# Nonlinear Backward Tracking Control of an Articulated Mobile Robot with Off-Axle Hitching

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*Abstract:* - This paper proposes a nonlinear path control for backward driving of a car-like mobile robot pushing trailer with off-axle hitching. The control law is constructed using high-gain design techniques and involves error coordinates expressed which are partially linked to the trailer and invariant properties with respect to the vehicle speed. The stability of the closed-loop system is analyzed using Lyapunov stability theory. Simulation results illustrate the effectiveness of the proposed controller.

*Key-Words:* - Articulated mobile robot, off-axle hitching, path following, nonlinear control, high-gain design

## 1 Introduction

In this paper, we consider the path following problem for backward driving of an articulated mobile robot with off-axle hitching (tractor-trailer with off-axle hitching). In recent years, significant advances have been made in designing feedback controllers for nonholonomic mobile robots. Several specific classes of nonholonomic control systems such as systems in chained form have been studied. Kinematic models of many mobile robots can be converted into chained form by applying the procedures proposed in [1] (in the case of wheeled mobile robots without/with a single trailer) or in [2] (in the case of mobile robots with n-trailers with on-axle hitching). In this way, the controller constructed via chained form is applicable for a large scale of mobile robots. Although the tractor-trailer system with off-axle hitching is differentially flat [3] and can be converted into chained form [4], in contrast to the tractor-trailer systems with on-axle hitching, the explicit derivation of this transformation is far from being obvious. The difficulty consists in finding functions which generate a chained set of coordinates for articulated vehicles with off-axle hitching. The existence of a feedback change of coordinates that transforms the kinematic model of a tractor-trailer with off-axle hitching into chained form was also shown in [5], however functions which generate a chained set of coordinates were not presented.

To overcome the difficulties associated with controlling articulated mobile robots of this kind, the idea of using alternate reference points for forward and backward driving located at the tractor and the trailer, respectively, in combination with feedback control based on a reduced-order model of the system, becomes attractive. Previous work on path following of tractor-trailer systems with off-axle hitching for forward driving can be found in [6, 7], where the controller was designed by using linear control techniques.

However, in contrast to the case of forward driving with reference point located at the tractor where, in general, the trailer needs not be stabilized, and we may concern only with the control of the tractor, when the tractor-trailer starts moving in the backward direction, the control strategy is not so obvious. Previous work on path following of tractor-trailer systems with off-axle hitching for backward driving can be found in [8, 9, 10, 11, 12], where the controller was designed by using nonlinear control techniques.

In this paper, we present a nonlinear path controller for backward driving of a car-like mobile robot pushing trailer with off-axle hitching. The controller is constructed using high-gain design techniques, and is based on a reduced-order model of the system. The control strategy consists in controlling the trailer using as control input the steering angular velocity of the front wheel of the car-like robot (the tractor). We prove asymptotic stability of the internal dynamics associated with the orientation of the tractor with respect to the trailer which has not been taken into account in the feedback control design.

The paper is organized as follows: In Section 2, the kinematic model of the articulated mobile robot is presented. In Section 3, we state the path following problem using error coordinates expressed in a moving reference frame partially linked to the trailer. In Section 4, the design of the proposed controller and stability analysis are given. Simulation results are presented in Section 5. Section 6 contains some conclusions.

## 2 Articulated Mobile Robot Kinematics

A plan view of the articulated mobile robot is shown in Fig. 1. The wheels of the robot are assumed to roll without lateral sliding. Point $P$ located at the center of the wheel axle of the trailer is used as reference point of the system. The coordinates of point $P$ with respect to an inertial frame $Fxy$, are denoted by $(x_P, y_P)$. The angles $\theta_1$ and $\theta_2$ are the orientation angles of the robot and the trailer, respectively, with respect to the frame $Fxy$. Angle $\alpha$ is the front-wheel steering angle of the mobile robot measured with respect to the robot body. Angle $\varphi$ is defined as the difference between the orientation angles $\theta_1$ and $\theta_2$, i.e. $\varphi = \theta_1 - \theta_2$. The lengths of the robot and the trailer are denoted by $l_1$ and $l_2$, respectively. The kingpin length is denoted by $h$.

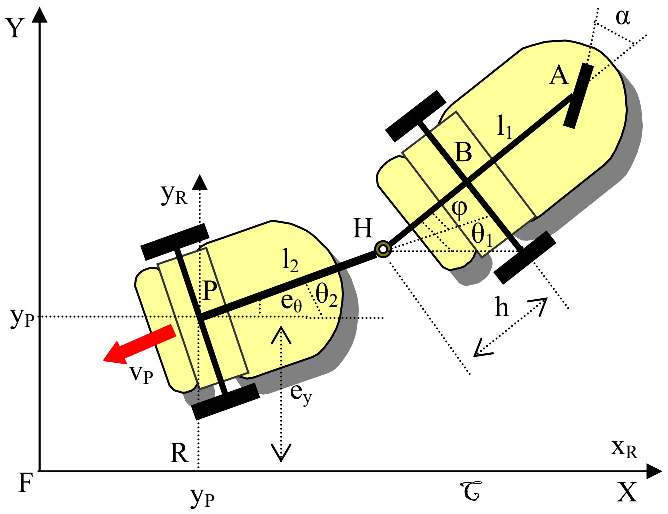


Fig. 1 A plan view of the articulated mobile robot

Using the coordinates of the reference point $P$, the configuration of the system is described by five generalized coordinates, $q = [x_P, y_P, \theta_2, \varphi, \alpha]^T$. Taking into account the nonslipping constraints, the tractor-trailer kinematics can be written in the following affine driftless control form [8]

$$
\begin{aligned}
\dot{x}_P &= v_P \cos\theta_2 \\
\dot{y}_P &= v_P \sin\theta_2 \\
\dot{\theta}_2 &= \frac{v_P \tan(\varphi+\beta)}{l_2} \\
\dot{\varphi} &= v_P\left(-\frac{\tan(\varphi+\beta)}{l_2} - (\cos\varphi + \sin\varphi.\tan(\varphi+\beta))\frac{\tan\beta}{h}\right) \\
\dot{\beta} &= \omega_\beta
\end{aligned}
\tag{1}
$$

where

$$
\begin{aligned}
\beta &:= -a\tan\left(\frac{h}{l_1}\tan\alpha\right), \qquad \left(-\frac{\pi}{2} < \beta < \frac{\pi}{2}\right) \\
\omega_\beta &:= -\frac{h\cos^2\beta}{l_1\cos^2\alpha}\omega_\alpha
\end{aligned}
\tag{2}
$$

In (1), $v_P$ is the velocity component of the reference point $P$ taken along the longitudinal axis of the trailer. $\omega_\alpha = \dot{\alpha}$ is the steering angular velocity of the front wheel of the mobile robot about a vertical axis and together with $v_B$ are the actual control inputs of the system, ($v_B = v_P\cos\beta/\cos(\varphi+\beta)$).

## 3 Problem Formulation

The path following geometry used in this paper is represented in Fig. 1. Consider an articulated mobile robot of the type "tractor-trailer" with off-axle hitching moving on a flat surface. We assume that the path $\mathcal{C}$ is a straight line which for simplicity coincides with the $Fx$ axis of the inertial frame $Fxy$. A reference coordinate frame $Rx_ry_r$ is defined such that the $Rx_r$ axis moves on the $Fx$ axis and is oriented in the direction of $Fx$, and the $Ry_r$ axis passes through the reference point $P$, as shown in Fig. 1. Let $[x_P, y_P, \theta_2]^T$ and $[x_r, y_r, \theta_r]^T$ be the real and reference posture coordinates of the trailer with respect to an inertial frame $Fxy$, where $y_R(t) \equiv 0$ and $\theta_r(t) \equiv 0$. The error posture $[e_x, e_y, e_\theta]^T$, i.e., the position and orientation of the trailer with respect to the moving reference frame $Rx_ry_r$ is given by [13]

$$
\begin{bmatrix} e_x \\ e_y \\ e_\theta \end{bmatrix} = \begin{bmatrix} \cos\theta_r & \sin\theta_r & 0 \\ -\sin\theta_r & \cos\theta_r & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_P - x_r \\ y_P - y_r \\ \theta_2 - \theta_r \end{bmatrix}. \tag{3}
$$

where $e_x(t) \equiv 0$ since the $Rx_R$ axis passes through the reference point $P$.

Differentiating (3) and using (1), after some work we obtain a kinematic model of the system for path following applications

$$
\begin{aligned}
\dot{e}_y &= v_P \sin\theta_2 \\
\dot{\theta}_2 &= \frac{v_P \tan(\varphi+\beta)}{l_2} \\
\dot{\varphi} &= v_P\left(-\frac{\tan(\varphi+\beta)}{l_2} - (\cos\varphi + \sin\varphi.\tan(\varphi+\beta))\frac{\tan\beta}{h}\right) \\
\dot{\beta} &= \omega_\beta
\end{aligned}
\tag{4}
$$

We assume that $v_P$ together with its derivative are bounded and also, that the following inequalities hold:

$0 < v_{Pmin} \leq |v_P(t)| \leq v_{Pmax}$. In this case, using the parameterization ($e_y$, $e_\theta$) and given a path $\mathcal{C}$, the path following problem consists of finding a feedback control law for the system (4) with control input $\omega_\beta$, (respectively $\omega_\alpha$, using the inverse transformation of (2)), such that the state vector $[e_y, e_\theta, \varphi, \beta]^T$ tends to $[0, 0, 0, 0]^T$, as $t \to \infty$.

## 4 Feedback Control Design

In this Section, we present a path following controller for backward driving, ($v_P < 0$), of a mobile robot pushing trailer (tractor-trailer system) with off-axle hitching given by equations (4). The control objective is to regulate the state vector $[e_y, e_\theta, \varphi, \beta]^T$ to zero.

We proceed with the following change of coordinates

$$z = e_y$$
$$\xi_1 = e_\theta \tag{5}$$
$$\xi_2 = \frac{\tan(\varphi+\beta)}{l_2}$$

and input

$$u = \frac{1}{l_2\cos^2(\varphi+\beta)}\left\{-\frac{\tan(\varphi+\beta)}{l_2} - \left[\cos\varphi + \right.\right. \tag{6}$$
$$\left.\left. + l_2\sin\varphi\frac{\tan(\varphi+\beta)}{l_2}\right]\frac{\tan\beta}{h} + \frac{\omega_\beta}{v_R}\right\}.$$

Furthermore, since $v_P(t)$ is assume to be strictly negative ($v_P(t) = -|v_P(t)| < 0$), to obtain a time-invariant system, the differentiation with respect to time is replaced by differentiation with respect to $s$ ($ds = v_P dt$), where $s$ is the real path length drown by the reference point $P$. In that way, we express the vehicle's equations of motion in terms of $s$ and denote the derivation with respect to $s$ by " ′ ". Using (6), the system (4) can be written in the form

$$\begin{aligned} z' &= -\sin\xi_1 \\ \xi_1' &= -\xi_2 \\ \xi_2' &= -u \\ \varphi' &= \xi_2(\cos\varphi + l_2\xi_2\sin\varphi)\frac{\tan[a\tan(l_2\xi_2)-\varphi]}{h} \end{aligned} \tag{7}$$

The high-gain control design technique is based on a reduced-order system composed of the first three equations of (7). For this end, we rewrite the first equation of (7) in the form

$$z' = -\frac{\sin\xi_1}{\xi_1}\xi_1. \tag{8}$$

For (8), the control $\gamma_0(z)$ given by

$$\gamma_0(z) = \xi_1 = z \tag{9}$$

achieves global asymptotic and local exponential stability of $z = 0$.

For the (z, $\xi_1$, $\xi_2$) subsystem of (7), we propose the following feedback control

$$u = K\left[a_1\xi_2 - Ka_0(\xi_1 - \gamma_0)\right]. \tag{11}$$

where $K$, $a_1$ and $a_2$ are positive constants. The control (11) achieves *semi-global* stabilization of (z, $\xi_1$, $\xi_2$) = (0, 0, 0). The semi-global stabilization [14] means that for any compact neighborhood $\Gamma$ of ($z$, $\xi_1$, $\xi_2$) = (0, 0, 0), there exists $K^*$ such that for all $K \geq K^*$, the region of attraction contains $\Gamma$.

Indeed, let introduce the following change of coordinates

$$\begin{aligned} z &= z \\ \eta_1 &= \xi_1 - \gamma_0(z). \\ \eta_2 &= \frac{\xi_2}{K} \end{aligned} \tag{11}$$

In these new coordinates, the closed-loop subsystem system of (7) is

$$\begin{aligned} z' &= -\frac{\sin(\eta_1+\gamma_0)}{\eta_1+\gamma_0}(\eta_1+\gamma_0) \\ \eta' &= KA\eta + d(-\gamma') \end{aligned} \tag{12}$$

where

$A = \begin{bmatrix} 0 & -1 \\ a_0 & -a_1 \end{bmatrix}$ is a Hurwitz matrix and $d = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

For the system (12) we consider the following Lyapunov function

$$V = \frac{1}{2}z^2 + \eta^T P\eta \tag{13}$$

where $P > 0$ is the solution of the Lyapunov equation

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$A^T P + PA = -I$.

Using (12), the derivative of $V$ becomes

$$\dot{V} = z\dot{z} + 2\eta^T P\dot{\eta} = -Rz^2 - K\|\eta\|^2 - z\eta_1 R + 2\eta^T PdR\eta_1 + 2\eta^T PdRz \le -Rz^2\left(1 - \frac{1}{4k_1} - \frac{2\sqrt{p_{11}^2 + p_{12}^2}}{4k_2}\right) - \|\eta\|^2\left[K - 2(1+k_2)\sqrt{p_{11}^2 + p_{12}^2} - k_1\right] \quad (14)$$

where

$$R = \frac{\sin(\eta_1 + z)}{\eta_1 + z} > 0$$

and $k_1$ and $k_2$ are positive constants obtained from the following inequality, which is used in (14): $xy \le kx^2 + \frac{y^2}{4k}$ for all $(x, y) \in R^2$ and any $k > 0$.

From (14), we obtain the following inequalities

$$1 > \frac{1}{4k_1} + \frac{2\sqrt{p_{11}^2 + p_{12}^2}}{4k_2} \quad (15)$$

$$K > 2(1+k_2)\sqrt{p_{11}^2 + p_{12}^2} - k_1 \quad (16)$$

and any choice for the constants $k_1$, $k_2$ and $K$ according to (15)-(16) renders $\dot{V}$ negative definite.

Since the dynamics of $\varphi$ has not been taken into account into the feedback control design, the next step in the stability analysis is to establish asymptotic convergence of $\varphi$ to zero. We analyze the zero dynamics of $\varphi$ assuming that $z(t) = \xi_1(t) = \xi_1(t) \equiv 0$ for all time. In this case, using that $\xi_2 = \frac{\tan(\varphi + \beta)}{l_2}$ from (5), for the last equation of (7) we have

$$\varphi' = -\frac{\sin\varphi}{h}. \quad (17)$$

Choosing a lower bounded function

$$W = 1 - \cos\varphi \quad (18)$$

its derivative along the solutions (17) is given by

$$W' = -\frac{\sin^2\varphi}{h} < 0 \quad (19)$$

since $\varphi(t) \to 0$ as $t \to \infty$.

The actual control for the front-wheel steering angular velocity $\omega_\alpha$ of the mobile robot is obtained by using the inverse transformations of (2) and (6).

## 5 Simulation Results

Simulation results were performed to illustrate the effectiveness of the proposed controller. The control law designed in Section 4 was implemented in MATLAB. A straight line reference path, which coincides with the *Fx* axis of an inertial frame, was chosen for the simulations. The velocity of the mobile robot (the tractor) was $v_B(t)$ = -1*m/s*. The velocity of the trailer is obtained according to the following relationship: $v_P = [\cos(\varphi + \beta)/\cos\beta]v_B$. The articulated vehicle parameters were chosen to be: the base lengths of the tractor and trailer $l_1 = l_2 = 1m$; the kingpin length $h = 0.2m$. The control gains were: $a_0 = 1$, $a_1 = 1.8$ and $K = 3$. Initial conditions were chosen to be: $e_y(0) = -0.5m$, $e_\theta(0) = 0rad$, $\varphi(0) = 0rad$ and $\alpha(0) = 0rad$.

Evolution of the state coordinates $[e_y, e_\theta, \varphi, \beta]^T$ with respect to the variable $s = \int_0^t |v_P|\, d\tau$ is depicted in Fig. 2.

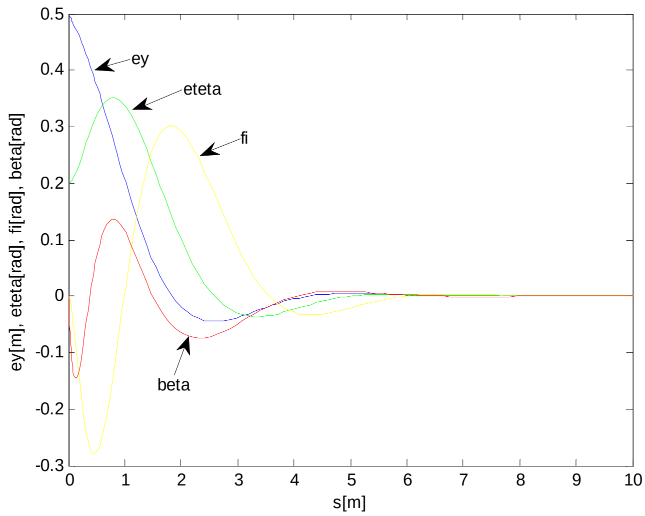


Fig. 2. Backward rectilinear path following: Evolution of the state coordinates of the closed-loop system $[e_y, e_\theta, \varphi, \beta]^T$

Evolution of the front-wheel steering angle of the robot is depicted in Fig. 3.

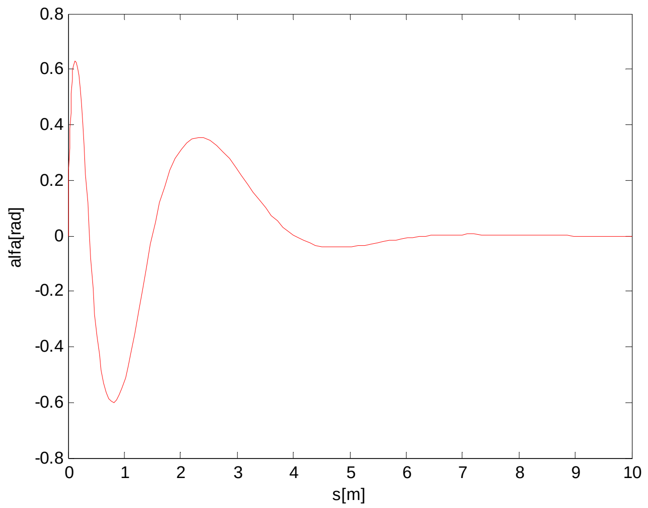


Fig. 3. Backward rectilinear path following: Evolution of the front-wheel steering angle $\alpha$ of the mobile robot

## 6 Conclusion

In this paper, we have presented a nonlinear feedback path controller for backward driving of an articulated mobile robot with off-axle hitching. The construction of the controller is based on high-gain design techniques and involves error coordinates expressed in a moving reference frame partially linked to the trailer. Asymptotic convergence to zero for the error coordinates has been established. The controller design has been based on a reduced model of the system. It has been proved that the zero dynamics associated with the orientation of the tractor with respect to the trailer which has not been taken into account in the feedback control design is asymptotically stable. The simulation results have shown the effectiveness of the proposed controller. Our future work will address the problems associated with the dynamical extension of the proposed controller.

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