} x_1 & x_2 & g - \frac{C}{m}\left(\frac{x_3}{x_1}\right)^2 \end{bmatrix}$$

In order to ensure transformation is invertible the system state is restricted to the region x1>0 and x3>0. In new coordinates, system equation becomes;

$$\dot{z}_1 = z_2$$

$$\dot{z}_2 = z_3$$

$$\dot{z}_3 = \alpha(x) + \beta(x).u$$

Where

$$\alpha(x) = \frac{2C}{m} - \frac{4C^2}{mL}\left(\frac{x_3^2 x_2}{x_1^3}\right) + \frac{2CR}{mL}\left(\frac{x_3}{x_1}\right)^2$$

$$\beta(x) = -\frac{2Cx_3}{mLx_1^2}$$

Using state feedback

$$u = \frac{\upsilon - \alpha(x)}{\beta(x)} \quad \text{Where} \quad \upsilon = \dot{z}_3$$

Linear state space representation can be written as

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \dot{z}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} z_1 \\ z_2 \\ z_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ \upsilon \end{bmatrix}$$

As the above system is in control canonical form, it is simple to choose the feedback gains to place the closed loop poles on the left half plane.

4 Simulation Results

In order to verify the proposed linear and nonlinear controller, the system was simulated using MATLAB and parameter values are presented in Table 1.

Table 1: Physical parameters of Magnetic Levitation system

Parameter	Unit	Value
m	Kg	0.05
g	m/s^2	9.8
R	ohms (Ω)	1
L	H	0.01
C	------	0.0001
x_{01}	m	0.012
x_{02}	m/s	0
x_{03}	A	0.84

In state space form we can write the linear model as under

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 1633.33 & 0 & -23.33 \\ 0 & 116.66 & -100 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 0 \\ 100 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

The response of the linear system with step input and sinusoidal input is given in Fig 3 and 4 respectively.

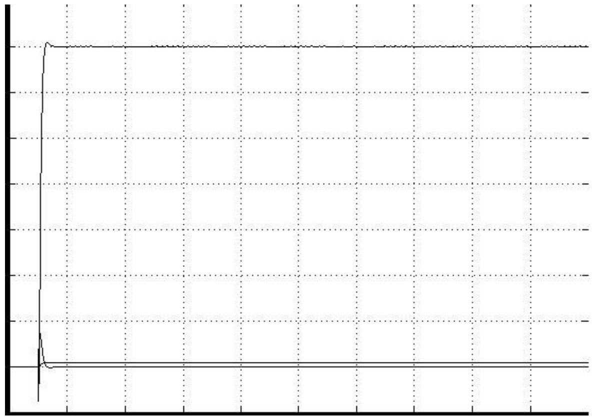

Figure 3: Unit step input response of the linear system

Figure 4: States of the linear system with sinusoidal input

For nonlinear system there are two loops in the control system in which inner loop linearized the input-sate relation, and the outer loop stabilize the closed loop dynamics and in shown in Fig 5.

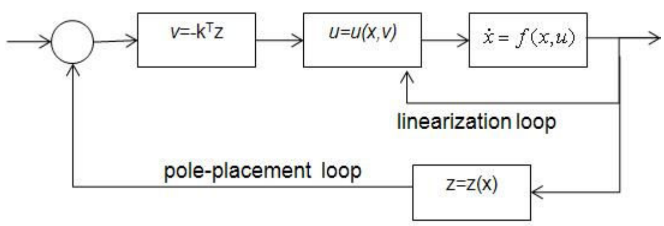

Figure 5: Feedback Linearization

5 Conclusions

While developing a model for Megnetic Levitation system, various approximations for coil inductance can be made. Model developed in this paper is based on the assumption that the inductance varies with the inverse of ball's position and is given in (4), however, in literature various approximations are presented. Thus one can

particularize the dynamic model of the levitation system for each mode of calculating the inductivity.
The results of exact feedback Linearization controller are valid in a large region, however it is not global. The control law is not well defined at $x_1=0$, thus if initial states are at singularity points, the controller can not bring the system to equilibrium point.

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