Image Segmentation Using a Generalized Fast Level Set Method

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Abstract: - Image segmentation is one of the most important and fundamental tasks in image processing. In this paper we address the drawbacks of the previous level set methods for segmentation problems and propose a Generalized Fast Level Set Method to cope with the limitations. We formulate a new level set function, study its stability, introduce a relationship matrix, a modified Chan-Vese model and a novel filtering criterion to construct a novel and effective segmentation technique. The experimental results show that this technique can be used in segmenting individual objects or individual parts of an object in images, which is useful in reducing the heavy image noises. These results also demonstrate that the proposed method is more effective and efficient than the classical Level Set Methods and the original Fast Level Set Method.

Key-Words: - Generalized Fast Level Set Method, Chan-Vese model, filtering criterion, relationship matrix, local stopping criteria, parent-child boundary relationship, image segmentation

1. Introduction

The aim of this paper is to describe the existing level set methods with their limitations and introduce a new level set method for the solution of plane object boundary segmentation problems in image analysis domains. The level set framework, which was first proposed by Osher and Sethian in [1], has made a profound impact on image segmentation. The geometric approach brings an important advantage of the ability to adjust to the current image topology. Unfortunately, the main shortcoming in the computational process is that an increase of dimensions will inevitably lead to low speed. Many solutions have been proposed (Cremers et al. [2]). Among them, the fast level set method (Shi Y. et al. [3], Jakub K. et al. [4]), which falls into the category of narrow band algorithms, is extremely fast for image segmentation. Although very powerful, both the classical level set method and the fast level set method cannot segment more than two disjoint regions, such that the regions correspond to the objects in the image. A number of approaches have been proposed for segmentation of multiple objects or multiple components of an object. In Zhao et al. [5], Samson et al. [6], and Paragios et al. [7], a level set function is assigned to each region. In [8], Chan and Vese proposed the multiphase level set framework using Mumford-Shah model (Mumford et al. [9]). The idea of the multiphase approach is to use a combination of level set functions to represent multiple regions or multiple phases in the images. Based on the Chan-Vese model, many algorithms have been introduced. Although these methods only use $\log_2 N$ level set functions to represent $N$ regions, they will result in empty regions, if less than $N$ regions are presented in the images. Another important limitation of level set formulations is sensitivity to noise and a failure to capture prior shape-driven knowledge on the structure to be segmented. Moreover, the classical level set method and the original fast level set method take longer to segment the objects which are empty in the inner parts such as head or skull CT images because they cannot integrate empty parts’ information into their curve evolution.

To overcome these limitations, we propose a new level set function, extend the Chan-Vese segmentation criterion and introduce a novel filtering criterion into the geometric active contour framework. The Generalized Fast Level Set Method, which will be mentioned in the following sections, is more effective and efficient than the classical level set methods and the fast level set method. First, our method obtains the segmentation results better than the fast level set method based on the modified Chan-Vese energy function. Additionally, our proposed algorithm enables a significant decrease in computational time, compared with the fast level set method, because our algorithm can get the best initial value formulation based on the novel level set function (approximately two orders faster than the fast level set method in our experiments). Finally, our approach is proved to be
resistant to a particular noise environment, which means that it is less sensitive to noise.

The rest of the paper is organized as follows. Section 2 provides a detailed description of the Generalized Fast Level Set Method. Next, section 3 shows some experimental results. Finally, section 4 concludes the paper with a discussion of the results and future improvements to the algorithm.

2. Generalized Fast Level Set Method

The Generalized Fast Level Set Method introduced in this paper extends and generalizes the fast level set framework which was previously proposed by the authors in Shi Y. et al. [3] and Jakub K. et al. [4] based on the theory of narrow band approaches.

The idea behind this method is that we use the novel level set function that is derived from multiple discrete fast level set functions in order to integrate the prior knowledge of segmented objects with the initial value formulation of the level set function. Moreover, the alternative Chan-Vese segmentation criterion leads to a background subtraction of an image. Therefore, we can segment multiple regions or objects based on the object’s topology and its relationships. In addition, we introduce the local stop criterion for reducing computational time. Finally, we incorporate geometric knowledge into the evolution process in order to reduce the presence of noise in the image.

2.1. The relationship matrix

Based on the theory of narrow band approaches, the fast level set method uses a closed curve $\mathbb{C}$ which is represented by the pair of neighborhood grids $\{L_{in}, L_{out}\}$ and applies the outward (inward) speed function to the points in these lists. However, it’s very difficult to get the best initial value formulation of level set function with only one curve in the complex images. The idea here is that, we initialize the local level set function at each part of complex images, and then we try to find the relationship between them in order to make the initialization of global level set function from these local level set functions.

As a starting point, we assume that we have $n$ initial level set functions $\{\varphi_1, \varphi_2, ..., \varphi_n\}$ which are based on the values of $\varphi_k (k = 1, n)$ as given in (1) (Shi et al. [3]):

$$
\varphi_k(x) = \begin{cases} 
+3 & x \in B_{out} \\
+1 & x \in L_{out} \\
-1 & x \in L_{in} \\
-3 & x \in B_{in} 
\end{cases}
$$

(1)

where:

$$
\begin{align*}
B_{out} &= \{x | x \in outside(\mathbb{C}) \text{and} \ x \notin L_{out} \} \\
B_{in} &= \{x | x \in inside(\mathbb{C}) \text{and} \ x \notin L_{in} \}
\end{align*}
$$

Let $L = \{l_k\}_{k=1}^{n}$ be the set of the zero level set functions $l_k = \partial \varphi_k (k = 1, n)$. We begin by specifying our representation of the level set functions’ relationship through the relationship matrix $RM$ and two relationship sets $R_0^i$ and $R_1^i$.

Definition 1: A pair of two ordered zero level set functions $(l_1, l_2)$ is in parent-child boundary relationship (PBR) if and only if:

$$
l_2 \subset l_1 \text{ and } \forall l_k \subset l_1: l_2 \nsubseteq l_k (k = 1, n)
$$

where $l_1$ is parent boundary and $l_2$ is child boundary.

Definition 2: A pair of two zero level set functions $(l_1, l_2)$ is in independent boundary relationship (IBR) if and only if:

$$
\forall x \in l_1, y \in l_2: x \notin l_2 \text{ and } y \notin l_1
$$

Using these two definitions, the relationship matrix $RM$ and the relationship sets $R_0^i$ and $R_1^i$ are defined as:

$$
RM(l_1, l_j) = \begin{cases} 
-1 & (l_1, l_j) \text{ is PBR} \\
0 & (l_1, l_j) \text{ is IBR} \\
1 & (l_1, l_j) \text{ is PBR}
\end{cases}
$$

(2)

(3)

(4)

(5)

(6)

(6)

(k is called the relationship level)

Now we can design a novel framework for modeling the evolution of boundaries in the Generalized Fast Level Set Method by using the new level set function. The differences between the classical level set function with the fast level set function and our proposed level set function are illustrated by Fig.1.
From Fig. 1 we can see the level set functions in the classical level set method and the fast level set method cannot be represented in form:

\[ \Phi = f(l_1, l_2, ..., l_n) \]  

which must satisfy: \( \exists i, j \in \{1, 2, ..., n\}; l_i \subset l_j \)

where \( \{l_1, l_2, ..., l_n\} \) are the boundaries or zero level set functions. Therefore, the two methods cannot take full advantage of geometric shape information and integrate into initial value formulation of the level set function.

The level set function in our approach can be represented in this form. For example, in the case of Fig. 1c, we use 4 level sets \( \{l_1, l_2, l_3, l_4\} \) and the level set function is:

\[ \Phi = \min \left\{ \varphi_{l_1}, -\min\{\varphi_{l_2}, \varphi_{l_3}\}, \varphi_{l_4} \right\} \]

which is derived from (5) where the relationship matrix and the relationship sets are as follows:

<table>
<thead>
<tr>
<th>RM</th>
<th>( l_1 )</th>
<th>( l_2 )</th>
<th>( l_3 )</th>
<th>( l_4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l_1 )</td>
<td>0</td>
<td>-1</td>
<td>-1</td>
<td>0</td>
</tr>
<tr>
<td>( l_2 )</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( l_3 )</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( l_4 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

\[ R^0 = \{l_1, l_4\}; \ R^1_0 = \emptyset; \ R^1_1 = \{l_2, l_3\} \]

The relationship matrix proposed in this paper has three characteristics: a flexibility characteristic, a simplicity characteristic which is inherited from the fast level set method, and a connectivity characteristic which is inherited from multiphase level set approaches. The foremost important characteristic is to get the best initial value formulation of the level set function; so that it enables a significant decrease in computational time compared with the fast level set function. The latter characteristic establishes a quiet simple level set function with its value which is chosen from a limited set of integer \( \{-3, -1, 1, 3\} \). It’s known that the idea of the multiphase level set method is to use a combination of level set functions to represent multiple objects. Currently, these methods need \( n \) or more level set functions in order to segment \( 2^n \) objects. The connectivity characteristic of relationship matrix allows the new level set function to inherit this property from the multiphase level set method; moreover, our approach can easily segment multiple objects in the images with the unique level set function.

**2.2. Modified Chan-Vese segmentation criterion**

As pointed out earlier, many level set based segmentation approaches use the Chan-Vese energy function to separate the image into two regions: the background and the foreground. But, the images usually have multiple objects with their intensity variations surrounding the background. In these cases, some objects of the foreground are incorrectly identified as the background. In this paper, we extend the Chan-Vese segmentation criterion to overcome this problem. The alternative Chan-Vese energy function can segment the image into three regions: the high-intensity foreground, the low-intensity foreground and the background.

Suppose \( \{\Omega_1, \Omega_2, ..., \Omega_n\} \) are \( n \) regions in the foreground (inside(\( \mathcal{C} \))). Let \( \{c_j\}_{j=1}^{n} \) be the averages of the intensities in each region \( \{\Omega_i\}_{i=1}^{n} \), and \( c_b \) is the average of the intensities in the background (outside(\( \mathcal{C} \))). Now, we partition the foreground into 2 parts: a high-intensity foreground with \( n_1 \) regions \( \{\Omega_j\}_{j=1}^{n_1} \) which must satisfy: \( \forall j \in \{1, ..., n_1\}: c_j \geq c_b \); and a low-intensity foreground with \( n_2 \) regions \( \{\Omega_k\}_{k=1}^{n_2} \) which must satisfy: \( \forall k \in \{1, ..., n_2\}: c_k < c_b \). Note that: \( n = n_1 + n_2 \). Then, let \( c_{fh} = \min\{c^h_j\}_{j=1}^{n_1} \) and \( c_{fl} = \max\{c^l_k\}_{k=1}^{n_2} \) where \( c^h_j \) and \( c^l_k \) are the averages of the intensities in the high-intensity regions and the low-intensity regions, respectively. Therefore, the...
alternative Chan-Vese segmentation criterion is derived from two these values:

\[
E(c_{fb}, c_{fl}, c_b) = \lambda_1 \int_{\text{inside}(G)} \left( (u(x) - c_{fb})^2 H(\Delta x) + (u(x) - c_{fl})^2 (1 - H(\Delta x)) \right) dx + \lambda_2 \int_{\text{outside}(G)} (u(x) - c_b)^2 dx + \mu \text{Length}(G) + \nu \text{Area}(\text{inside}(G))
\]

where \( H(\Delta x) \) is the Heaviside function:

\[
H(\Delta x) = \begin{cases} 
1 & \Delta x = u(x) - c_b \geq 0 \\
0 & \Delta x = u(x) - c_b < 0 
\end{cases}
\]

Hence, the speed function is defined as follows:

\[
F(x) = \begin{cases} 
1 - \lambda_3 \left( (u(x) - c_b)^2 H(\Delta x) + (u(x) - c_a)^2 (1 - H(\Delta x)) \right) & \lambda_3 (u(x) - c_b)^2 \geq 0 \\
-1 & \lambda_3 (u(x) - c_b)^2 < 0 
\end{cases}
\]

Experimental results show that the accuracy of segmentation is significantly with the modified speed function.

After the background is eliminated altogether by the modified Chan-Vese segmentation criterion, we segment and label individual regions (objects) or individual components of an object based on the object’s topology and its relationships. In this case, the algorithm can automatically segment individual objects and delineate an arbitrary number of regions by adding and removing the pairs of neighborhood grid points in the segmentation process. Currently, the fast level set method uses two lists of grid points adjacent to the evolving curve and applies the stopping criterion to these level sets. If the stopping condition is not satisfied, this method scans all points on the two lists. In our approach, we consider and extend this stopping criterion to \( n \) local stopping criteria \( \{P_k\}_{k=1}^n \) which corresponds to \( \{l_{in,k}, l_{out,k}\}_{k=1}^n \) and is applied to each stopping criterion \( P_k \) to two corresponding level sets \( L_{in,k} \) and \( L_{out,k} \). Note that the value of \( P_k \) is determined by (11):

\[
P_k = \gamma \left( \|L_{in,k}\| + \|L_{out,k}\| \right)
\]

where: \( \gamma = 10^{-3} \), \( k = 1, n \)

By using the local stopping criteria, our method only scans the points in the level sets \( L_{in,k} \) and \( L_{out,k} \) whose local stopping criterion is not reached, and therefore speeds up the curve evolution.

2.3. Noise reduction filter

In this work, we proposed the new approach for reducing the noise in the case of noisy images based on the properties of our method. We incorporate partial shape information into the Generalized Fast Level Set Method in order to reduce the presence of noise in the image. More specifically, in our algorithm, each pair of \( \{l_{in,k}, l_{out,k}\}_{k=1}^n \) represents an object or a noise in the image. By integrating shape information, the pairs of \( \{l_{in,k}, l_{out,k}\}_{k=1}^n \) that represent the noises can be filtered out. Note that in the current works, prior geometry knowledge is only required for the object size, the object area, and the object boundary. For each pair of \( \{l_{in,k}, l_{out,k}\}_{k=1}^n \), the filter criterion for reducing noisy image is defined as follows:

\[
F = H_{size} \cap H_{area} \cap H_{bound}
\]

where:

\[
H_{size} = \begin{cases} 
1 & T_w \leq \text{wid}_k \leq T_i \\
0 & \text{otherwise}
\end{cases}
\]

\[
H_{area} = \begin{cases} 
1 & \text{area}_k \geq T_a \\
0 & \text{otherwise}
\end{cases}
\]

\[
H_{bound} = \begin{cases} 
1 & \text{bound}_k \geq T_b \\
0 & \text{otherwise}
\end{cases}
\]

with \( T_i, T_w, T_a \) and \( T_b \) being the threshold values; and the values of \( \text{len}_k, \text{wid}_k, \text{area}_k \) and \( \text{bound}_k \) being determined by (13), (14), (15) and (16), respectively.

\[
\text{len}_k = \max_{x,y \in L_{out,k}} \left\{ \max_{j \in [1,d]} |x_j - y_j| \right\}
\]

\[
\text{wid}_k = \min_{x,y \in L_{out,k}} \left\{ \max_{j \in [1,d]} |x_j - y_j| \right\}
\]

\[
\text{area}_k = \| \varphi_k(x) \|^2_{\varphi_k(x) < 0}
\]

\[
\text{bound}_k = \| L_{out,k} \|
\]

Our experiments show that the accuracy of the proposed method is significantly improved from the original fast level set method in these images.

3. Experimental results

Experimental results from the proposed method are given in this section. Firstly, we use different images to demonstrate our algorithm for segmentation of individual objects when they appear in the group. Fig.2 compares the accuracy of multiple objects segmentation between our proposed method and the fast level set method. Here the image contains 4 regions: 3 objects and 1 background. The averages of the intensities of these regions are \( c_{\varphi_2} = 110, c_{\varphi_3} = 156, c_{\varphi_3} = 0 \) and \( c_{\varphi_4} = 193. \) In the fast level set method, two mean intensity values are \( c_1 = 139 \) and
Three mean intensity values in the Generalized Fast Level Set Method are $c_{f_h} = 193$, $c_{f_l} = 110$ and $c_b = 156$. From Fig.2 (c), the region ($\phi_4$) of the foreground is incorrectly identified as background. The segmentation results in Fig.2 (c)-(d) show that our method is more effective than the fast level set method.

Next, we show the results of skull and head segmentation and compare computation time between three different level set methods: the classical level set method (LSM), which is described in (Osher and Sethian [1]), the fast level set method (F-LSM), which is described in (Shi Y. et al. [3]), and our proposed method (GF-LSM). Fig.3 and Fig.4 show skull and head CT image segmentation results by using the F-LSM in two cases: the outside initial value formulation of level set function with the inward speed function (F-LSM1) and the inside initial level set formulation with the outward speed function (F-LSM2). Fig.5 shows the segmentation results by using our algorithm. Note that we use the skull and head CT images with each image size of 512x512. Moreover, we initialize the curves in our algorithm with two pairs of level sets $\{ L_{in,1}, L_{out,1} \}$, $\{ L_{in,2}, L_{out,2} \}$ which satisfy: $L_{out,1} \subset L_{in,2}$, whereas the other two methods only use one pair of level sets $\{ L_{in,1}, L_{out,1} \}$ because they cannot be represented in (7).

From these figures we can see that the results of the proposed method are more effective and efficient than the fast level set method. The fast level set method cannot segment the outer boundary of skull images in two first cases in Fig.4. The computational time of the classical level set method, two cases of the fast level set method and our algorithm for the four segmentation experiments is listed in Table 1. For all experiments, we can see that our algorithm is approximately two orders faster than the fast level set method. Note that F-LSM2 cannot segment the outer boundary in the Skull01 and Skull02 images.

**TABLE 1: SEGMENTATION TIME**

<table>
<thead>
<tr>
<th>Image</th>
<th>Skull01</th>
<th>Skull02</th>
<th>Skull03</th>
<th>Head</th>
</tr>
</thead>
<tbody>
<tr>
<td>LSM</td>
<td>4.1407e+003</td>
<td>6.1597e+003</td>
<td>5.3502e+003</td>
<td>3.1628e+003</td>
</tr>
<tr>
<td>F-LSM1</td>
<td>71.8197</td>
<td>84.442</td>
<td>81.7069</td>
<td>35.4766</td>
</tr>
<tr>
<td>F-LSM2</td>
<td>Wrong result</td>
<td>Wrong result</td>
<td>124.9553</td>
<td>80.9545</td>
</tr>
<tr>
<td>GF-LSM2</td>
<td>39.4952</td>
<td>43.8154</td>
<td>47.5773</td>
<td>24.0372</td>
</tr>
</tbody>
</table>

Finally, we apply these methods to real medical images in order to further examine the algorithms’ effectiveness in noisy environments. Though the main challenge for medical image segmentation is the noisy source image and the low contrast between objects, our method renders satisfactory results as it can combine the prior geometry knowledge in the level set literature. Fig.6 illustrates the segmentation of the fast level set method and our algorithm with different prior knowledge of geometry.
which is useful in reducing the heavy image noises and also enables a significant decrease in computational time. Our future work will involve applying this algorithm to multiple overlapping objects segmentation problems and improving the algorithm based on the prior shape-driven knowledge.

References:


4. Conclusion

We have described the limitations of object segmentation based on the classical level set methods and the fast level set method. These problems naturally arise in many situations including the segmentation of CT skull and head images, hand bones and wrist segmentation, detection of multiple people in crowds, cell segmentation, and so on. We have developed a framework of level set based segmentation for solving these limitations. Using the new level set function, the proposed method now can be performed more easily and in a way that allows integrating an arbitrary number of level sets. Our experiments indicate that this approach can be used in segmenting individual objects or individual parts of an object in images,