LQR OPTIMIAL CONTROL FOR MICROMACHINED TUNNELING ACCELEROMETER

H.E.A. Ibrahim
Electrical and Computer Control Engineering Department AAST and Maritime Transport EGYPT

Abstract:- The micromachined tunneling accelerometer has wide applications in the automotive industry. The most important problems that face such accelerometers are range, bandwidth, sensitivity and accuracy. This paper presents new technique for improving system behavior, by using optimal LQR controller. Adequate desired performance is obtained by using the proposed technique.

Key-Words:- MEMS, LQR, Optimal Control, Accelerometer, Tunneling Mechanism.

1. Introduction

All accelerometers share a basic structure consisting of an inertial mass suspended from a spring [1-8]. However, they differ in sensing the relative position of the inertial mass as it displaces under the effect of an externally applied acceleration. A common sensing method is the measure of the variable capacitance between the two plates, where the mass forms one side of a two-plate capacitor and the other plate is fixed. This approach requires the use of special circuits to detect minute changes in capacitance and to translate them into an amplified output voltage. Another common method uses piezoresistors to sense the internal stress induced in the spring. The primary specifications of an accelerometer are range, often given in G the earth’s gravitational acceleration (1 G = 9.81 m/s^2), sensitivity (V/G), resolution, bandwidth (Hz), cross-axis sensitivity, and immunity to shock. The range and bandwidth required vary significantly, depending on the application. Accelerometer for airbag crash sensing are rated for a full range of ± 50 G and a bandwidth of about 1 kHz. In contrast, devices for measuring engine knock or vibration have a range of about 1 G, but most resolve small accelerations (< 100 μG) over a large bandwidth (> 10 kHz) [1]. A new category of micromachined sensors has emerged recently that, it utilizes a constant tunneling current between one tunneling tip (attached to a movable microstructure) and its counter electrode to sense deflection. Owing to the exponential relation of the tunneling current to the tunneling distance, these microsensors potentially have a small device area, high sensitivity, and wide bandwidth with very simple readout electronics [2,3]. A number of micromachined tunneling sensors have been developed for the high-resolution and wide-bandwidth applications. However, high-voltage operation (a few 10 V or even higher) is required for these devices, thus limiting their applications.

The operating principle of the micromachined tunneling accelerometer is simple. As the tip is brought sufficiently close to its counter electrode (within a few angstroms) using electrostatic force generated by the bottom deflection electrode, tunneling current (I_{tun}) is established and remains constant if the tunneling voltage (V_{tun}) and distance between the tip and counter electrode are unchanged. Once the proof mass is displaced due to acceleration, the readout circuit responds to the change of the tunneling current and adjusts the bottom deflection voltage (V_o) to move the proof mass back to its original position, thus maintaining a constant tunneling current.
Acceleration can be measured by reading out the bottom deflection voltage in this closed-loop system. The top deflection electrode (Vi applied) is used to simulate acceleration electrostatically for self-test and protect the accelerometer against acceleration over range [4, 5].

For best performance, tunneling-based devices need to be operated in a closed loop mode [4,5], the reasons are as follows: 1) the ratio of the change in the tunneling current to change in the tunneling distance is extremely large, thus limiting their measurement range if they are operated in an open loop mode and 2) the tunneling barrier height for two tunneling electrodes may vary by one order of magnitude in air [1,5], thus affecting open loop device sensitivity directly, which is undesirable. In closed loop operation, the movable structures are virtually stationary so that a large distance to current gain does not limit the measurement range of tunneling based devices. Furthermore, closed loop device sensitivity is determined only by the electrostatic feedback force rather than the tunneling barrier height [4, 5].

2 Accelerometer Analysis and Mathematical Model

A micromachined tunneling accelerometer system presented in [6,7] consists of four subsystems, suspended proof mass; tunneling based mechanism, current to voltage amplifier, and electrostatic feedback, the overall mathematical model can be developed as follows.

The above subsystems can be summarized as in Fig.1 the transfer function of the closed loop sensor system can be expressed as in equation (1).

\[
G(s) = \frac{X(s)}{a(s)} = \frac{s^2 + 1}{m_p s^4 + (m_p + \frac{h}{a_b}) s^3 + (b + k) s + k + K_f A_b A_o} \tag{1}
\]

Where, 
\[
K_f = \alpha \rho \varepsilon_o \sqrt{\Phi} \quad K_f = \frac{e_o A_p V_o}{A_o} \frac{1}{h_b}
\]

\[
\omega_o = 2 \pi f_o = \frac{1}{m_p s^2 + b s + k}
\]

Fig.1 Closed loop diagram for the overall system

Where \(a\) is the input acceleration, \(X\) is the proof mass displacement, \(m_p\) is the proof mass, \(k\) is the spring constant, \(b\) is the damping coefficient, \(\omega_o\) is the cut off frequency for the current to voltage amplifier in rad/sec, \(A_o\) is the current to voltage amplifier dc gain, \(\alpha\) is a tunneling mechanism constant 1.025 Å⁻¹ eV⁻¹/₂, and \(\Phi\) is the tunneling barrier height for two tunneling electrodes, \(h_b\) is the distance between the proof mass and bottom deflection electrode, \(A_b\) the electrode area, \(\varepsilon_o\) is the permittivity of the free space, where the media between the two electrodes is the air and \(V_o\) is the applied voltage on the plates to generate and control the electrostatic force to bring the proof mass back to its original position. \(\rho\) and \(t_p\) are the material density and thickness of the proof mass, respectively, and \(X_o\) is the initial distance between the two tunneling electrodes.

Table 1: Design Parameters for the Tunneling Accelerometer

<table>
<thead>
<tr>
<th>Nominal Design Parameters</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>(m_p)</td>
<td>5.576 µg</td>
</tr>
<tr>
<td>(k)</td>
<td>4.42 N/m</td>
</tr>
<tr>
<td>(b)</td>
<td>1.097 * 10⁻³ N.S/m (air)</td>
</tr>
<tr>
<td>(f_o)</td>
<td>5 kHz</td>
</tr>
<tr>
<td>(K_f A_b A_o ) / (k)</td>
<td>Loop gain</td>
</tr>
<tr>
<td>(K_f A_b A_o ) / (k)</td>
<td>22.1 N/m</td>
</tr>
</tbody>
</table>

In addition, it is assumed that the proof mass and bottom deflection electrode have the same area \(A_b\). Table (1)
presents the nominal design parameters for the accelerometer taken from [6].

2.1 System Results and Simulation

Intensive simulation has been done, the results obtained from the simulation are declared as follows. The system has three poles at \((-31542, -35 \pm 2176i)\), and a zero at \(-31542\), which assures the system stability.

Fig. 2 shows the step response for the system, it reaches 4% from the reference value at steady state, which means 96% error. The step input here represents the system acceleration input (to be sensed) and the system output is the mass displacement, which will be measured, by means of the variations in the tunneling current, to give the accelerometer sensor reading. Based on investigating system response in time and frequency domains and the above equations, we can find that.

1. Accelerometer has sensitivity proportionally increases with the proof mass thickness as well as the square root of the proof mass area. But from the simulation it concludes that, the closed loop system bandwidth decreases with large increase in the proof mass. Fig.3 shows the bandwidth versus different mass dimensions.

2. Accelerometer sensitivity is inversely proportional to the square root of the spring constant \(k\), while the bandwidth of the closed loop system increases with the spring constant. When the spring constant of the device is increased by a large amount, simulation results show that, the bandwidth of the overall system goes down until it becomes equal to the bandwidth of the current-to-voltage amplifier.

3. An increase in the bandwidth of the current-to-voltage amplifier does not affect the accelerometer sensitivity, where it increases the bandwidth of the closed loop system significantly (it brings the system from an overdamped to underdamped state), one can find that. The closed loop frequency response, for 50 kHz bandwidth current to voltage amplifier, the closed loop system bandwidth is about 28 kHz. And when the current to voltage amplifier bandwidth has been reduced to 1 Hz, the closed loop system bandwidth became 100 kHz.

4. By increasing the loop gain \(\frac{K_T A_o K_{FB}}{k}\), the system goes to unstable state, which causes large increase in the closed loop system bandwidth. However, the large value in the loop gain will make the system unstable. So that, it needs to be controlled carefully. If the closed loop gain increased 100 times that would increase the closed loop system bandwidth almost 4 times. The loop gain is linearly proportional to the dc amplifier gain \(A_o\). One can
conclude that, accelerometer bandwidth has nonlinear relationship with the closed loop gain and accelerometer mass.

From the above analysis and discussion we can conclude that the system needs controller to assure the optimal performance and preventing the system to be unstable. So, the next section will present a new approach for using linear quadratic regulator (LQR) optimal controller to assure tracking the input reference and optimum performance (accurate sensing i.e. mass moves to the predetermined position according to the input, making sure the mass gets back to the original position after removing the input signal, faster behavior, stable system, oscillations reduction for any mass displacement, desired bandwidth and sensitivity).

3. LQR Optimal Control

The micromachined tunneling accelerometer model is a linear time invariant system and can be put as in equation (3).

\[
\dot{x}(t) = Ax(t) + Bu(t), \quad y(t) = Cx(t), \quad z(t) = Dx(t)
\]

(3)

Where \( x(t) \) is the state vector, \( u(t) \) is the control input, \( y(t) \) is the observed or measured vector, \( z(t) \) is the tracking vector that is required or follow a reference vector \( r(t) \), and \( A, B, C, D \) are constant matrices of compatible dimensions. Let \( r(t) \) be a reference input that \( z(t) \) must track or follow, such that, the tracking error \( e(t) \) is represented as in equation (4) and the integral error \( \alpha(t) \) in equation (5).

\[
e(t) = r(t) - z(t) \rightarrow 0 \text{ as } t \rightarrow \infty
\]

(4)

\[
\alpha(t) \equiv \int_0^t e(t), dt + \alpha(0)
\]

(5)

From equation (3 and 4) let,

\[
\begin{bmatrix}
\dot{x}(t) \\
\alpha(t)
\end{bmatrix} =
\begin{bmatrix}
A & 0 \\
-D & 0
\end{bmatrix}
\begin{bmatrix}
x(t) \\
\alpha(t)
\end{bmatrix} +
\begin{bmatrix}
B \\
0
\end{bmatrix} u(t) +
\begin{bmatrix}
0 \\
I
\end{bmatrix} r(t)
\]

(6)

Or in another form as in equation (7)

\[
\dot{w}(t) = \bar{A}w(t) + \bar{B}u(t) + \bar{E}r(t)
\]

(7)

Where \( w(t) = \begin{bmatrix} x(t) \\ e(t) \end{bmatrix} \), \( \bar{A} = \begin{bmatrix} A \\ -D \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \), \( \bar{B} = \begin{bmatrix} B \\ 0 \end{bmatrix} \), \( \bar{E} = \begin{bmatrix} 0 \\ I \end{bmatrix} \)

And \( I \) is 1*1 unit matrix

The control objective is to have \( z(t) \) to track \( r(t) \) and in the same time minimize a LQ performance measure. Using the performance measure \( J \) in equation (8) can readily accommodate the control objectives, Fig.4 shows the block diagram for the control system.

\[
J = \left[ w(t_f) - r(t_f) \right]^T P_f \left[ w(t_f) - r(t_f) \right] + \int_{t_i}^{t_f} \left[ [z(t) - r(t)]^T Q [z(t) - r(t)] \right] u^T(t) R u(t) dt
\]

(8)

Where \( J \) is the optimal control performance measure, \( K_1 \) is the integral gain, \( Q \) is n*n symmetric, positive semidefinit matrix, weighting matrix \( Q \geq 0 \), \( R \) is l*l symmetric, positive definite weighting matrix \( R > 0 \), \( P \) is n*n symmetric positive semidefinit constant matrix can be found from riccati equation, \( ()^T \) means the transpose of the matrix, \( (t_i , t_f) \) is the integration period and \( n \) is the number of the state variables, \( l \) is the number of inputs.

![Fig.4 Optimal feed back control system](image)

So, in this case \( Q \) and \( R \) matrices could be represented as in equation (9)

\[
Q = \begin{bmatrix}
q_{11} & 0 & 0 \\
0 & q_{22} & 0 \\
0 & 0 & q_{33}
\end{bmatrix}, \quad R = [1]
\]

(9)

To start the solution we have to assume \( R = 1 \), and assuming any magnitudes for \( q_{11} \), \( q_{22} \) and \( q_{33} \) then doing the simulation and checking the state variables performance to get the desired state variables performance.

The optimal control, which minimizes \( J \), is given by equation (10).
\[ u_{opt}(t) = -R^{-1}\bar{B}^TP(t, t_f)w(t) \] (10)

Where \( P(t, t_f) = P_f \) is the \( n \times n \) matrix obtained from the Riccati equation (11).

\[-\dot{P}(t, t_f) = \bar{A}^TP(t, t_f) + P(t, t_f)\bar{A} + \bar{Q} - \bar{P}(t, t_f)\bar{B}\bar{R}^{-1}\bar{B}^TP(t, t_f) \] (11)

from equation (10) and (4)

\[ \dot{w}(t) = \bar{A}w(t) + \bar{B}u_{opt}(t) + Er(t) \\
= (\bar{A} + \bar{B}K(t))w(t) + Er(t) \] (12)

Where \( K(t) \) is the optimal feedback controller gain \( K(t) = -R^{-1}\bar{B}^TP(t, t_f) \).

4. System Stability

Using Lyapunov indirect method for linear systems, the system stability can be checked, we are going to concentrate our attention on the eigenvalues;

\[ |\lambda I - \bar{A}| = 0 \] (13)

Where \( \lambda \) is the eigenvalues. It is known, for stable system, the real parts of the eigenvalues have to be in the right side in the S-plan. After using the optimal control to check the stability, we replace \( \bar{A} \) by \( \bar{A} = (\bar{A} - \bar{B}R^{-1}\bar{B}^TP(t, t_f)) \) and use the same equation (13).

5. Results and Simulation after Using LQR

Simulation using the optimal LQR has been implemented. Fig. (5) shows the unit step system response. As shown from the figure the displacement reaches its final value in 3*10^{-6} sec., gives zero tracking error and the system has better stability. Comparing that with the results have been obtained previously [5,6,7]. It is quit clear using LQR has much better response.

Fig. (6, 7) show unit ramp and unit acceleration system response. As shown from the figure the displacement reaches its final value in few microseconds, and gives zero tracking error, which means much better sensitivity and accuracy for the micro accelerometer compared to previous approaches [5,6,7].

The speed of reaching the final values depends on choosing the values of matrix \( Q \), as choosing high values of \( Q \) as having faster response for any input signal and having better stability.

Finally, for all inputs, the mass speed when using the LQR is faster and almost has no oscillations, for example, for unit step input the average speed is 1.824*10^{-3} m/sec. Compared to 1.7111*10^{-4} m/sec in [7]. That makes, the mass reaches its final position faster with no oscillations. Using LQR has increased and improved the accelerometer bandwidth, stability, accuracy and response time.
6. Conclusion

Micro accelerometer analysis has been presented with optimization objective in mind. Micro accelerometer model is presented and simulated using MATLAB. Device Sensitivity, Bandwidth and stability for the micro accelerometer closed loop system are investigated. LQR algorithm has been presented and designed. The system has been simulated using the LQR system. Comparing results (system bandwidth, stability, mass speed and system accuracy) among systems with LQR, without LQR and the previous work has been presented. The results using the LQR were much better than previous work. LQR guarantees wide bandwidth, good accuracy, faster response and stable system.

Reference:


