Time-Frequency Localisation Analysis for Extracting Fatigue Damaging Events

S. ABDULLAH¹, T. E. PUTRA, M. Z. NUAWI, Z. M. NOPIAH, A. ARIFIN AND L. ABDULLAH
Department of Mechanical and Materials Engineering
Universiti Kebangsaan Malaysia
43600 UKM Bangi Selangor
MALAYSIA
¹ shahrum1@gmail.com

Abstract: The fatigue feature extraction using the Short-Time Fourier Transform (STFT) and wavelet transform approaches is presented in this paper. The transformation of the time domain signal into time-frequency domain computationally implemented using the STFT and Morlet wavelet methods provided the signal energy distribution display with respect to the particular time and frequency information. In this study, cycles with lower energy content were eliminated, and this selection was based on the signal energy distribution in the time representation. The simulation results showed that the Morlet wavelet was found to be a better approach for fatigue feature extraction. The wavelet-based analysis obtained a 59 second edited signal with the retention of at least 94% of the original fatigue damage. The signal was 65 seconds (52%) shorter than length of the edited signal that was found using the STFT approach. Hence, this fatigue data summarizing computational algorithm can be used for accelerating the simulation works related to fatigue durability testing.

Key-Words: Fatigue strain signal, fatigue damage, STFT, Morlet wavelet, edited signal.

1 Introduction

In a fatigue life assessment, fatigue signal extraction is described as a method to summarize a fatigue signal. The method is performed by segment identification and extraction that contributes to the more fatigue damaging events to a metallic material. On the other hand, segments containing lower amplitude cycles are omitted, since these data type theoretically gave minimal or no fatigue damage. This process generates a new shortened signal, for which this signal type can be used to reduce the testing time and costs for fatigue testing [1].

One of the new approaches that were developed for the fatigue signal extraction is the one in time-frequency domain. Previously, the time-frequency approach had been applied to the problem of fatigue signal extraction, but only for the purpose of spike removal and de-noising [2]. The STFT or windowed Fourier transform is one of the methods for transforming the time domain signal into the time-frequency domain. In addition, the STFT adopted the Fourier transform to analyze only a small section of the signal at one specific time. It provides information on when and at what frequency a signal occurs. However, this information is only obtained with limited precision determined by the size of the window. Many signals require a more flexible approach, in order to determine more accurately either time or frequency [3].

During the last decade, an improved signal processing technique, called the wavelet transform (WT), has been frequently used in the field of vibration diagnostic and fault detection. This approach is probably the most recent solution to overcome the nonstationary signals. The WT is applied by cutting time domain signal into various frequency components through the compromise between time and frequency-based views of the signal. It presents information in both time and frequency domain in a more useful form [4-6].

The WT analysis is started with a basic function (called the mother wavelet) scaled and translated to represent the signal being analyzed [7]. The transform shifts a window along the signal and calculates the spectrum for every position. The process is repeated many times with a slightly shorter (or longer) window for every new cycle. In the end, the result will be a collection of time-frequency representations of the signal with different resolutions. The WT provides information on when and at what frequency the change in signal behavior occurs [4].

In order to obtain the appropriate technique for the fatigue strain signal extraction, the STFT and WT approaches were utilized to transform the time domain signal into time-frequency domain and trace the lower energy cycles contained in the original signal. Those segments were then removed from the original signal in order to gain a new edited signal having the higher energy cycles. Segments which have been removed have minimal or no fatigue damaging potential. Therefore, the original fatigue damage can be retained in the edited
signal produced at the end of the process. The effectiveness of these techniques was validated based on the fatigue damaging retention in the shortened signals.

This STFT-based fatigue feature extraction algorithm was previously developed by Abdullah et al [8]. Since the WT has been found to be theoretically better than the STFT in the time-frequency localization, it gave a motivation to the authors for developing a similar data extraction approach in the WT. Therefore, a new algorithm for fatigue feature extraction using the Morlet wavelet was developed. The WT results were compared to the findings using the STFT extraction approach in order to see the suitability approach in fatigue history editing.

2 Literature Background

2.1 The STFT

The STFT is composed by the local spectra of segments of the primary function, as viewed through a translating window of fixed shape. The local spectra at all points on the primary time axis constitute the STFT. Generally, the STFT is expressed as [9]:

\[
STFT(t, f) = \frac{1}{\pi} \int_{-\infty}^{\infty} h(t)w(t-\tau)\exp(-2\pi if\tau)d\tau
\]

(1)

where \(h\) is the primary function, \(\tau\) is the time, and \(f\) is the frequency. The position of the translating window \(w\) is determined by \(t\), which has the same units as \(\tau\). If \(w\) is replaced with the value of 1, the STFT reduces to \(H\), i.e. the Fourier transform of \(h\). The modulus of the STFT is also known as the spectrogram.

2.2 The Morlet wavelet

The Morlet wavelet is one of functions that are generally used in the Continuous Wavelet Transform (CWT) analyzes [10]. Basically, the name of the wavelet family is written “morl”. The wavelet decomposition calculates a resemblance index between signal being analyzed and the wavelet, called coefficient. It is a result of a regression of an original signal produced at different scales and different sections on the wavelet. It represents correlation between the wavelet and a section of the signal. If the index is large, the resemblance is strong, otherwise it is slight [3].

The WT of any time-varying signal \(f(t)\) is defined as the sum of all of the signal time multiplied by a scaled and shifted version of the wavelet function \(\psi(t)\) [11]. The CWT is expressed by the following integral:

\[
CWT_{(a,b)} = \int_{-\infty}^{\infty} f(t)\psi_{a,b}(t)dt
\]

(2)

The parameter \(a\) represents the scale factor which is a reciprocal of frequency, the parameter \(b\) indicates the time shifting or translation factor, and \(t\) is time. \(\psi_{a,b}(t)\) denotes the mother wavelet, i.e [12]:

\[
\psi_{a,b}(t) = \frac{1}{\sqrt{|a|}}\psi\left(\frac{t-b}{a}\right)\quad a, b \in R; a \neq 0
\]

(3)

\[
CWT_{(a,b)} = \int_{-\infty}^{\infty} f(t)\frac{1}{\sqrt{|a|}}\psi\left(\frac{t-b}{a}\right)dt
\]

(4)

In addition, the wavelet coefficient indicates how energy in the signal is distributed in the time-frequency plane [13]. The energy spectrum (the energy density over frequency) is plotted in order to observe the signal behavior and its content gives significant information about the random signal pattern.

2.3 Fatigue Life Assessment

Three major approaches to predicting fatigue life namely stress-life, strain-life, and fracture mechanics. At below the transition point (approximately 1000 cycles), the \(\varepsilon-N\) - based approach is appropriate method and is commonly used to predict fatigue life for ductile materials at relatively short fatigue life. The crack initiation method relates the plastic deformation that occurs at a localized region where fatigue cracks begin to the durability of the structure under influence of mean stress [14].

For strain - based fatigue life prediction, current industrial practice uses the Palmgren-Miner linear cumulative damaging rule normally associated with the established strain-life fatigue damaging models. The common model is the Coffin-Manson relationship, defined as [15]:

\[
\varepsilon_a = \frac{\sigma' f}{E}\left(\frac{2N_f}{\varepsilon'}\right)^c + \varepsilon' f\left(\frac{2N_f}{\varepsilon'}\right)^b
\]

(5)

where \(\varepsilon_a\) is the true strain amplitude, \(\sigma' f\) is the fatigue strength coefficient, \(E\) is the material modulus of elasticity, \(N_f\) is the numbers of cycle to failure for a particular stress range and mean, \(b\) is the fatigue strength exponent, \(\varepsilon' f\) is the fatigue ductility coefficient, and \(c\) is the fatigue ductility exponent.

The fatigue damage caused by each cycle of repeated loading is calculated by reference to material life curves, such as S-N or \(\varepsilon-N\) curves. The fatigue damage \(D\) for one cycle and the total fatigue damage \(\Sigma D\) caused by cycles are expressed respectively as [1]:
\begin{align}
D &= l/N_f \\
\Sigma D &= \Sigma(N_i/N_f)
\end{align}

where \( N_i \) is the numbers of cycle within a particular stress range and mean.

3 Methodology

In this study, the input signal was measured at a front lower suspension arm of a passenger car driven over a public road surface. The signal was a variable amplitude loading sampled at 200 Hz for 32,000 data points. It gave the total signal record length of 160 seconds. The collected signal recorded using a fatigue data acquisition system contained many small amplitude and high frequency in the signal background, as illustrated in Fig. 1 for the time series and Fig. 2 for its Power Spectral Density (PSD). For the fatigue damaging calculation, the selected material for the simulation purpose was the SAE1045 carbon steel shaft. This material was chosen because it was commonly used in automotive industries for fabricating a vehicle lower suspension arm structure [16]. The material properties and their definitions are given in Table 1 [17].

4 Results and Discussions

In this analysis, the signal in time domain was converted into time-frequency domain using the STFT method. The time history signal was separated into a number of windows using the Gaussian window with 128 of window size. The 120 numbers of overlap were used in order to provide the high resolution in the time representation. For each window, the Fourier transform was applied for the calculation of the signal energy contained in each window. The energy calculation was gained from the PSD that produced the spectrogram of the STFT.

Using a specific commercial software package, the STFT plot of the original fatigue signal showed a two dimensional view of the energy distribution, as observed in the time-frequency plane. This result is plotted in Fig. 3a. The level was presented by a colour contour, where the red colour showed the highest energy content and followed by yellow, green and blue. Based on the energy parameter, the spectrogram value was decomposed into a time domain display in order to represent the energy distribution in time history. The energy display provided the time location containing the lower energy cycle, as illustrated in Fig. 3b.

<table>
<thead>
<tr>
<th>Properties</th>
<th>Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Ultimate tensile strength, ( S_u ) (MPa)</td>
<td>621</td>
</tr>
<tr>
<td>Modulus of elasticity, ( E ) (GPa)</td>
<td>204</td>
</tr>
<tr>
<td>Fatigue strength coefficient, ( \sigma' ) (MPa)</td>
<td>948</td>
</tr>
<tr>
<td>Fatigue strength exponent, ( b )</td>
<td>-0.092</td>
</tr>
<tr>
<td>Fatigue ductility exponent, ( c )</td>
<td>-0.445</td>
</tr>
<tr>
<td>Fatigue ductility coefficient, ( \epsilon' )</td>
<td>0.26</td>
</tr>
</tbody>
</table>
In this STFT study, nine signals (the original and 8 edited signals) were simulated for the purpose of the verification the efficiency of fatigue data editing using STFT method. The edited signals were produced at eight gate values, i.e. 5 µε, 10 µε, 15 µε, 20 µε, 25 µε, 30 µε, 35 µε, and 40 µε. From the fatigue damaging calculation results, as shown in Fig. 4, it was found that 5 µε to be an optimum gate value since the total fatigue damaging value produced from this edited signal had only 6 % deviation when compared to the original signal. The new edited signal was produced at the 124 seconds, which was 36 seconds shorter than the original signal length, as shown in Fig. 5. This value gave 22 % of the original time length reduction.

For the Morlet wavelet - based edited signal, it was started by analyzing the wavelet coefficients, as shown in Fig. 6a using Equation (4). With the newly Morlet wavelet - based developed algorithm, the wavelet coefficients were transposed into time domain signal, as shown in Fig. 6b.

This extraction process only involved 150 µε and 200 µε. From the total fatigue damaging calculation results, it was found that 200 µε was selected to be the optimum gate value giving lower than 10 % difference of the fatigue damage, as can observed in Fig. 7. The total fatigue damaging produced from this edited signal had only 6 % deviation compared to the original signal. At this fatigue damaging ratio, the new edited signal was produced of 59 seconds, which was 101 seconds shorter than the original signal length, as shown in Fig. 8. This value gave 63 % of the original time length reduction.

Based on these two approaches, finally, the applicability of fatigue data editing with the adaptation of the Morlet wavelet method was proven for the situation to shorten the signal length with the retention of the majority of the original fatigue damage. The energy spectrum showed relatively adequate with
damaging event in the fatigue signal and was a very useful tool for damaging detection in the fatigue signal. The extraction of fatigue damaging events successfully removed the lower energy cycles in the time history and created a new edited signal which retains higher fatigue damaging segments containing the majority of the fatigue damage. In overall, the analysis findings of this paper suggested that the Morlet wavelet was the best approach for the fatigue data editing.

Fig. 8. The 59 seconds of the Morlet - based edited signal

5 Conclusion
This paper discussed on the study of a fatigue data editing technique in time-frequency domain by using the STFT and Morlet wavelet methods. Based on the simulation analysis, it was found that the Morlet wavelet was more suitable for the fatigue data editing. The Morlet wavelet - based edited signal contained at least 94 % of the original fatigue damage in the 59 second edited signal, i.e. only 37 % of the original signal time length. Whereas the STFT-based edited signal contained 94 % of the original fatigue damage in the 124 second edited signal, i.e. 78 % of the original signal time length. In terms of the applicability of the shortened signal, this kind of signal was normally used for the purpose of accelerated laboratory fatigue testing.

Acknowledgment:
The authors would like to express their gratitude to Universiti Kebangsaan Malaysia through the fund of UKM-GUP-BTT-07-25-158, for supporting the research.

References: